ION-ION INELASTIC SCATTERING CROSS SECTIONS OF LASER INTEREST

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Abstract

The first purpose of this work is to report the ion-impact excitation cross sections in Al^{10+} ion. The generalized oscillator strength has been calculated and numerical results obtained for Al Li-like ion, $1s^2$ *nl* configuration The approximation used for atomic data calculation is presented.

1. Introduction

We extend the procedure discussed by E.J. McGuire [1] to calculate the ion-impact excitation cross sections in Li-like ions, $1s^2nl$ configuration. In the intermediate and high centre-of mass energy region where the impact phenomena are dominated by elastic scattering of the projectile with the excitation of the target ion, our interest is mainly directed to the following processes:

$$1s^2\,2l + 1s^2\,2l' \rightarrow 1s^2\,2l + 1s^2\,nl'' \qquad (1)$$

2. Method of calculation

The concept of a generalized oscillator strength (GOS), $f_n(K)$, was first defined by Bethe [2] to clarify the relationship between excitation by electron-impact and optical absorption measurements. $f_n(K)$ is an even function of Ka_0, and for the excitation by electron impact from the initial state $\langle 0|$ to the final state $|n\rangle$ of energies E_0 and E_n, respectively, is defined by:

$$f_{0n}(K) = \frac{8\pi^2 m(E_n - E_0)}{h^2K^2}\ \left|\langle 0|\sum_s exp(i\mathbf{K}\mathbf{r}_s)|n\rangle\right|^2 \qquad (2)$$

In the equation (2) $\mathbf{r}_s$ is the position vector of the s-th electron in the scatterer, and **K** is the vector momentum change of the colliding electron. The contribution of different excitation transition mode Eλ (λ=0,1,2 for electric-dipole, quadrupole and octupole transition, respectively) to the GOS can be obtained by expanding the exponential in powers of $(Ka_0)^2$ for small (Ka_0), [3]:

$$f_n(K) = \sum_{\lambda=0}^{\infty} (Ka_0)^{2\lambda} f_{0n}^{(\lambda)}/\lambda! \quad (3)$$

with

$$f_{0n}^{(\lambda)}/\lambda! = (\lambda!)^{-1} [d/d(Ka_0)^2]^{\lambda} f_{0n}(K)\Big|_{K=0} = \frac{E_n}{Ryd} \sum_{\mu=1}^{2\lambda+1} \frac{(-1)^{\lambda-\mu+1}}{\mu!(2\lambda+2-\mu)!} \Phi_n^{(\mu)} \Phi_n^{(2\lambda+2-\mu)*} \quad (4)$$

$$\text{where:} \quad \Phi_n^{(\mu)} = \frac{1}{a_0^{\mu}} \sum_j \int \Psi_n^* z_j^{\mu} \Psi_0 \, d\boldsymbol{r}_1 \ldots d\boldsymbol{r}_N \quad \text{and} \quad z_j = \left(\boldsymbol{K r}_j\right)/ K.$$

The first expansion coefficient $f_{0n}^{(0)}$ is the optical oscillator strength f_n. For optical allowed transitions $\Phi_n^{(\mu)}$ vanishes for all even μ, and for optically forbidden transitions it vanishes either for all odd or all even μ, depending on the parity of the excited states.

In the center-of mass system we consider the projectile an ion Al^{10+} of mass M_1, nuclear charge $Z_1 e$ and carrying N_1 bound electrons described by the electronic eigenstate $|n_0\rangle$. The other ion, also Al^{10+}, acting as target, is rising from the state $|m_0\rangle$ to the state $|m\rangle$ by the impact. Within the range of validity of the first approximation of Born's theory, the cross section and energy loss for ion-ion scattering is obtained from integrals over the squared matrix elements of the interaction. We wrote the matrix element for elastic scattering of the projectile, 1, and the excitation of target, 2, in momentum transfer variable of the projectile:

