CROSS-FIELD PLASMA TRANSPORT AND POTENTIAL FORMATION IN AN ELECTRON BEAM-PLASMA SYSTEM

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Abstract. It has been proved theoretically that anomalous electron transport along and across a magnetic field and an electric field across a magnetic field can be induced by localized electrostatic waves.

I. INTRODUCTION

Cross-field anomalous electron transport induced by the localized electrostatic waves excited linearly or nonlinearly in an electron beam-plasma system has been investigated theoretically based on the quasilinear transport equations derived from Vlasov-Maxwell equations [1-3]. They show that particle transport along and across a magnetic field can be induced by almost perpendicularly propagating electrostatic waves in a magnetized plasma. In order to analyze this anomalous transport numerically, the velocity distribution function of plasma electrons is assumed to be the two-dimensional drifted Maxwellian distribution. The cross-field electron drift is given by $E_0 = v_{e\perp} B_0 / c$ **. It was verified that the parallel and perpendicular electron transport creates a strong cross-field electric field and a large inhomogeneity of the electron density and temperature along and across the magnetic field [4, 5].**

II. TRANSPORT EQUATIONS

We consider the electron transport which arises from quasilinear velocity-space diffusion due to the almost perpendicularly propagating electrostatic waves. The transport equations for plasma electrons are given by

$$\frac{\partial U_e}{\partial t} = -2\gamma_{\mathbf{k}}^{(e)} U_{\mathbf{k}} \tag{1}$$

$$\frac{\partial \mathbf{P}_e}{\partial t} = -\frac{2\gamma_{\mathbf{k}}^{(e)} \mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} \quad , \tag{2}$$

where $U_{\mathbf{k}} = (1/8\pi) \quad \varepsilon'_{\mathbf{k}}\omega_{\mathbf{k}})/\partial\omega_{\mathbf{k}})|E_{\mathbf{k}}|^2$ **is the wave energy density,** $\mathbf{k} U_{\mathbf{k}}/\omega_{\mathbf{k}}$ **the wave momentum density,** $U_e = \int d\mathbf{v}\, \tfrac{1}{2} n_e m_e v^2 g_e$ **and** $\mathbf{P}_e = \int d\mathbf{v}\, n_e m_e \mathbf{v} g_e$ **are**

energy and momentum densities of plasma electrons, $\gamma_{\mathbf{k}}^{(e)}=-\varepsilon_{\mathbf{k}}^{\prime\prime(e)}/(\partial\varepsilon_{\mathbf{k}}^{\prime}/\partial\omega_{\mathbf{k}})$ ($\gamma_{\mathbf{k}}=\Sigma_s\gamma_{\mathbf{k}}^{(s)}$) is the linear damping rate ascribed to plasma electrons, and $\mathbf{P}_e=(P_{e\perp},\ 0,P_{e\parallel})$, $\mathbf{k}=(k_\perp,\ 0,k_\parallel)$, $P_{e\perp}=m_en_ev_{e\perp}$, $P_{e\parallel}=m_en_ev_{e\parallel}$. The dielectric constant $\varepsilon_{\mathbf{k}}=1+\Sigma_s\varepsilon_{\mathbf{k}}^{(s)}=\varepsilon_{\mathbf{k}}^{\prime}+i\varepsilon_{\mathbf{k}}^{\prime\prime}$ ($\varepsilon_{\mathbf{k}}^{(s)}=\varepsilon_{\mathbf{k}}^{\prime(s)}+i\varepsilon_{\mathbf{k}}^{\prime\prime(s)}$, s denotes the plasma species) is expressed by

$$\varepsilon_{\mathbf{k}}^{(s)}=-\frac{\omega_{ps}^2}{k^2}\sum_{r=-\infty}^{\infty}\int d\mathbf{v}\frac{J_r^2(\mu_{\mathbf{k}})U_r(\mathbf{k})g_{s0}}{k_\parallel v_\parallel+k_\perp v_d-\omega_{\mathbf{k}}+r\omega_{cs}}\quad, \tag{3}$$

$$\begin{aligned}g_s&=a_s\sum_{m=0}^{\infty}\frac{1}{m!}v_x^m\left(-\frac{v_d}{v_\perp}\frac{\partial}{\partial v_\perp}\right)^m g_{s0}(v_\perp,v_\parallel,t)\\&=a_sg_{s0}((v_\perp^2-2v_xv_d)^{1/2},\ v_\parallel,t)\quad,\end{aligned} \tag{4}$$

where

$$U_r(\mathbf{k})=k_\parallel\frac{\partial}{\partial v_\parallel}+\frac{r\omega_{cs}}{v_\perp}\frac{\partial}{\partial v_\perp}\quad, \tag{5}$$

