RANDOM AND REGULAR LOWER–HYBRID FIELDS IN PLASMAS

S.I. Popel

Institute for Dynamics of Geospheres
Leninsky prospekt 38, Building 6, Moscow 117979, Russia

1. The significant feature of the observations in the Earth's magnetosphere carried out by the instrumented spacecraft FREJA was the possibility to determine the value of the threshold of excitation of regular lower–hybrid (LH) fields on the basis of the data observed. Regular LH fields observed in the experiment FREJA are associated [1, 2] with the envelope LH solitons which can be formed due to the development of LH wave modulational interaction. Thus the problem of comparison of the theoretical magnitude of the modulational instability threshold (calculated under the conditions of the experiment FREJA) with the observed one is quite natural. Here we will present the results of investigation of the modulational instability of arbitrary LH wave spectra which are based on the universal nonlinear formalism for description of the dynamics of random and regular fields. This formalism allows us, in particular, to determine the thresholds of LH wave modulational instability. We will use the results [1] of the observations of regular LH fields in the Earth's magnetosphere by the instrumented spacecraft FREJA for comparison of the observed values of the LH wave threshold energy density and the theoretical ones.

2. The concept of the modulational instability thresholds is of crucial importance in the description of transition from weak to strong plasma turbulence (see, e.g., [3]), because it is the modulational interaction that results in formation of regular fields in a plasma (even in the case when initially there exist only random fields) and, correspondingly, in formation of the strongly turbulent state. The modulational instability threshold can naturally be introduced in the following manner: if the value of the wave energy is lower than the threshold value then no modulational processes occur, while in the opposite case the modulational perturbations are generated.

The example of the modulational instability of a monochromatic wave in a collisionless plasma (where the thresholds of the instability are absent) shows that the problem of definition of the modulational instability threshold is closely connected with the problem of investigation of the instability of wave spectra and the threshold should depend on the width of the spectra. The straightforward formalism for investigation of the modulational instability of wave spectra [4 – 8] assumes the expansion of the values characterizing plasma particles and waves in their random (turbulent) and regular components. This expansion for some value A is

$$A = \delta A + A^{\text{reg}}, \tag{1}$$

where A^{reg} is the regular component, δA is the random one: $\langle \delta A \rangle = 0$, $\langle A \rangle = A^{\text{reg}}$; the angular brackets $\langle \rangle$ denote the averaging over a statistical ensemble. This formalism shows clearly the importance of interrelation between the modulational interaction and the generation of the regular fields.

It allows us to obtain the equations for the modulational instability (see for the case of a plasma without magnetic field in [4] and for the case of a plasma in the presence of an external magnetic field in [8]). These equations are valid in the near–threshold domain for

any relationships between the plasma and wave parameters as well as the parameters of the wave spectrum and the modulational perturbations. The main steps in the development of the formalism are the following. We separate the random and regular components of the electric field and the distribution function of plasma particles. Averaging the kinetic equation over a statistical ensemble we obtain separate equations for the random and regular quantities. We distinguish the positive and negative frequency harmonics of the fields. Taking into account interactions via low–frequency "virtual" (beat) fields we obtain equations both for these fields and for the high–frequency wave fields. The (nonlinear) spectrum of the weak turbulence is the solution of the corresponding unperturbed (nonlinear) equation. Equations describing the modulational instability can be written for the correlation functions $G^{\pm}_{k,k'} = \left\langle \delta' E^{\pm}_{\pm k + k'} \delta E^{\mp (0)}_{\mp k} \right\rangle$, where $\delta' E$ is the random field perturbation due to the modulational interaction, $\delta E^{\pm (0)}$ is the initial random field, the superscript "+" ("−") denotes the positive (negative) frequency component of the field, $k = \{\omega, \mathbf{k}\}$ is the four–vector, ω is the frequency, and $\mathbf{k}$ is the wave vector. For a general case these equations take the form:

$$\varepsilon_{\pm k + k'} G^{\pm}_{k,k'} = -2 \int \Sigma^{\text{eff}}_{\pm k + k', \pm k_1, \mp k_1} G^{\pm}_{k,k'} |E^{+(0)}|^2_{k_1} dk_1$$

