DYNAMICS OF PARTICLES IN AN ANNULAR PURE ELECTRON PLASMA UNDER STATIC MAGNETIC PERTURBATIONS

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1. Introduction

In magneto-electrostatic (Penning-Malmberg) traps the radial confinement of charged particles is provided by an axial uniform magnetic field and the axial confinement by electrostatic potentials applied to azimuthally symmetric electrodes.

Collisionless radial transport of particles can be driven by electrostatic perturbations and/or by perturbations of the confining static magnetic field. To characterize the confinement properties of such devices, the dynamics of a single particle under the action of external and space charge fields has been analyzed in two simple cases.

At first, the radial drift driven by a current flowing through a coaxial cylindrical conductor, which provides an azimuthal component of the magnetic field, has been considered. In this case the system remains azimuthally symmetric, and is characterized by the conservation of the canonical angular momentum. Then, the case of static magnetic perturbations (field errors) has been addressed. Here, instabilities in the particle motion can arise, due to the presence of resonances in the unperturbed motion (stochastic diffusion), and a degradation of the radial confinement of the particles has to be expected.

2. Annular plasma with an added poloidal field

The case of a pure electron plasma contained radially between two coaxial conducting cylinders is considered here. The external cylinder is grounded, while the internal conductor provides an externally controllable radial electric field. The volume available for confinement is then annular.

In the standard cylindrical Penning-Malmberg trap [1], the magnetic field is assumed to be constant and uniform, $\mathbf{B} = B_0\mathbf{e}_z$, and perfectly aligned with the axis of the trap, which has been assumed to be the z-axis of a cylindrical coordinate system (r, θ, z), with the origin at the center of the trap. The drift motion of the guiding centers (only the electric drift $E_r \times B_z$ is present) lies within cylindrical flux surfaces, with an azimuthal velocity which depends in general on the position along z. A current flowing through the central conductor provides an azimuthal component of the magnetic field. The combined poloidal and axial magnetic fields produce field lines which spiral at constant radius. Plasma loss along these field lines is prevented by the applied electrostatic potential. The addition of a poloidal magnetic field creates a radial drift which comes from the $E_z \times B_\theta$ term. Since the sign of E_z changes at the midplane of the trap, the radial drift is outward during one half of the axial bounce period, and inward during the other half. The magnetic field is written as $\mathbf{B} = B_0\mathbf{e}_z + (2I/cr)\mathbf{e}_\theta$, where I represents the current flowing in the central conductor, and c is the speed of light. The guiding center drift equations can be written in Hamiltonian form. The Hamiltonian function is simply the total energy of the guiding center, i.e., the sum of its kinetic and electrostatic energy:

$$H = mv_\parallel^2/2 + \mu B - e\Phi_e \tag{1}$$

(the case of electrons has been explicitly considered). Here μ is the magnetic moment, and $\Phi_e = \Phi_e(r, z)$ is the electrostatic potential. The Hamiltonian has two degrees of freedom, $H = H(\psi, \theta_p, \rho, \chi)$, where the canonical variables are defined as $\psi = -(eB_0/2c)r^2$, $\theta_p =$

$\theta - (2I/cr^2)z$, $\rho = mv_\parallel/B$, and $\chi = B_0 z + (2I/c)\theta$. The drift surface is given through the conservation of the canonical angular momentum of the particle by

$$r^2 = (4mI/eB_0)\,(v_\parallel/B) + r_b^2, \tag{2}$$

where r_b represents the radial coordinate of the tips of the banana orbits (the banana center drifts then in the azimuthal direction, with a velocity, which is in general a function of the z coordinate). In the adiabatic limit, where the axial bounce frequency, ω_b, is much larger than the azimuthal rotation frequency, ω_d, the bounce-averaged drift is within a (cylindrical) flux surface (it results $v_\parallel = v_\parallel(r, s)$, where s is the coordinate along the magnetic field lines).

These results are independent of the explicit form of the electrostatic potential, and are therefore valid for a general external potential (and an arbitrary value of the potential on the internal conductor), and for a plasma with an arbitrary profile of the density, provided that the system remains azimuthally symmetric.

