

TOROIDAL DIELECTRIC TENSOR-OPERATOR FOR ARBITRARY ASPECT RATIO AND WAVE FREQUENCY: AN ANISOTROPIC-RESISTIVITY, FINITE PRESSURE MHD FORMULATION

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Abstract

The derived dielectric tensor elements are based on a two-fluid, finite pressure weakly collisional plasma description, with the Hall term included. They are characterized by the following features: (i) They are casted in a form evidencing the dielectric (non-operator) and operator contributions; (ii) They are **not subject to any limitation** on the magnitude of the toroidal effects; (iii) They include **anisotropic** — parallel and perpendicular to the magnetic field — contributions to the plasma resistivity; (iv) They are **not limited by any restriction** on the value of the wave frequency; (v) No limitation on the electron or ion equilibrium flow velocity is imposed — it is the center-of-mass equilibrium velocity which is taken to be zero.

1. Linearized Resistive Two-Fluid Equations.

We characterize the plasma perturbations by the set of functions $(\mathbf{V}, \mathbf{j}, \mathbf{B}, \mathbf{E}, n, p)$, and an equilibrium state is described by the functions $(0, \mathbf{j}_0, \mathbf{B}_0, 0, n_0, p_0)$. Linearized two-fluid resistive MHD equations read:

$$-i\omega n + \nabla \cdot (n_0 \mathbf{V}) = 0, \quad (1)$$

$$-i\omega n_0 \left(\sum m_\alpha \right) \mathbf{V} = - \sum (\nabla p_\alpha) + \mathbf{j}_0 \times \mathbf{B} + \mathbf{j} \times \mathbf{B}_0, \quad (2)$$

$$\begin{aligned} -i\omega \mathbf{j} = & -\nabla \left(\sum \frac{q_\alpha}{m_\alpha} p_\alpha \right) + n_0 e^2 \left(\sum \frac{1}{m_\alpha} \right) (\mathbf{E} + \mathbf{V} \times \mathbf{B}_0) + \\ & + s (\mathbf{j}_0 \times \mathbf{B} + \mathbf{j} \times \mathbf{B}_0) + e \mathbf{R}_{ei} \left(\sum \frac{1}{m_\alpha} \right), \end{aligned} \quad (3)$$

where $s \equiv (\sum q_\alpha / m_\alpha)$.

2. Dielectric Tensor-Operator

After a lengthy algebra, we obtain the following form of the tensor-operator:

$$\hat{\epsilon} = \hat{\Omega}_1 + \hat{\Omega}_2 \cdot \hat{M}, \quad (4)$$

where $M_{jk} = (\text{rot})_{jk} = \beta_{jk} - \varepsilon_{jkl} \nabla_l$, $\beta_{jk} \equiv \mathbf{e}_j \cdot (\nabla \times \mathbf{e}_k)$ and ε_{jkl} is the Levi-Civita anti-symmetric tensor. The tensor $\hat{\Omega}_1$ represents a dielectric plasma response independent of the equilibrium plasma current, \mathbf{j}_0 , and $\hat{\Omega}_2$ results from the Hall effect proportional to \mathbf{j}_0 . The matrix \hat{M} contains operators that act on the rf electric field.

3. Illustrative Case

To illustrate the result obtained in Section 2, we present a numerical evaluation of the tensor elements for the parameters characterizing an equilibrium state of small aspect ratio device, e.g. START [1]. The equilibrium functions (n_0 , $T_{e,i;0}$, B_0 , j_0 and ψ) are taken from the results of numerical self-consistent equilibrium simulation for START by means of the code SCENE [2]. Next, we take the best fit of these functions.

Thus, the main equilibrium parameters used are: $R_0/a = 1.435$, $R_0 = 0.165 \text{ m}$, $n_0 = 2.5 \cdot 10^{20} \text{ m}^{-3}$, $T_{e,0} = 180 \text{ eV}$, $T_{i,0} = 150 \text{ eV}$, $I_0 = 0.065 \text{ MA}$, $B_0 = 0.483 \text{ T}$ (on the magnetic axis); elongation — 1.3 and triangulation — 0.3. The frequency of rf wave $\omega = 1.5 \cdot 10^7 \text{ s}^{-1}$.

