NONLINEAR LANDAU DAMPING AND COLLISIONLESS HEATING IN BOUNDED PLASMA

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1. Introduction

The processes of collisionless power dissipation are of fundamental interest in plasma physics. The principal examples are Landau damping of longitudinal waves [1] and anomalous skin effect of transversal waves [2].

The linear theory of collisionless damping breaks down for times longer than the bounce time of trapped resonance electrons $\tau_r \equiv (m/eEk)^{1/2}$, where k is the wave number and E is the amplitude of the electric field. For finite perturbations, when $\gamma_L \tau_r < 1,$ where γ_L is the linear Landau damping [1] , the problem treatment is essentially non-linear. It is generally believed, that in this regime of nonlinear Landau damping the initial decay of the wave amplitude will soon turn into nonlinear oscillations and eventually approach a Bernstein Green Kruskal (BGK) steady state with lower value of wave amplitude [3]. In a real plasma, electron collisions with neutral atoms, electrons and ions have to be taken into the account. Though the collision frequency is small it is the only remaining mechanism providing the wave damping at nonlinear regime. The decrement of the longitudinal wave (nonlinear Landau damping γ_{nl}) was calculated in [4] under the conditions $\gamma_L \tau_r << 1$ with taking into account rare Coulomb collisions. Unlike the Landau decrement the resulting decrement depends on the amplitude of the wave and collision frequency.

In the present article a partially ionised plasma is considered, where electrons collide mainly with neutral atoms. These conditions are met for a gas discharge plasma, in which Landau damping has frequently been measured. The differential cross section of electron-atom scattering has no singularity at small angles in the range of energies up to about 30 eV, so that small angle scattering does not contribute to the total cross section. This makes it possible to calculate the decrement of nonlinear Landau damping analytically for any collision frequency. The approximation of the analytical calculation for the decrement of non-linear Landau damping gives, within an error of less than 5 percent $\gamma_{nl} = \gamma_L th(2\nu\tau_r)$, where γ_L is the linear Landau damping, ν is the total collision frequency, and τ_r is the bounce time of trapped electrons. The obtained result is applied for the calculation of collisionless heating in a bounded plasma. In contrast to the traditional theory of anomalous skin-effect the collisionless heating with account of non-linear effects is not constant when the collision frequency is tending to zero, but it is tending to zero with ν.

The cause of collisionless damping is the interaction of resonant electrons with a wave. The average scattering angle $< \theta >$ is considered to be not small $< \theta > >> \Delta u/v_f$, where

$\Delta u = (e\Phi_0/m)^{1/2}$, Φ_o is the potential-field amplitude of the wave and $v_f = \omega/k$ is its phase velocity, $\Delta u/v_f << 1$.

This allows to assume that after scattering in elastic collisions the resonant electrons immediately leave the resonance region. To get the damping coefficient, the rate of increase of kinetic energy of resonant electrons has to be calculated.

2. Exact solution of nonlinear Landau problem

We consider a stationary wave in a coordinate system of moving with its phase velocity. We look at stationary electron distribution function (EDF) at time larger than the collision time. The EDF is close to isotropic everywhere at velocities far from the resonance velocity v_f. To find the matching with the resonance region where strong interaction with a wave occurs, one has to solve the kinetic equation taking collisional integral inot account:

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} - eE(t,x)\frac{\partial f}{m\partial v_x} = \int (f' - f) v d\sigma \tag{1}$$

We are searching the EDF mainly in the resonance region: $|v - v_f| \leq 2 * \Delta u$. Outside the resonance region the EDF is close to isotropic $f_0(w)$. Inserting it into the collision integral income term gives $\int f' v d\sigma = \nu f_0$, where ν is total collision frequency (not transport as in the BGK integral). Let f^1 be the difference between f and $f_0(w)$, then Eq. (1) takes the form :

$$\frac{\partial f^1}{\partial t} + v_x \frac{\partial f^1}{\partial x} - eE(t,x)\frac{\partial f^1}{m\partial v_x} - eEv_x \frac{\partial f_0(w)}{\partial w} = -\nu f^1 \tag{2}$$