$$-2 \int \Sigma^{\text{eff}}_{\pm k_1, \pm k + k', \mp k_1} G^{\pm}_{k,k'} |E^{+(0)}|^2_{k_1} dk_1$$

$$-2 |E^{+(0)}|^2_k \int G^{\pm}_{k_1, k'} (\Sigma^{\text{eff}}_{\pm k_1 + k', \pm k, \mp k_1} + \Sigma^{\text{eff}}_{\pm k, \pm k_1 + k', \mp k_1}) dk_1$$

$$-2 |E^{+(0)}|^2_k \int G^{\mp}_{k_1, k'} (\Sigma^{\text{eff}}_{\pm k, \pm k_1, k' \mp k_1} + \Sigma^{\text{eff}}_{\pm k_1, \pm k, k' \mp k_1}) dk_1, \tag{2}$$

where $\Sigma^{\text{eff}}_{k_1, k_2, k_3}$ is the effective third–order (in the wave field) response, ε_k is the linear dielectric function, $\left\langle \delta E^{+(0)}_k \delta E^{-(0)}_{k_1} \right\rangle = -|E^{+(0)}|^2_{k_1} \delta(k + k_1)$.

The effective nonlinear response for the case of LH waves is (see, e.g., [9])

$$\Sigma^{\text{eff}}_{k_1, k_2, k_3} = -\frac{e^2}{2 m_e^2} \frac{|\mathbf{k}_2 + \mathbf{k}_3|^2}{|\mathbf{k}||\mathbf{k}_1||\mathbf{k}_2||\mathbf{k}_3|} \frac{\varepsilon^{(i)}_{k_2 + k_3}}{\varepsilon_{k_2 + k_3}} (\varepsilon^{(e)}_{k_2 + k_3} - 1)$$

$$\times \left[\frac{k_{\parallel} k_{1\parallel}}{\omega^2} + \frac{i(\mathbf{k}_{\perp} \times \mathbf{k}_{1\perp})_{\parallel}}{\omega \omega_{Be}} \right] \left[\frac{k_{2\parallel} k_{3\parallel}}{\omega^2} - \frac{i(\mathbf{k}_{2\perp} \times \mathbf{k}_{3\perp})_{\parallel}}{\omega \omega_{Be}} \right], \tag{3}$$

where $\varepsilon^{(e)}$ ($\varepsilon^{(i)}$) is the electron (ion) part of the dielectric function, $-e$ is the electron charge, m_e is the electron mass, ω_{Be} is the electron gyrofrequency.

The consideration [8] of the modulational instability for the case of spectra of LH waves which propagate under the angle θ with respect to the external magnetic field (so that $\cos \theta \geq (m_e/m_i)^{1/2}$, where m_i is the ion mass) and have the disperion law

$$\omega_{\mathbf{k}} \approx \omega_{LH} \left(1 + \frac{m_i}{m_e} \cos^2 \theta \right)^{\frac{1}{2}}, \tag{4}$$

where $\omega_{LH} = \omega_{pi} / \left(1 + \omega_{pe}^2 / \omega_{Be}^2 \right)^{1/2}$ is the frequency of LH resonance, $\omega_{pe(i)}$ is the electron (ion) plasma frequency, allows us to make the following conclusions:

(a) The "long–scale" modulational instability of wave spectra (when the length of the waves in the spectrum is less significantly than the wave length of the modulational perturbations $2\pi/|\mathbf{k}'|$, and the width in $\mathbf{k}$–space of the wave spectrum $|\delta \mathbf{k}|$ is larger than $|\mathbf{k}'|$) is suppressed compared with that of a monochromatic (with a fixed frequency and wave vector) pump wave.

(b) The maximum growth–rates of the "short–scale" modulational instability of wave spectra (when the length of the waves in the spectrum is larger than the wave length of the modulational perturbations, and the width in the wave vector space of the wave spectrum is smaller than $|\mathbf{k}'|$) coincide in definite cases with the maximum rates of the monochromatic pump wave modulational instability.

(c) The modulational instability thresholds appear only for broad spectra.