In Figs.1–2, we present the evaluated elements of $\hat{\Omega}_1$ and $\hat{\Omega}_2$. Note that $\hat{\Omega}_1$ and $\hat{\Omega}_2$ have different dimensions: the first is dimensionless and the second one represents the coefficient of a matrix which has the dimension of j/r and contains operatorial terms. For an order of magnitude comparison, one can normalize the elements of the matrix $\hat{\Omega}_2$ by the value j_{0m}/a (j_{0m} is the maximum equilibrium current density in the plasma and a — the minor plasma radius); this makes $\hat{\Omega}_2$ also dimensionless and accounts for the characteristics of the device ($j_{0m} = 3.5 \text{ MA/m}^2$). The illustrative results presented in Figs.1–2, are plotted as functions of cylindrical coordinates R and Z ($R = 0$ corresponds to the main axis of the torus and $Z = 0$ — to its equatorial plane). The results of evaluation allows one to conclude that **the Hall term contribution to the dielectric tensor can be important** under the conditions considered in this work.

References

- [1] Sykes, A.: Plasma Phys. Contr. Fusion **36**, Suppl. (12)B, B93 (1994)
- [2] Wilson, H.R.: Euratom-UKAEA Report, Culham, Abingdon, UK (1994).

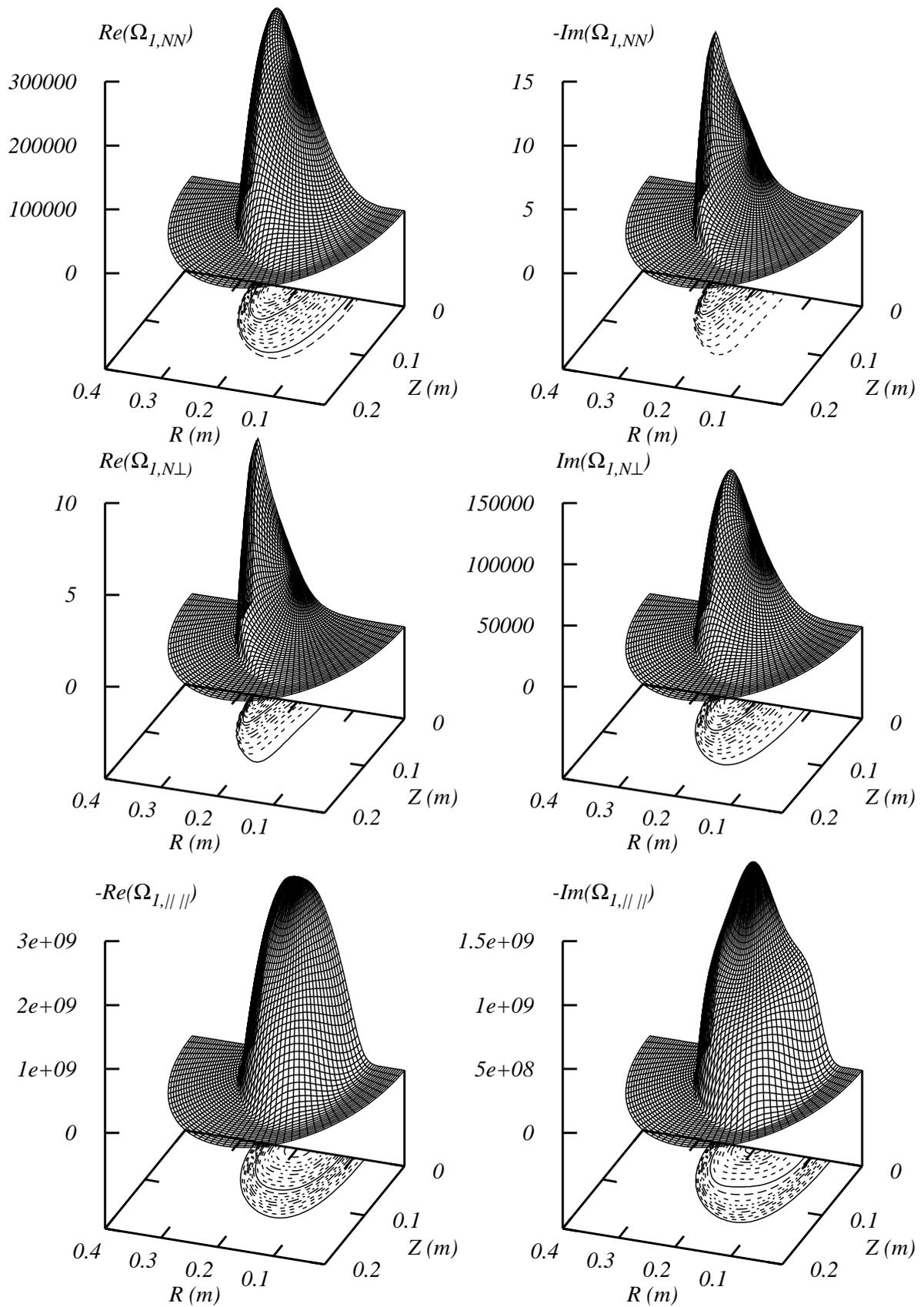


Fig.1. Spatial dependence of the j_0 -independent term, Ω_1 , for the parameters of START-like device.

