NON-LOCAL THEORY OF EXCITATION OF LOWER HYBRID WAVES BY A DENSITY MODULATED ELECTRON BEAM IN A PLASMA CYLINDER

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Abstract

A density modulated electron beam propagating through a plasma cylinder drives electrostatic lower hybrid waves to instability via Cerenkov interaction. We have developed a non-local theory of this process. The earlier theories of the lower hybrid wave excitation are valid only for unmodulated electron beam. The growth rate γ of the lower hybrid instability increases with the modulation index Δ and is maximized for Δ=l. For $\Delta=0$, γ turns out to be $\sim 0.412\times10^{8}$ sec^{-1} .The growth rate scales as the one- third power of the beam density. As the modulation index increases, the growth rate of the instability increases and this implies the enhancement in the efficiency of excitation.

1. Introduction

Electrostatic lower hybrid waves [1] have been observed by modulating the ion beam at different frequencies. In this case the largest growth rate occurred when the modulation frequency was close to the most unstable wave frequency. Krafft *et al.* [2] have studied emission of whistler waves by a density modulated electron beam in a laboratory plasma and results have been compared to the excitation by loop antenna.

2. Instability analysis

Consider a cylindrical plasma column of radius a, equilibrium density n^{o}_{op} immersed in a static magnetic field $B_s\hat{z}$. The plasma is cold and collisionless. A density modulated electron beam with velocity $v_{ob}\hat{z}$, density $n^{o}_{ob}+n_s\exp[-i(\omega_0 t-k_{oz}z)]$ (where $n_s=n^{o}_{ob}\Delta$; Δ being a modulation index (its value ranges from 0 to 1) and radius r_o propagates through the plasma. The equilibrium is perturbed by an electrostatic perturbation with potential

$$\phi = \phi_0\exp[-i(\omega t-k_z z)]. \quad (1)$$

The response of plasma electrons to the perturbation is governed by the equations of motion and continuity, which on linearization yields density perturbation

$$n_{1e} = -n_{op}^{o}\frac{e}{m}[-\frac{\nabla_{\perp}^{2}\phi}{\omega^{2}-\omega_{c}^{2}}+\frac{k_{z}^{2}\phi}{\omega^{2}}] \ , \qquad (2)$$

where $-e, \omega_c$ are the electron charge and electron cyclotron frequency, respectively. The response of the plasma ions can be obtained from Eq.(2) by replacing -e, m, ω_c by e, m_i, ω_{ci}, respectively.

$$n_{1i} = n_{op}^{0}\frac{e}{m_i}[-\frac{\nabla_{\perp}^{2}\phi}{\omega^{2}-\omega_{ci}^{2}}+\frac{k_{z}^{2}\phi}{\omega^{2}}] \ . \qquad (3)$$

The response of beam electrons can be obtained by solving the fluid equations of motion and continuity and which on linearization gives beam density perturbation

$$n_{1b} = -\frac{n_s(\omega_0-k_{oz}v_{ob})}{\omega - k_z v_{ob}} - \frac{n_{ob}^{o} e \ k_z^2\phi}{m(\omega - k_z v_{ob})^2} \ . \qquad (4)$$

Using Eqs.(2), (3) and (4) in Poisson’s equation, we obtain

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+p_1^2\phi = -\frac{4\pi \ e \ n_{ob}^{o}\ \Delta(\omega_0-k_{oz}v_{ob})^2}{(1-\frac{\omega_p^2}{\omega^2-\omega_c^2}-\frac{\omega_{pi}^2}{\omega^2-\omega_{ci}^2})(\omega - k_z v_{ob})^2} -$$

$$\frac{4\pi \ n_{ob}^{o}\ e^2 \ k_z^2\phi}{m(1-\frac{\omega_p^2}{\omega^2-\omega_c^2}-\frac{\omega_{pi}^2}{\omega^2-\omega_{ci}^2})(\omega - k_z v_{ob})^2} \ , \qquad (5)$$

where $$p_1^2 = \frac{\left(\frac{\omega_p^2}{\omega^2} - 1\right)k_z^2}{\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}\right)} \; ; \quad \omega_p^2 = \frac{4\pi n_{op}^o e^2}{m}, \; \omega_{pi}^2 = \frac{4\pi n_{op}^o e^2}{m_i} \quad . \tag{6}$$

