

AMPLITUDE COLLAPSE OF NONLINEAR DOUBLE LAYER OSCILLATIONS

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Abstract

The negative differential resistance of a strong electric double layer in connection with an external inductive current circuit can give rise to large, self excited oscillations [1]. Here an experimental investigations of these oscillations at low oscillation frequencies and weak nonlinearity is presented. The oscillation exists only in a finite interval of dc double layer voltages, closely connected to the interval of negative damping. The amplitude characteristics show a transition from continuous profiles, similar to that of the classical van der Pol oscillator, to characteristics containing a hysteresis. Their shape and the appearance of the hysteresis phenomenon can quantitatively be explained from asymptotic solutions of the circuit equation, which is a generalized van der Pol equation with a small nonlinear damping term. The material presented here has been published in Ref. [2].

1. Experimental device and the current circuit

The triple plasma machine maintaining a plasma column of 3 cm diameter and 60 cm length (Fig. 1) is described elsewhere [1]. When a steady voltage drop U , exceeding $k_B T_e / e \approx 8$ V is applied between the source chamber walls, a strong electric double layer forms in the central chamber, near aperture A_1 . Neglecting the resistive voltage drops across R and r , it follows for the voltage $U(t)$ between the source anodes (Fig. 1)

$$LC \frac{d^2}{dt^2} U + RC \frac{d}{dt} U + L \frac{d}{dt} I + U = U_{dc}. \quad (1)$$

R is the resistance of the inductor. Introducing the static current-voltage characteristic $I(U)$ and the normalized conductivity of the double layer $g(U)$ we write

$$\frac{d}{dt} I = I'(U) \frac{d}{dt} U = \frac{g(U)}{R_m} \frac{d}{dt} U \quad (2)$$

where the prime means the derivative with respect to U . $I'(U)$ is negative in a certain region and it assumes a minimum value, defined as $1/R_m$ at $U = U_m$, so that $g(U_m) = -1$ (Fig. 1). Introducing $V = U - U_{dc}$ and normalizing time by the resonance frequency of the external circuit $f_0 = \omega_0 / 2\pi = 1 / (2\pi\sqrt{LC})$ it follows

$$\frac{d^2}{d\tau^2} V + V = -\varepsilon [g(V + U_{dc}) + D] \frac{d}{d\tau} V, \text{ where} \quad (3)$$

$$\varepsilon = \frac{1}{R_m} \sqrt{\frac{L}{C}} \quad \text{and} \quad D = \frac{R}{R_m \varepsilon^2} = \frac{RR_m C}{L} \quad (4)$$

are the nonlinear coupling coefficient and the damping coefficient, respectively. Although we have $R \ll R_m$ the damping D is important when ε^2 is of the order R/R_m . When $V + U_{dc}$