J_r is the Bessel function of rth order, $\mu_{\mathbf{k}}=k_\perp v_\perp/\omega_{cs}$, $\omega_{ps}^2=4\pi n_se_s^2/m_s$, $\omega_{cs}=|e_s|B_0/m_sc$, $\mathbf{v}$ is given by the cylindrical coordinate in velocity-space, $\mathbf{v}_d=(v_d,\ 0,\ 0)=c\mathbf{E}_0\times\mathbf{B}_0/B_0^2=\mathbf{v}_{e\perp}$ equals $\mathbf{E}\times\mathbf{B}$ drift velocity, g_s is the background velocity distribution function containing the fluctuation-induced cross-field drift $v_d=\int d\mathbf{v}v_xg_s$, and a_s is the normalization constant. Equation (4) satisfies the unperturbed Vlasov equation $(\mathbf{E}_0+(\mathbf{v}/c)\times\mathbf{B}_0)\cdot(\partial g_s/\partial\mathbf{v})=0$, which leads to the generalized Ohm's law for a collisionless plasma $\mathbf{E}_0+\mathbf{v}_d\times\mathbf{B}_0/c=0$. This means that $\mathbf{E}_0=(0,E_0,\ 0)$ is produced by the dynamo effect of the cross-field particle drift arising from the acceleration due to electrostatic waves. Transport equations (1) and (2) predict clearly that the electrostatic waves generate anomalous transport or strong particle acceleration along and across the magnetic field ($P_{e\parallel}/P_{e\perp}=k_\parallel/k_\perp$ ($\mathbf{P}_s/\!/\mathbf{k}$) with $\mathbf{P}_e(0)=0$).

III. NUMERICAL ANALYSIS

A spatial and temporal development of the electron anomalous transport induced by the localized electrostatic wave excited in an unstable electron beam-plasma system shown in Fig. 1 has been investigated numerically assuming the two-dimensional drifted Maxwellian distribution function of plasma electrons

$$g_e=\frac{1}{\pi^{3/2}v_{te}^3}\exp\left[-\frac{(v_x-v_{e\perp})^2+v_y^2+(v_\parallel-v_{e\parallel})^2}{v_{te}^2}\right]\quad. \tag{6}$$

Here, g_e is obtained from $g_{e0}=(\pi^{3/2}v_{te}^3)\exp[-(v_\perp^2+(v_\parallel-v_{e\parallel})^2)/v_{te}^2]$ $a_e=\exp(-v_{e\perp}^2/v_{te}^2)$, and it is assumed that plasma ions are not magnetized. The energy and momentum densities of plasma electrons are given by $U_e = n_e m_e v_e^2/2 + 3n_e k_B T_e/2$, $\mathbb{P}_e = n_e m_e \mathbf{v}_e$ and $\mathbf{v}_e = (v_{e\perp}, 0, v_{e\parallel})$. The electrostatic waves are governed by the following kinetic wave equations:

$$\frac{\partial U_{\mathbf{k}}}{\partial t} = 2\gamma_N U_{\mathbf{k}} \quad , \tag{7}$$

$$\frac{\partial U_{\mathbf{k}}}{\partial z} = 2\gamma_{sN} U_{\mathbf{k}} \quad . \tag{8}$$

Here, z is the axial distance, and the linear or nonlinear growth rates are assumed to be determined by $\gamma_N = A\exp(-x^2/a^2) + \gamma_{\mathbf{k}}^{(e)}$ and $\gamma_{sN} = A_s\exp(-x^2/a^2) + \gamma_{s\mathbf{k}}^{(e)}$. The first terms of γ_N and γ_{sN} $(A, A_s > 0)$ express the linear or nonlinear growth rate ascribed to the electron beam. The second terms $(\gamma_{\mathbf{k}}^{(e)}, \gamma_{s\mathbf{k}}^{(e)} < 0)$ are the linear damping rate ascribed to plasma electrons. The quasilinear process by this linear damping accelerates and heats plasma electrons to generate the anomalous transport and the spatial inhomogeneity of the electron density and temperature. In the region of $\gamma_N, \gamma_{sN} > 0$ the electrostatic wave becomes linearly or nonlinearly unstatble and is localized in the region $|x| \lesssim a$, where $2a$ is nearly the transverse dimension of the electron beam. In addition we employ the following continuity equation:

$$\frac{\partial n_e}{\partial t} + \nabla\cdot\mathbf{\Gamma}_e = 0 \quad , \tag{9}$$

where $\mathbf{\Gamma}_e$ is the flux of plasma electrons given by

$$\mathbf{\Gamma}_e = n_e \mathbf{v}_e - \nabla_\parallel (D_e n_e + n_e v_{e\parallel}^2/\nu_e) \quad . \tag{10}$$

Here, $D_e = k_B T_e/m_e \nu_e$, $\nabla = (\partial/\partial x, 0, \partial/\partial z)$ and $\nabla_\parallel = (0, 0, \partial/\partial z)$ and $\nu_e \ll \omega_{ce}$. It can be proved from Poisson's equation that there exists only the cross-field electric field $\mathbb{E}_0$.