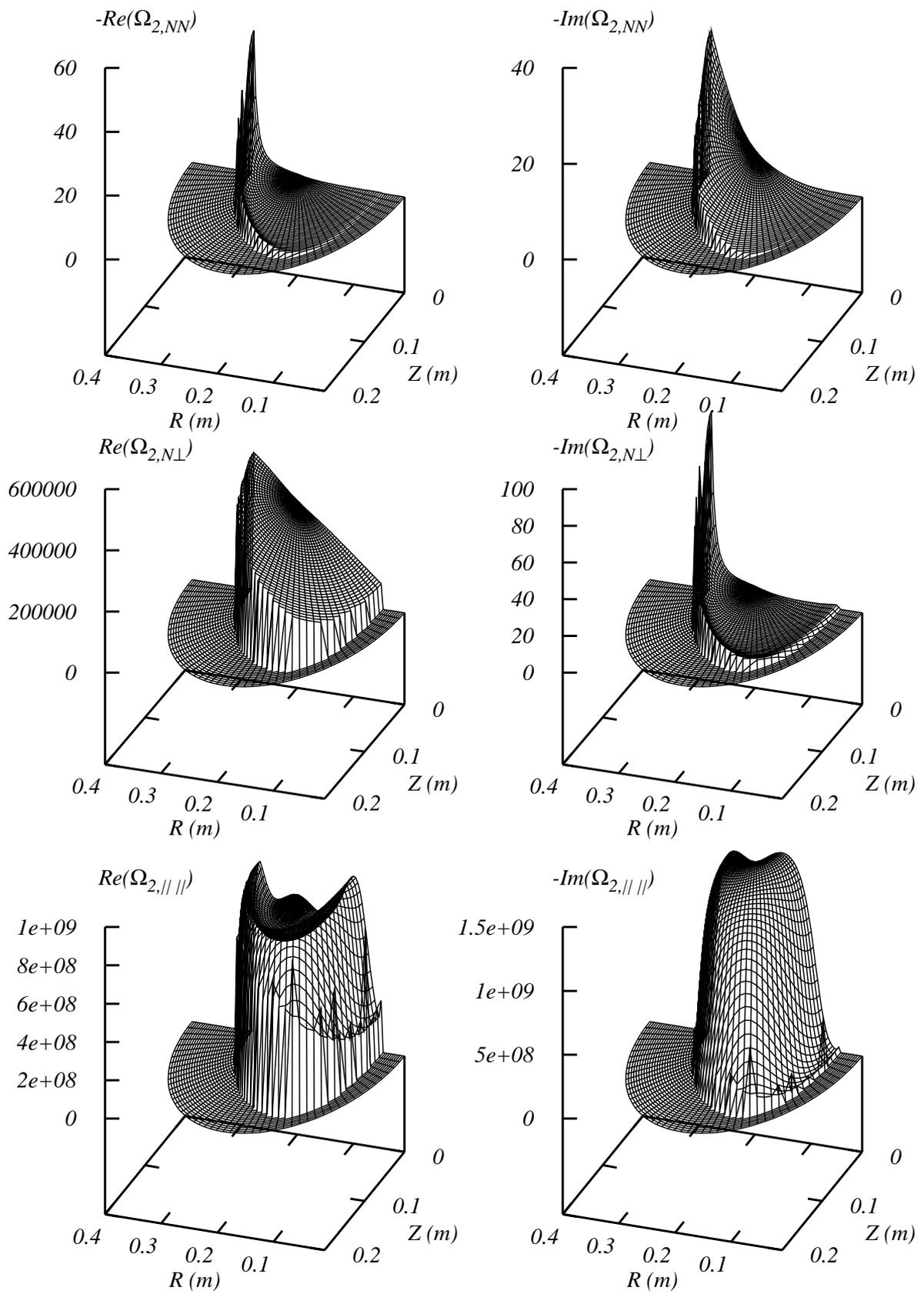


Fig.2. Spatial dependence of the j_0 -dependent term, Ω_2 , normalized to j_0/a , for the same parameters.