3. Magnetic field error in a Penning-Malmberg trap

In the absence of field errors the system is azimuthally symmetric, and the radial transport is constrained by the conservation of the total canonical angular momentum, which, in the low density case, is proportional to the mean-square radius of the plasma [2]. Processes that result in a net torque can degrade the confinement and cause plasma expansion and loss, as it has been verified in experiments on electron-neutral collisional transport at high background pressures [1]. At low background pressures, on the contrary, the plasma is lost by an anomalous transport mechanism (independent of pressure). The most likely cause of it is the presence of small azimuthally asymmetric magnetostatic or electrostatic field errors, coupling angular momentum into the plasma. In particular, magnetostatic errors could arise either from irregularities in the main solenoid, or from induced magnetization of the stainless-steel of the supports of the containment cylinders. On the other hand, magnetic perturbations could be deliberately induced from outside, in order to study their effects on the radial particle transport under controlled conditions.

The effect of a magnetic field error on the single particle trajectories has been studied here in a simple case. The analysis has been focused on an irrotational, z-independent field perturbation, characterized by the magnetic potential $\Phi_m = \sum_l \epsilon_l\, r^l\, \cos(l\theta)$. The cylindrical components of the perturbed magnetic field are given by $B_r = \sum_l \epsilon_l\, l\, r^{l-1}\, \cos(l\theta)$, and $B_\theta = -\sum_l \epsilon_l\, l\, r^{l-1}\, \sin(l\theta)$. Note that the case with only the $l = 1$ Fourier harmonic simply corresponds to a misalignment of the magnetic field with the axis of the trap, due, e.g., to the effect of the Earth's magnetic field.

In the central region of the plasma, Φ_e is approximately quadratic, and can be written as [2] $\Phi_e(r, z) = -(m\omega_z^2/2e)(z^2 - r^2/2) - (m\omega_p^2/4e)r^2$, where ω_z is the axial bounce frequency, and ω_p is the electron plasma frequency (for simplicity, a constant axial bounce frequency has been considered, and a simple model of uniform plasma density has been used).

A Hamiltonian analysis similar to the previous case can be carried on. ρ is of course the same, while $\chi = B_0 z + \sum_l \epsilon_l r^l \cos(l\theta)$. ψ and θ_p have rather complicated expressions. For the simple case of a misaligned magnetic field, it results $\psi = -(eB/2c)[r^2 + (z^2 - r^2\cos^2\theta)\sin^2\xi - rz\sin(2\xi)\cos\theta]$, and $\theta_p = \arctan[r\sin\theta/(r\cos\theta\cos\xi - z\sin\xi)]$. Here ξ denotes the angle between the direction of the (uniform) magnetic field and the axis of the trap. To obtain analytical results in the general case, the system has been analyzed at the first order in the perturbation. At this level of approximation, the guiding center equations can be described by $H = H_0(P_\theta, J_b) + H_1(\theta, P_\theta, \phi_b, J_b)$, with

$$H = J_b - \omega_d P_\theta + \sum_l \alpha_l \sqrt{J_b}(-P_\theta)^{l/2}[\omega_c \, \sin(l\theta)\sin(\phi_b) - \cos(l\theta)\cos(\phi_b)]. \qquad (3)$$

The Hamiltonian is written in terms of the canonical action-angle variables of the unperturbed system. $P_\theta = -\omega_c \, r^2/2$ is the canonical angular momentum of the electron, ω_c being the electron cyclotron frequency, J_b is the action relevant to the bounce motion along z, and ϕ_b is the corresponding phase. For the assumed magnetic perturbation, the effect of the ∇B- and the curvature drift are of the second order in the perturbation, and have therefore been neglected. $\omega_d = (\omega_p^2 - 1)/2\omega_c$ represents the frequency of the azimuthal rotation ($\simeq$ low density diocotron frequency). Adimensional quantities are used here. The time is normalized over ω_z^{-1}, the velocities over the thermal velocity $v_{\rm th} = (T_e/m_e)^{1/2}$, T_e being the plasma temperature, the magnetic field strength over B_0, the lenghts over v_{th}/ω_z, and the energy over T_e. It results: $\alpha_l = (\epsilon_l/B_0) \, l \, (v_{\rm th}/\omega_z)^{l-1}$.