The numerical analysis of Eqs. (1), (2), and (7)-(10) was carried out under the parameters of $kv_{te0}/\omega_{ce} = 1$, $k_\parallel/k_\perp = 0.2$, $A/|\gamma_{\mathbf{k}}^{(e)}| = 1.01$, $A_s/|\gamma_{s\mathbf{k}}^{(e)}| = 1.2$, $\gamma_{s\mathbf{k}}^{(e)} v_{te0}/\gamma_{\mathbf{k}}^{(e)} = 1$, $|\gamma_{\mathbf{k}}^{(e)}|/\omega_{ce} = 10^{-3}$, $\nu_e/|\gamma_{\mathbf{k}}^{(e)}| = 5$, $a\omega_{ce}/v_{te0} = 2$ and $U_{\mathbf{k}}(0)/n_{e0}k_B T_{e0} = 10^{-3}$. Figure 2 shows the axial evolution of the transverse profile of n_e/n_{e0} , T_e/T_{e0} , $v_{e\perp}/v_{te0}$ and $eE_0 a/k_B T_{e0}$ at $|\gamma_{\mathbf{k}}^{(e)}|t = 0.69$, where $z/z_e = |\gamma_{s\mathbf{k}}^{(e)}|z$. It is seen that the hollow profile of the electron density and the peaked profile of the electron temperature in the region of the hollow density profile develop temporally and spatially.

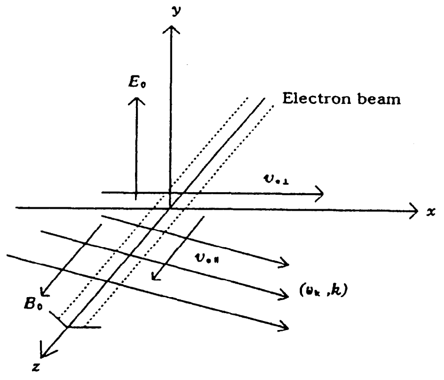

FIG. 1. Geometry for the electron beam, the electron drift velocity along and across the magnetic field, the cross-field electric field and the electrostatic waves.

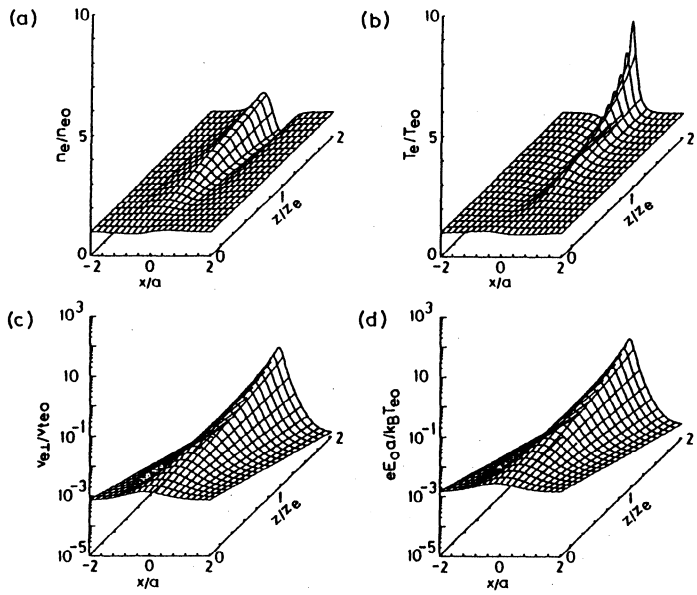

FIG. 2. Axial evolution of the transverse profiles of n_e , T_e , $v_{e\perp}$ and E_0 at $|\gamma_k^{(e)}|t=0.69$.

The typical parameters of the experiment reported by the author *et al.*[4, 5] are $T_{e0}=5$ eV , $B_0=7\times10^{-3}$ Tesla and $a=1.5\times10^{-3}$ m (beam radius), then $\omega_{ce}a/v_{te0}\simeq2$ is given. We set $v_{e\perp max}/v_{te0}\simeq0.2$, thereby $E_{0max}a/k_BT_{e0}=(\omega_{ce}a/v_{te0})(v_{e\perp max}/v_{te0})\simeq0.4$ is obtained. Thus we get $V_0=$ $=E_{0max}a\simeq2$ volt and $E_0\simeq1.3\times10^3$ volt/m . This value is very close to the experimental value of $V_{exp}\simeq10$ volt. Also the drift velocities $v_{e\perp max}\simeq$ $0.2v_{te0}\simeq1.9\times10^5$ m/s (0.1 eV) and $v_{e\parallel max}=(k_\parallel/k_\perp)v_{e\perp max}\simeq0.38\times10^5$ m/s (0.004 eV) are reasonably obtained.

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