BREMSSTRAHLUNG FROM HIGH-TEMPERATURE PLASMA

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For account of the single act of the bremsstrahlung radiation the quantum relativistic formulas for cross sections are used. The bremsstrahlung specific radiation power is calculated by integrating over a relativistic Maxwell-Boltzmann electron distribution function. Quantum relativistic bremsstrahlung power and spectrum are obtained and compared with classical ones. It is shown that quantum and relativistic effects bring in to bremsstrahlung significant correction for electron temperature T_e higher than 10 keV.

1. Introduction

Formula usually used for calculation of plasma bremsstrahlung power [1] is based on analysis of the Fourier-spectrum of the classical electron acceleration in nuclear field. Such approach supposes the electron motion is determined. But it is correct only for electrons, which kinetic energy does not exceed 27 eV [2]. The correction on quantum effects by using Bethe-Heitler dipole bremsstrahlung cross section formula does not allow to take into consideration a relativistic nature of fast electrons, that as calculations have shown plays a significant role in the bremsstrahlung. Besides, in a high-temperature plasma the quadrupole bremsstrahlung may be considerable.

2. The bremsstrahlung spectra

In quantum electrodynamics the cross sections taking into account all directions of the photon momentum and the secondary electron momentum for emitting photon with frequency between ω and $\omega+d\omega$ are characteristics of the single act of bremsstrahlung. For calculations in this work cross sections $d\sigma_\omega^{e-i}$ for e-i bremsstrahlung and $d\sigma_\omega^{e-e}$ for e-e bremsstrahlung are taken from [2]. The spectrum (or the radiation spectral distribution) of the bremsstrahlung from the plasma is

$$S^{e-i,e-e}(\omega) = \int \hbar\omega \frac{d\sigma_\omega^{e-i,e-e}}{d\omega} N_{i,e} N_e v f(v) dv \,, \tag{1}$$

where $\hbar$ is the Planck constant, N_e is the electron density, N_i is the ion density, integration is produced over range of the relative electron velocity v, where kinetic energy of the electron is exceeding the emitted photon energy and relativistic Maxwell electron velocity distribution function is

$$f(v) = \left[2\left(\frac{kT_e}{mc^2}\right)^2 K_1\left(\frac{mc^2}{kT_e}\right) + \frac{kT_e}{mc^2} K_0\left(\frac{mc^2}{kT_e}\right)\right]^{-1} c^{-3} v^2 \left(1-\frac{v^2}{c^2}\right)^{-\frac{5}{2}} \exp\left(-\frac{1}{kT_e}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right), \tag{2}$$

where k is the Boltzmann constant, m is the electron mass, c is the light velocity, K_1 and K_0 are the modified Bessel functions of the second kind.

Obtained dipole and quadrupole spectra $S^{e-i}(\omega)$ and $S^{e-e}(\omega)$ were compared with classical spectrum, which is

$$S^0(\omega) = \frac{16}{3}\sqrt{\frac{2\pi}{3}}\alpha r_e^2 \mathrm{ch} Z_{eff}^2 N_e^2 \sqrt{\frac{mc^2}{kT_e}} \exp\left(-\frac{\hbar\omega}{kT_e}\right), \qquad (3)$$

where α is the fine structure constant, r_e is the classical electron radius, Z_{eff} is the plasma ions effective charge, which is defined as

$$Z_{eff}^2 = \sum_Z Z^2 \frac{N_Z}{N_e} , \qquad (4)$$

where N_z is density of ions with charge Z.

In Fig. 1, the bremsstrahlung spectra divided by $S^0(0)$ are presented for Z_{eff}=1 and T_e=70 keV. Obtained quantum spectra have feature, which is consequence of infrared divergence, which appears in the low-frequency regions of quantum cross sections and spectra due to used approach is not correct there. The bremsstrahlung is supposed to be a single-photon process, but in the low-frequency region the multiphoton emitting is more probably [2]. The calculations shown, that the range and the influence of such long-wave photons to final results are insignificant.

3. The bremsstrahlung specific radiation powers

For account of energetic characteristics of plasma specific radiation power of bremsstrahlung is used. It is defined as