Now we attempt a solution of Eq. (5) and evaluate the growth rate of unstable mode in the beam plasma system using perturbation theory, we obtain the growth rate of the unstable mode

$$\gamma = \mathrm{Im}\,\delta = \left[\frac{a_1(a_1^2 - \omega_{ci}^2)}{2(a_1^2 - a_2^2)\left(1 + \frac{\omega_p^2}{\omega_c^2}\right)}\left(\frac{4\pi\, n_{ob}^o\, e\, \Delta(\omega_0 - k_{oz}v_{ob})^2 J_0(p_{1n}r_o)}{J_1^2(p_{1n}a)(p_{1n}^2 + k_z^2)A_n} + \frac{\omega_{pb}^2\, k_z^2\, J_0^2(p_{1n}r_o)}{J_1^2(p_{1n}a)(k_z^2 + p_{1n}^2)}\right)\right]^{1/3} \frac{\sqrt{3}}{2} \tag{7}$$

and the real frequency of the unstable wave in terms of beam voltage V_b

$$\omega_r = k_z\left(\frac{2eV_b}{m}\right)^{1/2} - \frac{1}{2}\left[\frac{a_1(a_1^2 - \omega_{ci}^2)}{2(a_1^2 - a_2^2)\left(1 + \frac{\omega_p^2}{\omega_c^2}\right)}\left(\frac{4\pi n_{ob}^o e\Delta(\omega_0 - k_{oz}v_{ob})^2 J_0(p_{1n}r_o)}{J_1^2(p_{1n}a)(p_{1n}^2 + k_z^2)A_n} + \frac{\omega_{pb}^2 k_z^2 J_0^2(p_{1n}r_o)}{J_1^2(p_{1n}a)(k_z^2 + p_{1n}^2)}\right)\right]^{1/3}, \tag{8}$$

where $a_1^2 = \omega_{LH}^2\left(1 + \frac{m_i}{m}\frac{k_z^2}{p_{1n}^2 + k_z^2}\right)$, $a_2^2 = \frac{\omega_{ci}^2}{1 + \frac{m}{m_i}\frac{p_{1n}^2 + k_z^2}{k_z^2}}$, $\omega_{pb}^2 = \frac{4\pi n_{ob}^o e^2}{m}$, and $A_n = \frac{\phi}{J_0(p_{1n}r)}$.

3. Results and discussion

Using Eq. (7) we have plotted in Fig. 1 the growth rate of the lower hybrid instability as a function of the modulation index for the following parameters: plasma density $n^{o}_{op}=10^{12}\ cm^{-3}$, beam density $n^{o}_{ob}=1x10^{8}\ cm^{-3}$, beam energy $E_b=300\ eV$, guide magnetic field $B_s=300$ Gauss, modulation frequency $\omega_0 \approx 5.65\times10^{8}$ rad/sec, axial wave number of the modulated beam $k_{oz}=0.54\ cm^{-1}$, unstable wave frequency $\omega=5.66x10^{8}$ rad./sec, axial wave number $k_z=0.53\ cm^{-1}$, ion plasma frequency $\omega_{pi}=2.028\times10^{8}$ rad/sec (argon plasma), radius of plasma cylinder $a=1.0$ cm, beam radius $r_o=0.5$ cm, radial distance $r=0.4$ cm and modulation index $\Delta=0.1$, 0.5, 0.9, 1.0. The growth rate increases with the modulation index and reaches maximum for Δ =l. For $\Delta=0$, γ turns out to be $\sim0.412x10^{8}\ sec^{-1}$. The real frequency of the unstable wave scales as one half-power of the beam voltage (cf. Eq. (8)).

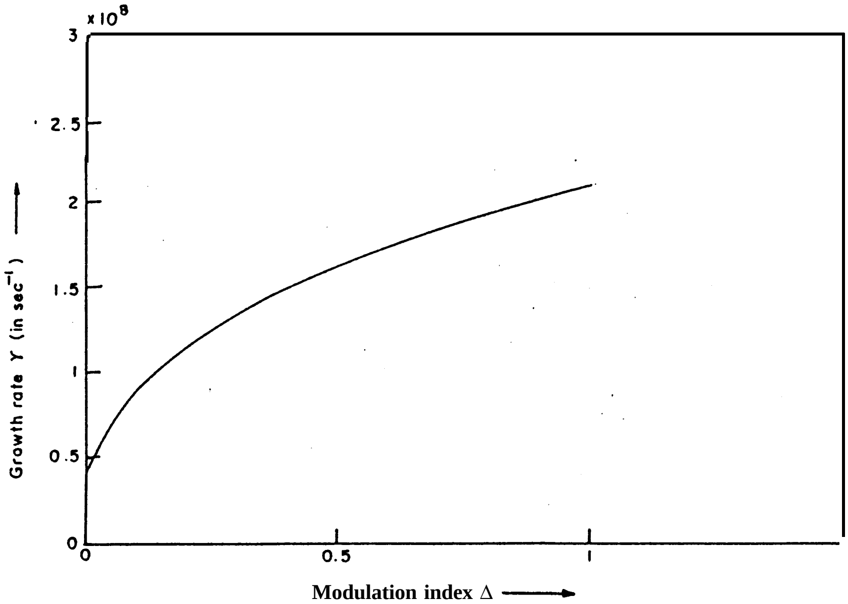

Fig.1 Growth rate of the lower hybrid instability (in sec^{-1}) as a function of modulation index Δ for $r_o=0.5$ cm and $r=0.4$ cm. The other parameters are given in the text.

References

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