ION–NEUTRAL COLLISION EFFECTS ON ALFVÉN SURFACE WAVES

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Abstract

In this paper we derive the dispersion equation for the surface waves in partially ionized plasma along a plasma-plasma interface and show that ion-neutral collisions can cause a drastic change in the Alfvén surface waves propagation characteristics. The effect of this on resonant absorption of surface waves is discussed briefly.

1. Introduction

It is well established that in a magnetized partially ionized plasma, the dispersion relation of the shear Alfvén waves is strongly influenced by the ion-neutral collisions [1,2]. The drastic change in the dispersion of Alfvén waves arises mainly due to the fact that in a partially ionized medium the Alfvén speed can have different time scales depending on the strength of the coupling of the neutral fluid and the charged fluid. The same criterion can arise in the case of Alfvén surface waves which can propagate along the discontinuities in partially ionized plasmas. In the case of inhomogeneous plasmas the study becomes important especially in understanding of resonant absorption of Alfvén waves. The aim of this paper is therefore to discuss the basic characteristic features of Alfvén surface waves in partially ionized plasmas.

2. Basic Equations

We consider the ideal MHD model for the plasma with interaction between the charged and neutral components modeled adequately by simple drag terms in the momentum equations. After linearization of MHD equations and considering incompressible perturbations in the y-z plane as $f(x)e^{i(\ell y+kz)}e^{\omega t}$ we get the following equations:

$$\left[\omega^2+\frac{\omega_A^2(x)}{\left(\dfrac{\nu_{in}}{\omega+\nu_{ni}}+1\right)}\right]\nabla^2 v_{ix}+\frac{(\omega_A^2(x))'}{\left(\dfrac{\nu_{in}}{\omega+\nu_{ni}}+1\right)}v'_{ix}=0 \tag{1}$$

where $\omega_A^2(x)=V_A^2(x)k^2=B_o^2x^2/4\pi\rho_{io}$ and $V_A(x)$ is the local Alfvén speed. $\rho,\overline{v},p$ are density, velocity and pressure respectively. Suffix 'i' stands for ions and 'n' stands for neutral particles. ν_{in} is the ion-neutral and ν_{ni} is neutral-ion collission frequency respectively. Momentum conservation requires, $\dfrac{\rho_n}{\rho_i}=\dfrac{\nu_{in}}{\nu_{ni}}\equiv\alpha\ (\mathrm{say})$. It is interesting to note that if we define modified

Alfvén velocity $\omega_{A'}(x)$ as

$$\omega_{A'}(x) = \frac{\omega_A(x)}{\left(\frac{\nu_{in}}{\omega + \nu_{ni}} + 1\right)^{1/2}} \tag{2}$$

then Eq. (1) becomes identical with the equation governing the propagation of Alfvén waves in fully ionized plasma with $\omega_A(x)$ replaced by $\omega_{A'}(x)$ (see for example Uberoi [3]).

3. Dispersion Relation

To study surface waves, let us consider plasma-plasma interface at $x = 0$.

Applying the boundary conditions as continuity of pressure $\tilde{p}$, neutral pressure p_{n1}, normal component of ion velocity v_{ix} and neutral velocity v_{nx} across the surface of discontinuity, we get the dispersion relation for the surface waves which can be written as:

$$\rho_{io_1}\left[\omega^2\left(1 + \frac{\nu_{in_1}}{\omega + \nu_{ni_1}}\right) + \omega_{A_1}^2\right] + \rho_{io_2}\left[\omega^2\left(1 + \frac{\nu_{in_2}}{\omega + \nu_{ni_2}}\right) + \omega_{A_2}^2\right] = 0 \tag{3}$$

4. Discussion of the Dispersion Relation

We shall now discuss Eq. (3) for special cases: namely the strongly coupled plasmas, where the neutral and charged fluid move together and weakly coupled plasmas, where the charged medium can behave as it is without friction.

For the case of strongly coupled plasmas, we take $\omega \ll \nu_{ni_1} < \nu_{ni_2}$, in that case (3) gives for $i\omega \equiv \omega$

$$\omega^2 = \frac{\rho_{io1}\omega_{A_1}^2 + \rho_{io2}\omega_{A_2}^2}{\rho_{io1}(1+\alpha_1) + \rho_{io2}(1+\alpha_2)} = \frac{\omega_s^2}{A} \tag{4}$$

where ω_s is the Alfvén surface wave frequency for fully ionized plasmas [3] and

$$A \equiv (1 + \rho_n/\rho)$$

where $\rho_n = \rho_{n1} + \rho_{n2}$, $\rho = \rho_{i1} + \rho_{i2}$.