For the case of LH waves considered the condition for the existence of the instability thresholds is very severe. The thresholds appear only for very broad spectra when their width $|\delta\mathbf{k}|$ exceeds the wave vector of any modulational perturbation, i.e., for spectra containing all possible (for the waves) magnitudes of $\mathbf{k}$ ($\omega/k_\parallel > (2-3)v_{Te}$, where the subscript $\parallel$ denotes the vector component parallel to the external magnetic field, v_{Te} is the electron thermal velocity). This is associated with the fact that the "short–scale" modulational instability of LH wave spectra develops effectively for any (as small as is wished) magnitudes of LH wave energy density. For sufficiently wide LH wave spectra in ω–space so that $\delta\omega \gg |\delta\mathbf{k}|v_s$ (where $\delta\omega$ is the width of the spectrum in ω–space, $v_s = (T_e/m_i)^{1/2}$ is the ion acoustic velocity, T_e is the electron temperature) the threshold is determined by the following condition of the modulational instability development [8]:

$$\int \frac{d\mathbf{k}W_{\mathbf{k}}^{LH}}{n_0 T_e} \frac{\omega_{pe}^2}{|\mathbf{k}|^2 v_{Te}^2} > \frac{m_e}{m_i}\left(\frac{\omega_{Be}}{\omega_{pe}}\right)^2 \left(1 + \frac{\omega_{pe}^2}{\omega_{Be}^2}\right)^2,$$ (5)

where $W^{LH} = \int W_{\mathbf{k}}^{LH} d\mathbf{k}$ is the energy density of LH waves, n_0 is the unperturbed electron density.

3. The case of LH waves corresponding to the dispersion law (4) is very important for numerous real situations in laboratory and space plasmas. Among them is the excitation of regular fields in Earth's magnetosphere observed by the FREJA satellite [1].

The purpose of the FREJA project was the investigation of the interaction between the hot plasma of the magnetosphere and upper atmosphere/ionosphere. The high resolution of the plasma and field measurements provided by a high telemetry rate (up to 512 kbits/s) as well as the burst memory capacity of many instruments have given a possibility of observation of electrostatic fields in the frequency range of LH waves. The measurements have been carried out at the altitudes of about 1760 km. Purely electrostatic fields have been observed (no perturbations of the magnetic field have been detected). The characteristic feature of these fields is the presence of density cavities in the regions of the field localization [1, 2, 10, 11]. The characteristic frequency range of the LH wave fields is $\omega \approx 2\pi \cdot (3.5 - 4.5)$ s^{-1} (see, Fig. 3 in [1]). The characteristic magnitude of the frequency of LH resonance is $\omega_{LH} \sim 2\pi$ s^{-1} [2]. We note that the latter magnitude is the estimate one. The measured magnitudes of the plasma parameters are the following [2]: $n_0 \approx 1.1 \cdot 10^3$ cm^{-3}, $\omega_{pe} \approx 1.87 \cdot 10^6$ s^{-1}, $\omega_{Bp} \approx 2\pi \cdot 400$ s^{-1}, where ω_{Bp} is the proton gyrofrequency. The latter magnitude corresponds to the external magnetic field $|\mathbf{B}_0| \approx 0.263$ G. The ion composition of the magnetospheric plasma in the region where the measurements have been fulfilled is O$^+$, He$^+$, H$^+$.

To obtain the magnitude of the threshold of excitation of regular LH fields one has investigated [1] the dependence of the LH field energy density on the magnitudes of relative density variations in the regions of their localization. It has been shown that the quantities of the LH field energy density and the density variations are statistically related. The threshold value has been determined by considering the limit where the magnitudes of the density variations tend

to zero. It has been found that the threshold of the excitation of regular structures corresponds to the magnitude of the LH field energy density of the order of $W^{LH} \sim 10^{-14}$ J/m$^3 = 10^{-13}$ erg/cm^3.