$$M(n_0\, m_0, n_0\, m) = -(1 - \delta_{m\,0\,m})\, \delta_{n\,0\,n}\, [Z_1 - F_1(k^2)]\, (4\pi/k^2)\, \langle m_0 \,|\, \left(\sum_j exp\, [\, i\, \boldsymbol{k}\, \boldsymbol{r}_{2j}]\right) |\, m \rangle \quad (5)$$

where $F_1(k^2)$ is the atomic form factor. We note by $\boldsymbol{r}_{ij}$ the position vector of the electron j of the projectile and target with respect to the correspondingly centres of mass, respectively, i = 1,2. For $|k| \to 0$, $F_1(k^2) \to Z_1 - z_1$, the number of electrons on the ion. Finally, the cross sections for the processes of type (1) were evaluated as :

$$\sigma(k,E_r / M_r) = [\, 4\pi a_0^2/(mE_r /M_r)] \int_{K^2_{min}}^{K^2_{max}} dk^2 \, \left| F_1(k^2) \langle m_0 | \left(\sum_i exp\, [\, -i\, \boldsymbol{k r}_i\,]\right) | m\rangle \right|^2 / k^4 \quad (6)$$

In the centre of mass system the reduced mass M_r and energy E_r are given by: $M_r = (M_1 M_2)/ (M_1 + M_2)$ and $E_r = (1/2)\, M_r\, (v_L)^2$, respectively, where v_L is the projectile initial velocity in the laboratory frame. With the energies in Rydbergs and lengths in Bohr radii, the cross section in the equation (6) is given in cm^2.

3. Atomic data calculation

The $1s^2 2l$, $1s^2 3l$, $1s^2 4l$ and $1s^2 5l$ states of Al^{10+} have been carefully considered. The target wave functions were represented by configuration interaction (C.I.) wavefunctions where radial part of each orbital is a sum of Slater-type functions. Fifteen orthogonal one-electron orbitals *1s, 2s, 2p, 3s, 3p, 3d, 4s, 4p, 4d, 4f, 5s, 5p, 5d, 5f, 5g* were used in our work. The large C.I.

computer code developed by Hibbert [5] was used in all of our bound state calculations. Using these C.I. wavefunctions we were able to calculate the energy difference, ΔE, and transition linestrengths, $S^{(\lambda)}$, λ=0, 1, 2 (dipole, quadrupole, octupole) between all 14 states as output from RMATRIX II [6]. Correspondingly matrix elements for atomic data calculations have been evaluated using the non-relativistic LS coupling code. To carry out finestructure linestrengths we used the properties of tensor operators and 6-j symbols [7].

4. Results

The equations (3) - (6) have been applied for a large number of electric - dipole, quadrupole and octupole transitions to calculate the generalized oscillator strength for Al^{10+} ion. The following results have been obtained:

the excited state	*the total generalized oscillator strength*
$1s^2\,2s_{1/2}$	$0.57865+0.142392155\ K^2+0.0012864\ K^4$
$1s^2\,2p_{1/2}$	$0.822528+2.466649^{-02}\ K^2+3.5472154^{-04}\ K^4$
$1s^2\,2p_{3/2}$	$0.870424+0.15499535\ K^2+0.010930757\ K^4$

where the values as 1.5^{-04} signify 1.5×10^{-04}.

The figures 1 and 2 present our results on the calculated cross section for Al^{10+}impact excitation of $1s^2\,2s_{1/2}$ and $1s^2\,2p_{1/2}$ states, respectively, as function of the projectile velocity.