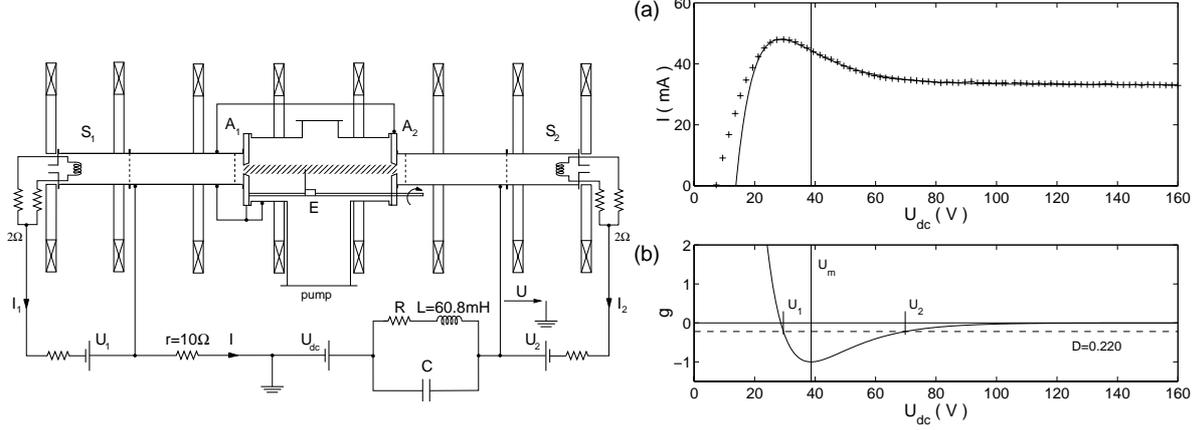


Figure 1. LEFT: Experimental setup: The plasma column in the central chamber is maintained by plasma diffusing from the sources S_1 and S_2 . plasma parameters: $n_e = 2 \times 10^{15} \text{m}^{-3}$, $T_e = 8 \text{eV}$, $\omega_{ce} = 2.3 \times 10^9 \text{rad/s}$, $\omega_{pe} = 2.5 \times 10^9 \text{rad/s}$, $\omega_{pi} = 9.3 \times 10^6 \text{rad/s}$. R is the resistance of the inductor. RIGHT: (a) Current-voltage characteristic $I(U_{dc})$ (+) and least square fit (solid) described in section 2. (b) Differential conductance $g(U_{dc})$ according Eq. (5). The dashed line gives the zero level of $g(U_{dc}) + D$ for $D = 0.220$. The oscillation is self-excited in the interval (U_1, U_2) .

is large, the damping is important for any value of ε since g vanishes for large values of its argument. Equation (3) is a generalized van der Pol equation. It can be reduced to the classical van der Pol equation only if $U_{dc} = U_m$ and D is close to unity so that $g(V + U_m)$ can be approximated by a parabola in V . In this case the amplitude characteristic is given by Eq. (10) below.

2. Experiments

A kinetic model of the current-voltage characteristic based of the existence of a current limiting potential minimum between the double layer and S_1 has been presented in [3]. $g(U)$ is here instead approximated by a least square fit to the measured current-voltage characteristic (Fig. 1) of the form

$$g(U) \approx \frac{1}{V_1 - V_2} \left[V_2 \exp\left(-\frac{U - U_m}{V_2}\right) - V_1 \exp\left(-\frac{U - U_m}{V_1}\right) \right]. \quad (5)$$

From the static current-voltage characteristic R_m has been determined to 1950Ω with a maximum error of 100Ω resulting from fluctuations in the plasma. The least square fit values of $\int g(U) dU / R_m + I_0$ to the static current-voltage characteristic, with $R_m = 1950 \Omega$ and I_0 being the plasma current at high dc voltages, are $V_1 = 14.6 \text{V}$, $V_2 = 7.2 \text{V}$, $U_m = 38.7 \text{V}$ (Fig. 1(b)).

With $L = 60.8 \text{mH}$ and capacitance C according to Fig. 2, f_0 takes values between 230 and 650Hz and $\varepsilon \ll 1$ is reasonably fulfilled. In spite of the small frequencies the resistance of the inductor is frequency dependent due to eddy current effects.

For $C = 7 \mu\text{F}$ ($D = 0.89$) the amplitude characteristic has a maximum of the order of $k_B T_e / e$ and has approximately the shape known from the van der Pol oscillator (Fig. 2). At $C = 1 \mu\text{F}$ to maximum amplitude reaches 70% of the dc voltage. For capacitances smaller than $4 \mu\text{F}$ ($D < 0.75$) the amplitude characteristic shows a hysteresis. Its width increases with decreasing C . The oscillation is harmonic and its frequency deviates maximally 1% from f_0 .