Note that in Eq. (2) the nonlinear term is included, in contrast to linear theory. The solution of Eq. (2) is

$$f^1 = -\int_{-\infty}^{t} eE(\tau, x(\tau)) e^{-\nu(t-\tau)} d\tau * v_x \frac{\partial f_0(w)}{\partial w}, \tag{3}$$

where $x(\tau)$ is the electron trajectory in the wave . Inserting the solution of Eq. (3) for f^1 into Eq. (2) and averaging over time and velocity direction one can find the slow evolution of the main part of the EDF $f_0(w)$:

$$\frac{\partial f_0}{\partial t} + \frac{\partial}{\sqrt{w}\partial w}(\sqrt{w} D(w) \frac{\partial f_0}{\partial w}) = St^*(f_0), \tag{4}$$

where $St^*(f_0)$ is collisional integral accounting for energy losses in elastic and inelastic collisions, and $D(w)$ is the energy diffusion coefficient:

$$D(w) = \int D_v m^2 v_x^2 \frac{d\cos\alpha d\beta}{4\pi}, \tag{5}$$

which is averaged over velocity angle ($\cos\alpha = v_x/v$); $\cos\beta = v_y/\sqrt{v_y^2 + v_z^2}$) diffusion coefficient in velocity space:

$$D_v = e^2 < E(\tau, x(\tau)) \int_{-\infty}^{t} E(\tau, x(\tau)) e^{-\nu(t-\tau)} d\tau >, \qquad (6)$$

where angular brackets $<>$ denote averaging over time t. According to the equation of motion in the wave $eE(\tau, x(\tau)) = d(v_x(t) - v_x(\tau))/d\tau$. Substituting the expression for the electric field and partially integrating the diffusion coefficient (6) takes the form:

$$D_v = \frac{\nu}{2} < \int_0^{\infty} (v_x(t) - v_x(t-\tau))^2 \, \nu e^{-\nu\tau} d\tau > . \qquad (7)$$

The expression for D_v (7) has very transparent physical meaning as the product of square of step in velocity $(v_x(t) - v_x(t-\tau))$ by the frequency of this step ν averaged with a probability to make the step, or to be in the resonance region without collisions for a time τ - $\nu e^{-\nu\tau}$.

Now we have to calculate the velocity kick of resonant electrons $\Delta v_x(v_x, \tau, \phi)$. The evolution of electron velocity is governed by the Hamiltonian:

$$H(v_x, x) = \frac{m}{2}(v_x - v_f)^2 - e\Phi_0 \cos kx \qquad (8)$$

The solution is to be found in elliptic functions [3]. After cumbersome calculations we find

$$D_v = \frac{\pi [u]^3 [\tau_r^{-1}] v_f^2 \Pi(\tilde{\nu})}{2 * v^3}, \qquad (9)$$

where the function $\Pi(\tilde{\nu})$ of the dimensionless collision frequency $\tilde{\nu} = \nu * \tau_r$ is defined as :

$$\begin{aligned} \Pi(\tilde{\nu}) &\equiv 128\tilde{\nu} \sum_{n=1} \int_0^1 \left[(\frac{q^n}{1+q^{2n}})^2 \frac{1}{1+\left(\frac{\tilde{\nu}}{\frac{n\pi}{K\chi}}\right)^2} \frac{1}{K\chi^4} + (\frac{q^{n-\frac{1}{2}}}{1+q^{2n-1}})^2 \frac{1}{1+\left(\frac{\tilde{\nu}}{\frac{(2n-1)\pi}{2K}}\right)^2} \frac{\chi}{K} \right] d\chi \\ &\approx \tanh 2\tilde{\nu}, \end{aligned} \qquad (10)$$

where q and K are standard elliptic integrals. The last approximation is valid within error less than 0.05. The plot of function $\Pi(\tilde{\nu})$ is shown in Fig. 1.