References

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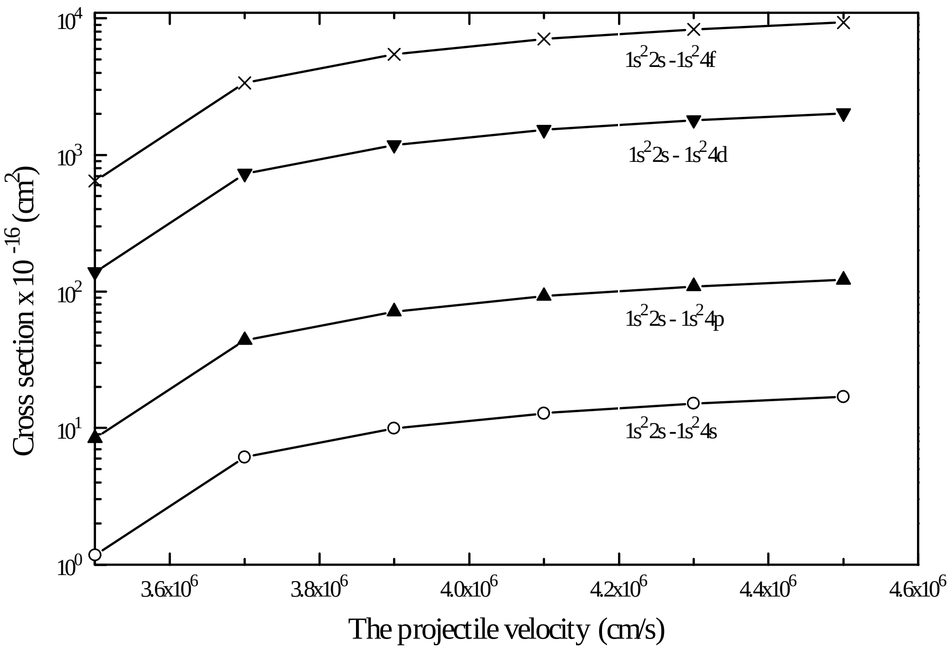

Fig.1. The cross section (cm^2) for Al^{10+} impact excitation of $1s^2 2s\,(^2S)$ state in Al^{10+} ion, plotted versus the projectile velocity (cm/s)

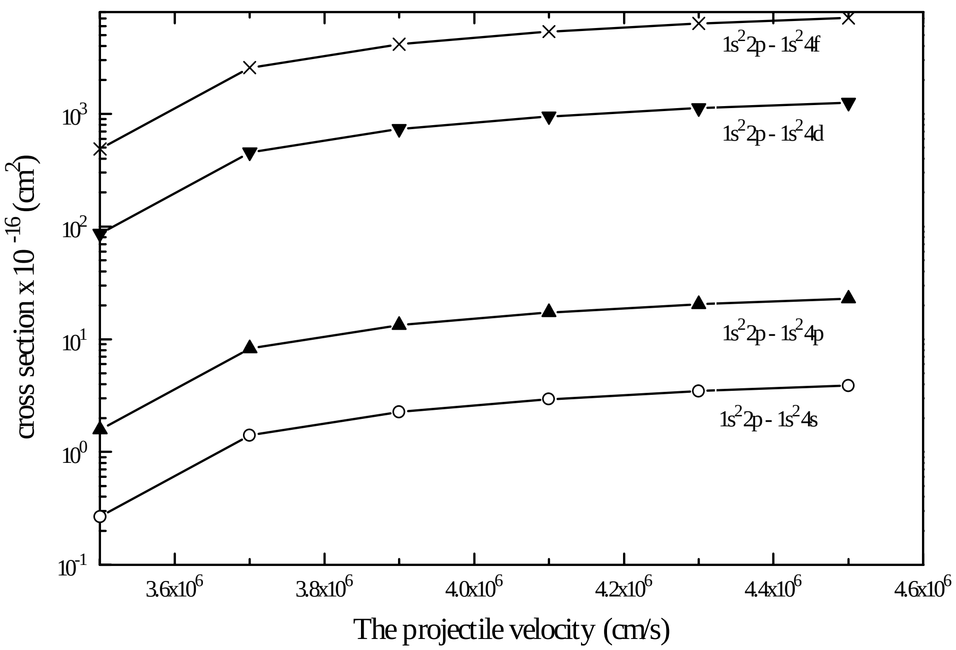

Fig.2. The cross section (cm^2) for Al^{10+} impact excitation of $1s^2 2p(^2P)$ state in Al^{10+} ion, as function of the projectile velocity (cm/s).