The unperturbed Hamiltonian is characterized by two frequencies, $\partial H_0/\partial J_b = 1$ (in the chosen units), i.e., the bounce frequency, and $\partial H_0/\partial P_\theta = -\omega_d$, i.e., the frequency of the azimuthal drift. In a pure electron plasma, the radial electric field, and hence ω_d, is set by the electron density. The bounce frequency depends in general on the radial position, but it can be externally adjusted by varying the confining potential, or by changing the position of the plug electrodes (the cylindrical containment conductor is usually divided into different sectors, at which different potentials can be applied).

If the perturbation is sufficiently high, stochastic diffusion can be present, as it has been verified by the Poincaré surface of section method (see. Figs. 1-3). Note that in any case the region of stochastic diffusion is limited, since its radial extension is constrained by the conservation of the Hamiltonian. The maximum allowed radius is $r_{\rm max} = (2H/\omega_d\omega_c)^{1/2}$. At the first order in the perturbation, it is possible to construct $(P_\theta - \theta)$ invariant curves in the surface of section $\phi_b = {\rm const}$. Using the method of global removal of resonances [3], they can be written as

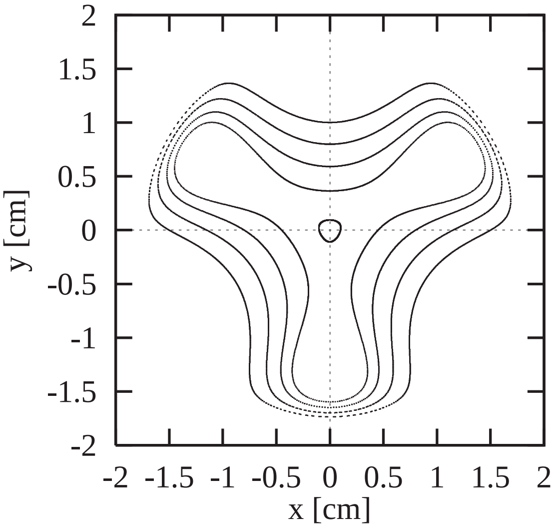
$$\mathcal{J}(P_\theta, \theta) = P_\theta \prod_l (l\omega_d + 1)(l\omega_d - 1) + \sum_l [a_-^l \cos(l\theta - \phi_b) + a_+^l \cos(l\theta + \phi_b)] = {\rm const}, \quad (4)$$

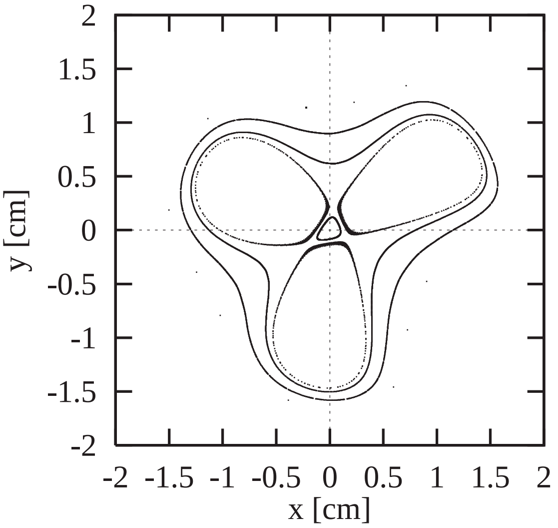
where $a_\pm^l = \pm(\alpha_l/2)\sqrt{J_b}(-P_\theta)^{l/2} l(\omega_c \pm 1)(l\omega_d \pm 1)\prod_{n\neq l}(n^2\omega_d^2 - 1)$. In these expressions J_b has to be evaluated using the unperturbed Hamiltonian, $J_b = H_0 + \omega_d P_\theta$. A plot of the level curves of the invariant $\mathcal{J}$ in the surface of section $\phi_b = \pi/2$ is shown in Fig. 4. The agreement with the Poincaré map shown in Fig. 2 is quite good. The unperturbed system is degenerate owing to r^2-, z^2-dependence of the electrostatic potential. The analytical treatment presented here can be extended to include a more general radial profile of the electron density, thus removing the degeneracy.