Considering $B_{o1} = B_{o2}$ and $\rho_{io2} = 0$,

$$\omega_{sn} = \frac{\sqrt{2}V_{A_1}k}{(1+\alpha_1)^{1/2}} < \omega_s. \tag{5}$$

For the case when neutrals and charged fluid is weakly coupled Eq. (3) for $\omega \gg \nu_{ni_2} > \nu_{ni_1}$ gives,

$$\omega^2 \approx \omega_s^2 \tag{6}$$

the surface wave frequency is the same as in the case of fully ionized plasmas.

For the case of strong coupling: $\omega^2 \ll \nu_{ni_1}\nu_{ni_2}$, The damping of Alfvén surface waves by collisions is given as:

$$\omega_I = \frac{-\omega_s^2(\nu_{ni_1} + \nu_{ni_2})}{2(\omega_s^2 + \nu_{ni_1}\nu_{ni_2}A)} \tag{7}$$

The damping depends on both collision frequencies and the strength of the magnetic field.

There is no surface wave propagation when

$$\omega_s^2 > \frac{4\nu_{ni_1}^2\nu_{ni_2}^2 A}{(\nu_{ni_1}\nu_{ni_2})^2}$$

For the case of weak coupling : $\omega^2 \gg \nu_{ni_1}\nu_{ni_2}$ There is no wave propagation when

$$\omega_s^2 < \frac{1}{4}\left[(\nu_{ni_1} + \nu_{ni_2} + \omega_\nu)^2 - 4A\nu_{ni_1}\nu_{ni_2}\right]$$

The damping frequency is given as

$$\omega_I = \frac{1}{2}(\nu_{ni_1} + \nu_{ni_2} + \omega_\nu). \tag{8}$$

We note that for weakly coupled plasma-neutral systems, the damping involves no Alfvén velocity.

5. Conclusion

From the dispersion equation we find that for the case when ion-neutral coupling is weak the wave propagates along the interface with the natural frequency of Alfvén surface wave in the charged medium without friction. When coupling is strong this frequency is determined by mass densities of both ions and neutrals in both the media. When ionization fraction is low these two frequencies can differ by several orders of magnitude. The damping of surface waves due to ion-neutral collisions can be very small in case of strong coupling. For weak coupling this damping can become large to large due to collision frequencies.

The variation of surface wave frequency with wave number k also shows interesting feature not present in the case of fully ionized system. The frequency shows a maximum value in the case of strongly coupled case before decreasing and then approaches the cut-off wave-number k_{c_1} above which it does not propagate (Fig. 1). Similarly for weakly-coupled plasmas there is a cut-off wave number k_{c_2} below which the surface wave cannot propagate (Fig. 2). But in the later case no maximum value of k is shown, ω_s increases with increase in k. Fig. 3 and Fig. 4 give the values of k_{c_1} and k_{c_2} varying with ν_{ni_1}/ν_{ni_2} respectively. k_{c_1} increases with the increase in collision frequency while k_{c_2} decreases with increase in the collision frequency and with the increase in fractional ionization factor. Finally, we like to mention that Alfvén wave resonances, as seen from Eq. (1) occur at $\omega^2 = \omega_{A'}^2(x)$. We like to remark that for strongly coupled plasmas, the resonant damping will be less than that in the case of fully ionized plasmas as surface wave frequency $\omega_{sn} < \omega_s$. Frictional damping in this case can become very small for large collisional frequencies, hence the resonant damping of surface wave can be important. However, in the case of weakly coupled plasma-neutral system the frictional damping is large for high collision frequencies and so the resonant damping may play a less important role.