$$P_{br}^{e-i,e-e} = \int_0^\infty S^{e-i,e-e}(\omega) d\omega \, . \qquad (5)$$

For comparison we use well known classical bremsstrahlung specific power

$$P_{br}^0 = C_0 Z_{eff}^2 N_e^2 \sqrt{\frac{kT_e}{mc^2}} , \qquad (6)$$

where

$$C_0 = \frac{16}{3}\sqrt{\frac{2\pi}{3}}\alpha r_e^2 mc^3 \, . \qquad (7)$$

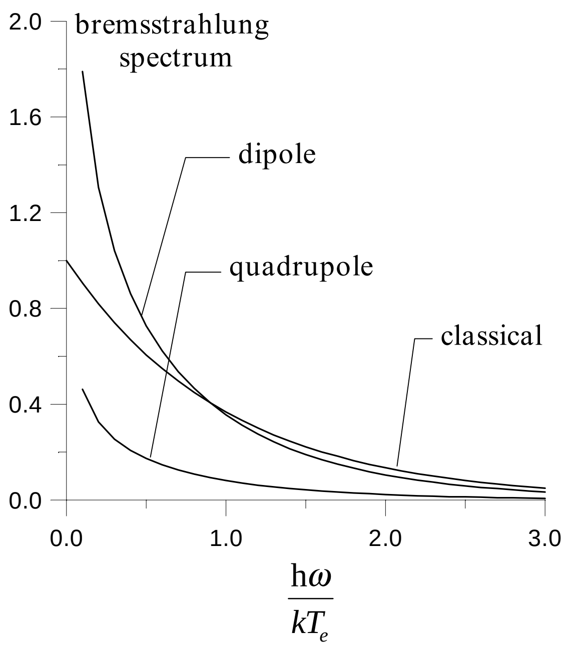

Figure 1. Bremsstrahlung spectra for T_e=70 keV.

As a results of calculations of specific powers P_{br}^{e-i} and P_{br}^{e-e} we obtain the corrections $K^{e-i}(T_e)$ for dipole and $K^{e-e}(T_e)$ for quadrupole radiation.

At last, total specific bremsstrahlung radiation power is

$$P_{br} = C_0 N_e^2 \sqrt{\frac{kT_e}{mc^2}} \left[Z_{eff}^2 K^{e-i}(T_e) + K^{e-e}(T_e) \right]. \tag{8}$$

In electron temperature range 1-500 keV fits for corrections $K^{e-i}(T_e)$ and $K^{e-e}(T_e)$ are

$$K^{e-i}(T_e) = 1.10 + 0.59y + 3.06y^2 - 2.56y^3 + 0.85y^4, \tag{9}$$

$$K^{e-e}(T_e) = 1.78y - 0.15y^2 + 0.58y^3, \tag{10}$$

where $y=kT_e/(mc^2)$. In extreme-relativistic case $K^{e-e}(T_e) = K^{e-i}(T_e)$ due to coincidence of corresponding cross sections [2].

To illustrate obtained results, ratios $P_{br}^0/(C_0 Z_{eff}^2 N_e^2)$ for classical, $P_{br}^{e-i}/(C_0 Z_{eff}^2 N_e^2)$ for dipole and $P_{br}^{e-e}/(C_0 N_e^2)$ for quadrupole are presented in Fig. 2.

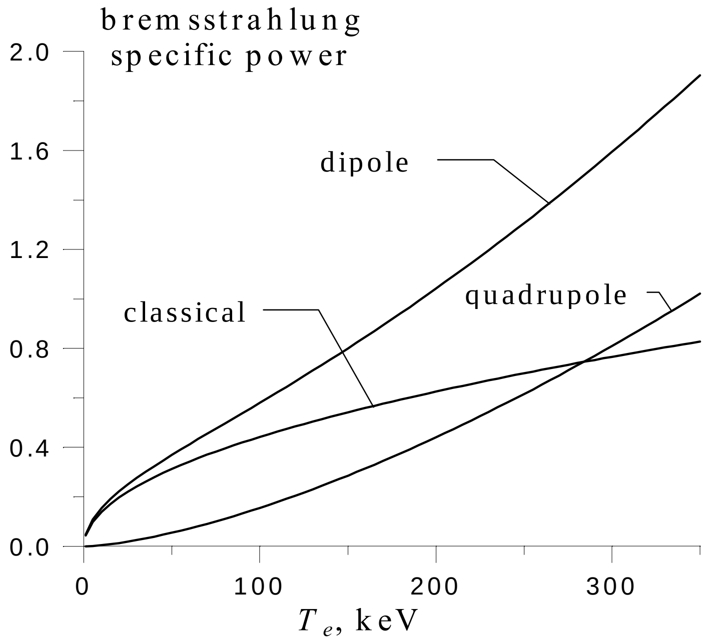

Figure 2. Bremsstrahlung specific powers.

4. Discussions and conclusions

For relativistic correction presented in [3] data of [4] was used to interpolate bremsstrahlung power between nonrelativistic and extreme-relativistic results. But this interpolation can be used only in case of very high Z_{eff}.

Obtained results are important for analysis of fusion plasmas using advanced fuels, e.g. D-^{3}He, D-^{3}He -^{6}Li, etc. Presented information about bremsstrahlung may be applied both in high-temperature plasma physics generally and in fusion plasma physics particularly.

References

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