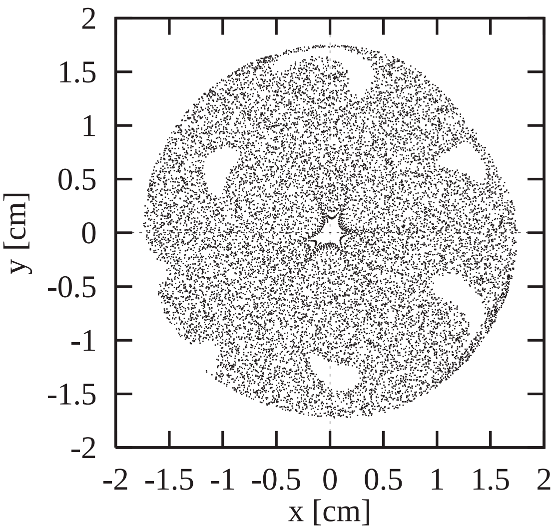
4. Conclusions

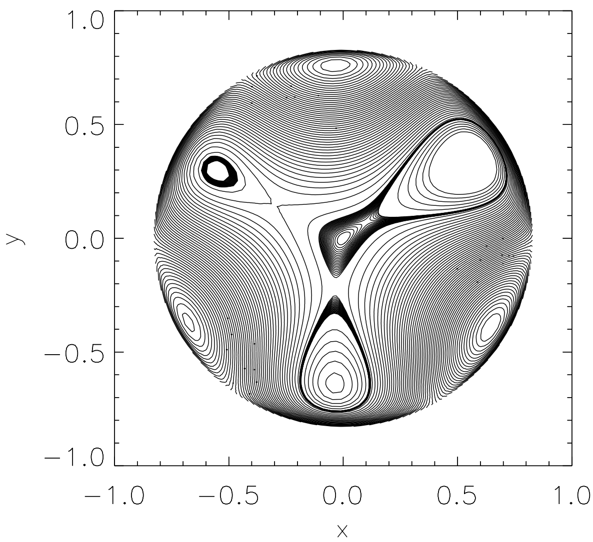
An annular version of the Penning-Malmberg trap with an added poloidal magnetic field has been considered. Experiments in this device could allow measurements of the transport coefficients. The electron density is determined by dumping the electrons onto a phosphor-coated screen and analyzing the resultant image. Experimental transport fluxes could then be calculated from the time evolution of these images. The ratio of the axial to poloidal field is a parameter, which can be used to adjust the width of the bananas, and hence in principle the ratio of neoclassical to classical transport (in analogy with ι in fusion devices).

When non-axisymmetric magnetic perturbations are applied to the system, instabilities in the particle motion can arise. The dominant contribution to the radial drift comes from particles which satisfy a resonant condition between their axial bounce motion and their rotational motion. The analysis of the numerical simulations indicates radial excursions of the order of the allowed range in some tens of bounce periods (time scale much lower than the collisional). Magnetic perturbations with a single or few l values could be produced by current currying external coils, lying between the vacuum chamber and the main solenoid. Primary resonance conditions could be achieved by changing the plasma density and/or varying the external confining potential.









Figs. 1, 2, 3: Poincaré surfaces of section $\phi_b = \pi/2$ (in Cartesian coordinates) for the Hamiltonian (3); $H = 100$, $\omega_d = 3.07$, 5 initial conditions. $\alpha_3 = 0.005$ (Fig. 1), $\alpha_2 = 0.01, \alpha_3 = 0.005$ (Fig. 2), and $\alpha_2 = 0.01, \alpha_3 = 0.015$ (Fig. 3).

Fig. 4: Level curves of the invariant $\mathcal{J}$ (see Eq. (4)) for $\phi_b = \pi/2$ in normalized Cartesian coordinates, for the same parameters as in Fig. 2.

References

[1] J.S. de Grassie and J.H. Malmberg: Phys. Fluids **23**, 63 (1980)
[2] T.M. O'Neil: Physica Scripta **T59**, 341 (1995)
[3] A.J. Lichtenberg and M.A. Lieberman: Regular and Stochastic Motion. Springer-Verlag, New York (1983)