Let us show that the parameters of the plasma and the waves for which the expression (5) has been calculated correspond to the conditions of the FREJA experiment. Indeed, the broad range of LH wave frequencies ($\delta\omega \sim \omega_{LH} \sim \omega_{\text{ch}}$, where ω_{ch} is the characteristic magnitude of the frequency in the spectrum) indicates that the role in the spectrum of the waves with the dispersion law (4) is great. As it has been mentioned the regular LH fields observed in the experiment FREJA are associated with the LH solitons. The soliton contains harmonics with all possible magnitudes of $\mathbf{k}$. This means that the LH wave spectrum can be considered as a broad one and it is meaningful to use the concept of the modulational instability thresholds. Furthermore, the following inequalities are valid: $|\delta\mathbf{k}|v_s \approx |\delta k_\parallel|v_{Te} (m_e/m_i)^{1/2} /|\cos\theta| \sim |k_\parallel|v_{Te} (m_e/m_i)^{1/2} /|\cos\theta| < \omega_{\text{ch}} \sim \delta\omega$.

Let us assume that the characteristic magnitude of $\left(1 + \omega_{pe}^2/\omega_{Be}^2\right)^{-1} \omega_{pe}^2/|\mathbf{k}|^2 v_{Te}^2 \approx \omega_{\text{ch}}^2/k_\parallel^2 v_{Te}^2$ is of the order of ten. Indeed, if the energy density of the regular fields tends to zero and the LH wave spectrum includes all possible $\mathbf{k}$ (for which the waves can exist) then the *only* characteristic wave vector of the problem considered is that satisfying the condition $(2-3)|k_\parallel|v_{Te} \approx \omega_{\text{ch}}$. Using also the characteristic magnitude of the electron temperature for the magnetospheric plasma at the altitude of about 1760 km, $T_e = (0.3-1)$ eV, we find from the condition (5): $W^{LH} \sim 2.1 \cdot 10^{-13}$ erg/cm^3 (where for definiteness sake the magnitude $T_e = 0.3$ eV is used and it is supposed that the main contribution to the ion composition is made by the hydrogen ions); $W^{LH} \sim 1.3 \cdot 10^{-14}$ erg/cm^3 (for $T_e = 0.3$ eV and the oxygen ions O$^+$). We see there is a good agreement between the theoretical (obtained on the basis of expression (5)) and observed in FREJA experiment magnitudes of the threshold energy density of LH fields. The example considered shows the association of the appearance of the regular fields (and, correspondingly, the formation of the strong turbulent state in the magnetospheric plasma) and the development of the modulational interaction.

References

[1] H.L. Pécseli, B. Lybekk, J. Trulsen, and A. Eriksson: Plasma Phys. Control. Fusion **39** A227 (1997)

[2] R. Lundin, L. Eliasson, O. Norberg et al.: in: Solar System Plasmas in Space and Time. Geophysical Monograph 84, American Geophysical Union, 1994

[3] V.D. Shapiro and V.I. Shevchenko: in: Basic Plasma Physics. A.A. Galeev and R.N. Sudan eds., Vol. 2, North–Holland Physics Publishing, Amsterodam, 1984

[4] S.I. Popel, V.N. Tsytovich, and S.V. Vladimirov: Phys. Plasmas **1**, 2176 (1994)

[5] S.I. Popel and V.N. Tsytovich: Contrib. Plasma Phys. **34**, 695 (1994)

[6] S.V. Vladimirov and S.I. Popel: Phys. Rev. E **51**, 2390 (1995)

[7] S.I. Popel: Kratk. Soobshch. Fiz. No. 7,8 **65** (1997) [Bulletin Lebedev Phys. Inst. No. 7,8 (1997)].

[8] S.I. Popel: *"Modulational Instability of Broad Spectra of Lower–Hybrid Waves"*. Fiz. Plazmy **24** (1998); Plasma Phys. Reports **24** (1998) *in press*

[9] S.V. Vladimirov, V.N. Tsytovich, S.I. Popel, and F.Kh. Khakimov: Modulational Interactions in Plasmas. Kluwer Academic Publishers, Dordrecht / Boston / London, 1995

[10] P.–O. Dovner, A.I. Eriksson, R. Boström, and B. Holback: Geophys. Res. Lett. **21**, 1827 (1994)

[11] A.I. Eriksson, B. Holback, P.–O. Dovner et al.: Geophys. Res. Lett. **21**, 1843 (1994)