| $C(\mu\text{F})$ | $f(\text{Hz})$ | $R(\Omega)$ | ε | D |
|------------------|----------------|-------------|---------------|-------|
| 1 | 646 | 6.87 | 0.126 | 0.220 |
| 2 | 457 | 5.18 | 0.089 | 0.333 |
| 3 | 374 | 4.62 | 0.073 | 0.445 |
| 4 | 326 | 4.34 | 0.063 | 0.557 |
| 5 | 290 | 4.17 | 0.057 | 0.669 |
| 6 | 266 | 4.06 | 0.052 | 0.781 |
| 7 | 246 | 3.98 | 0.048 | 0.893 |
| 8 | 230 | 3.92 | 0.045 | 1.005 |

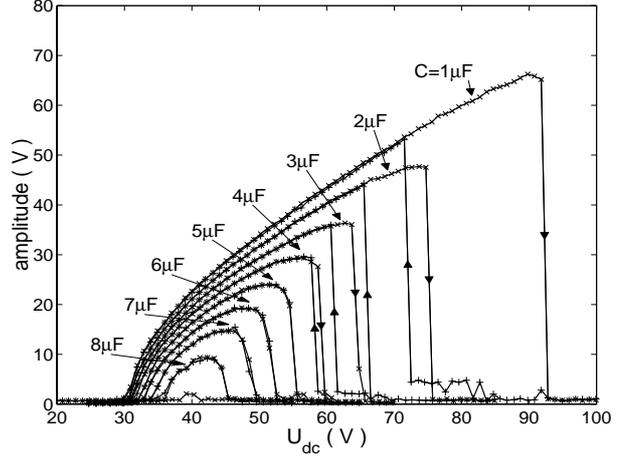


Figure 2. LEFT: External capacitance C , oscillation frequency f , nonlinearity coefficient ε and damping coefficient D according to Eq. (4). R is the resistance of the inductor. The error in ε and D is 5%. RIGHT: Amplitude characteristics showing a transition from continuous to hysteretic shapes.

3. Theory

The method of Bogoliubov and Mitropolsky [4] is a perturbation method to derive approximate solutions for differential equations of the form

$$\frac{d^2}{dt^2} V + \omega^2 V = \varepsilon h \left(V, \frac{d}{dt} V \right) \quad (6)$$

with $\varepsilon \ll 1$. The solution $V(t)$ is expressed in terms of the momentary amplitude $a(t)$ and phase $\psi(t)$ and is expanded in the small parameter ε . Taking the first order approximation only and applying it to Eq. (3), it follows

$$V = a(\tau) \cos \tau, \quad \frac{d}{d\tau} a = \varepsilon A_1(a), \quad \text{with} \quad (7)$$

$$A_1(a) = -\frac{1}{2\pi} \int_0^{2\pi} [g(a \cos \psi + U_{dc}) + D] a \sin^2 \psi d\psi. \quad (8)$$

Expanding $g(a \cos \psi + U_{dc})$ around U_{dc} it follows

$$A_1(a) = \frac{a}{2} [g(U_{dc}) + D] + \left[\frac{a^3}{16} g''(U_{dc}) + \right] \dots \quad (9)$$

4. Discussion

The behavior of A_1 can be understood from the shape of $g(U_{dc}) + D$. Stationary oscillation amplitudes a_{stat} are determined by the roots of A_1 . A stationary solution is stable if $\frac{d}{da} A_1(a = a_{stat}) < 0$, because then small fluctuation from a_{stat} decreases. The trivial solution $a = 0$ is stable if $g(U_{dc}) + D > 0$ (Eq. (7)), so that the double layer is stable if $U_{dc} < U_1$ or $U_2 < U_{dc}$. The oscillation is self-excited if $U_1 < U_{dc} < U_2$. Any small oscillation grows until it resides long enough in the regions $U_{dc} < U_1$ and $U_2 < U_{dc}$ to balance positive and negative damping. This amplitude corresponds to the stable root a_s of A_1 (Fig. 3).