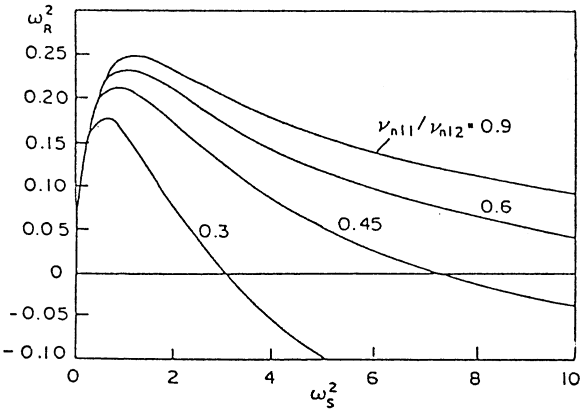

Figure 1. Dispersion curve showing variation of ω_R^2 with ω_s^2 for strongly coupled case. $\rho_{i1}/\rho_{i2} = 0.4$, $\alpha_1 = 0.2$, $\alpha_2 = 0.25$.

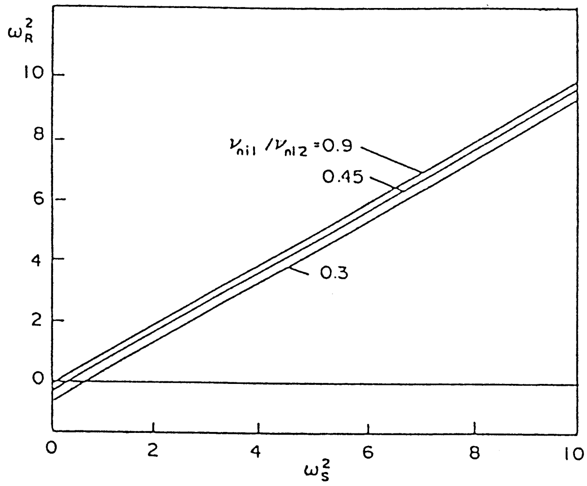

Figure 2. Dispersion curve between ω_R^2 and ω_s^2 for weakly coupled case. $\rho_{i1}/\rho_{i2} = 0.4$, $\alpha_1 = 0.2$, $\alpha_2 = 0.25$.

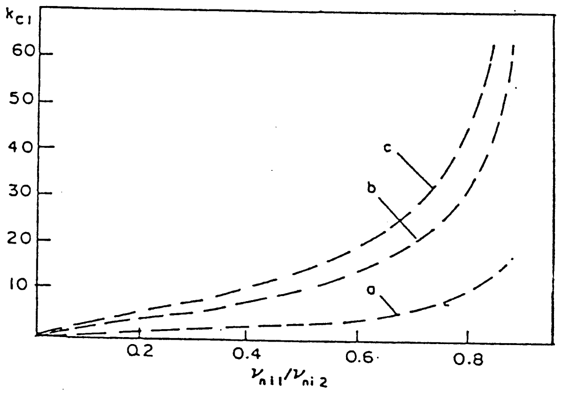

Figure 3. Variation of cut-off frequency (k_{c_1}) with ν_{ni_1}/ν_{ni_2} for strongly coupled case. $\rho_{i1}/\rho_{i2} = 0.4$, (a) $\alpha_1 = 0.2$, $\alpha_2 = 0.25$, (b) $\alpha_1 = 10, \alpha_2 = 12.5$, (c) $\alpha_1 = 20$, $\alpha_2 = 25$.

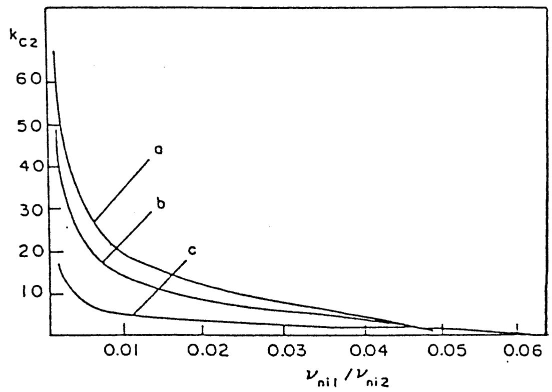

Figure 4. Variation of cut-off frequency (k_{c2}) with ν_{ni_1}/ν_{ni_2} for weakly coupled case. $\rho_{i1}/\rho_{i2} = 0.4$, (a) $\alpha_1 = 0.2$, $\alpha_2 = 0.25$, (b) $\alpha_1 = 10, \alpha_2 = 12.5$, (c) $\alpha_1 = 20$, $\alpha_2 = 25$.

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