3. Decrement of nonlinear wave

Surprisingly the complex function $\Pi(\tilde{\nu})$ can be very well approximated simply by $\tanh 2\tilde{\nu}$. At large $\tilde{\nu} >> 1$ the function $\Pi(\tilde{\nu}) \to 1$, and Equation (9) corresponds to quasi-linear theory and total power dissipation gives linear Landau decrement. We can then deduce that nonlinear damping in related to linear damping by:

$$\gamma_{nl} \cong \gamma_l * \tanh(2\nu \, \tau_r) \qquad (11)$$

As can be seen from Figure 1 at $\tilde{\nu} >> 1$ the main contribution is due to passing electrons (not trapped in the wave) electrons, see first term in r.h.s. of Eq. (10). For $\tilde{\nu} < 1$ the function

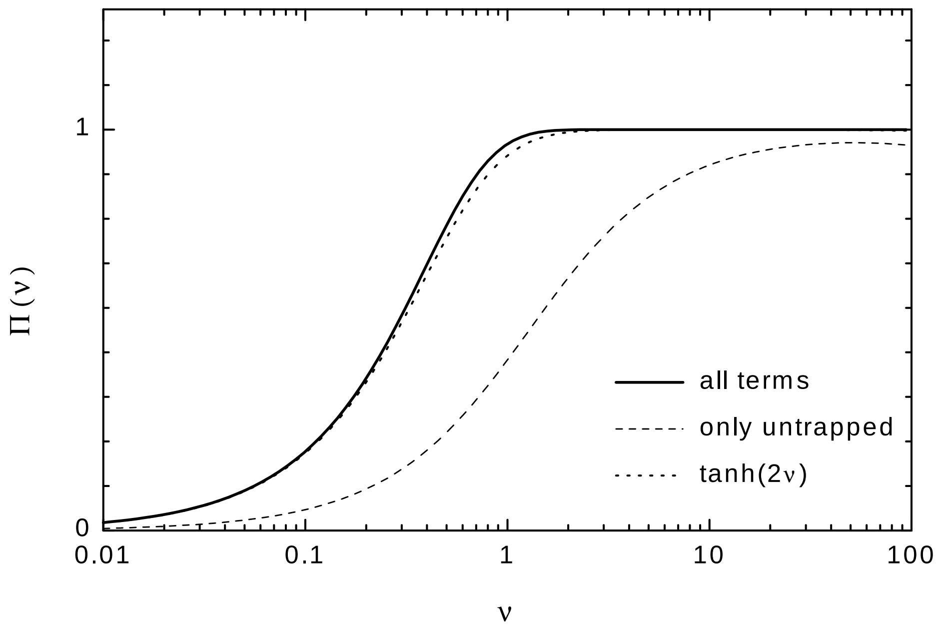

Fig. 1. Dimensionless function $\Pi(\nu)$

$\Pi(\tilde{\nu})$ is smaller than unity, and correspondingly the power dissipation and nonlinear decrement of the wave decreases. For very small $\tilde{\nu} << 1$ the function $\Pi(\tilde{\nu}) \cong 2\tilde{\nu}$ is proportional to the collision frequency, similar to results of [4]. Note that in contrast to the paper [4], where only the limit of rare collisions was considered, the resulting Eq. (11) is valid for arbitrary values of $\tilde{\nu}$. At small $\tilde{\nu}$ the main contribution to power dissipation is due to trapped-in-the-wave electrons (second term in r.h.s. (10)), the contribution of passed electrons (first term in r.h.s. (10)) is nearly 25% compared with that of trapped electrons.

We have concidered the electron heating by a single longitudinal wave. The obtained results can be applied for the calculation of the electron collisionless heating in a bounded plasma for an arbitary electric field. As a result, in contrast to general belief ([2]), collisionless power dissipation is not constant when the collision frequency is tending to zero, but it is tending to zero with ν.

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