For $D < 0.75$ a_s still exists in an interval $U_2 < U_{dc} < U_3$. Simultaneously an additional, unstable root a_u appear near $a = 0$ at $U_{dc} = U_2$ and grows with increasing U_{dc} until $a_u = a_s$ at

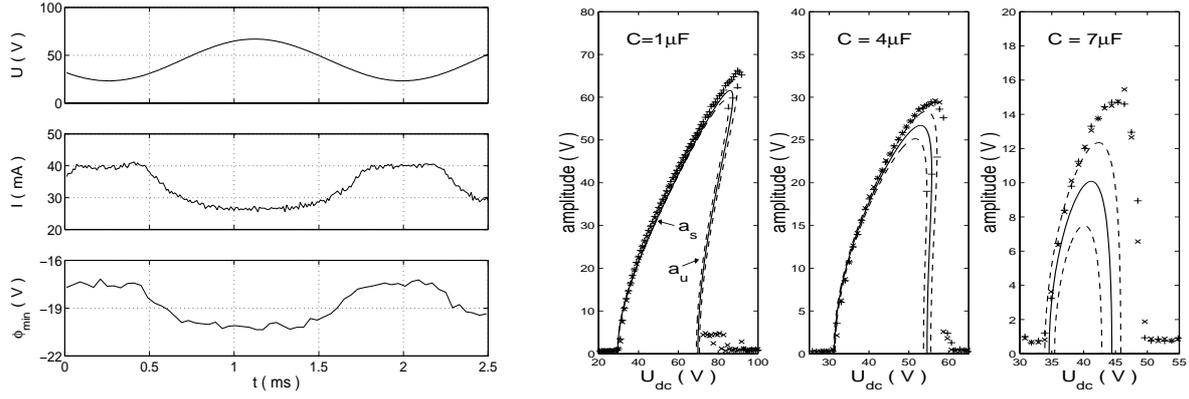


Figure 3. LEFT: Voltage and current oscillation ($C = 2\mu\text{F}$, $U_{\text{dc}} = 65.5\text{V}$, $f = 458\text{Hz}$). The voltage oscillation is harmonic and hysteresis effects in current can be neglected, so that a static current-voltage approach is valid. RIGHT: Measured amplitudes ($+ U_{\text{dc}}$ increasing, $\times U_{\text{dc}}$ decreasing) and roots of $A_1(a)$ (solid line) according to Eq. (8) with $R_{\text{m}} = 1950\Omega$. The dashed lines give the uncertainty caused by a 5% error in R_{m} . The theoretical amplitudes reproduce the transition from continuous to hysteretic characteristics. The difference between theoretical and experimental amplitudes is due to the overestimation of the damping by the fit function for $g(U_{\text{dc}})$ for small values of U_{dc} (Fig. 1).

$U_{\text{dc}} = U_3$. If U_{dc} is increased slowly the oscillation, being excited, follows the stable root a_s up to $U_{\text{dc}} = U_3$ (Fig. 3). Then A_1 turns negative for all $a > 0$ and the oscillation collapses to $a = 0$. If instead U_{dc} is decreased from large values, for which $a = 0$, oscillations are not expected until $U_{\text{dc}} = U_2$. The hysteresis exists only if a_u exists. Looking for a stationary amplitude a_{stat} when $U_{\text{dc}} \approx U_2$ by taking only the first two terms in Eq. (9) gives

$$a_{\text{stat}} = 2\sqrt{-\frac{2[g(U_{\text{dc}}) + D]}{g''(U_{\text{dc}})}}. \quad (10)$$

An a_{stat} in the region $U_{\text{dc}} > U_2$ exists only if $g''(U_{\text{dc}}) < 0$ and it is unstable then. The transition to hysteretic amplitude characteristics takes place when $g''(U_2) = 0$. This condition predicts hysteresis for $C < 4.6 \pm 0.5\mu\text{F}$, which agrees well with the observation.

Without external dissipation ($R = 0$) the oscillation amplitude would grow approximately linearly with U_{dc} and an amplitude collapse would not be observed. With a suitable choice of L and C high resonance frequencies and small nonlinearity can be reached simultaneously. However then the static current-voltage approach is not valid any longer and the ion dynamic would have to be taken into account.

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