# SPACE-CHARGE LIMITED EMISSION CURRENT FROM MATERIAL SURFACE IN A PLASMA

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**Abstract.** A precise expression of space-charge limited emission current from material surface in a plasma is given by the requirement of zero electric field on the material surface, and is compared with the experimental measurement and the modified Child-Langmuir expression.

## 1. Introduction

Electron emission from the material surface in a plasma reduces the sheath potential between the plasma and the material surface and thus influences the energy and particle fluxes to the surfaces. The emission current may be regulated by space charge effect in the sheath. The space-charge limited current in vacuum is described by the Child-Langmuir (CL) expression $j_{CL} = (4\varepsilon_0/9)(2e/m_e)^{1/2}\phi_s^{1.5}/d^2$, where $d$ is the distance between two electrodes. If we apply CL equation in a plasma, $d$ could be a sheath thickness which is a few times as large as the Debye length : $d = k\,\lambda_D$. It gives

$$j_{CL} = 4n_{se}e\,(2T_e/m_e)^{1/2}\Phi_s^{3/2}/(9k^2), \qquad (1)$$

where $\Phi_s = e\phi_s/T_e$. In the electrically floated condition, the modified CL expression coincides the formula given by Hobbs and Wesson when we employ $k$ =2.2[1-2]. However, Eq. (1) is not generally correct for arbitrary values of sheath voltage. It is necessary to analyze the electro-static structure in the sheath.

In this paper a precise expression of space-charge limited current through the sheath is given by the zero electric field condition on the material surface, and is compared with the modified CL expression, and its validity was clearly confirmed by the simulated experiments.

## 2. Model

We consider an infinite plane material surface, situated at $x = 0$ (see Fig. 1), in contact with a plasma filling the half-space $x > 0$. Far from the material surface there is a neutral plasma where the potential $\phi = 0$. At the sheath edge( $x = d$ ) $\phi$ decreases to $\phi_0$ to accelerate plasma ions up to a velocity $V_0$ and quasi-neutrality is still kept until this point.

The potential profile in the sheath is obtained by solving Poisson's equation,

$$\frac{d^2\phi}{dx^2} = -\frac{e}{\varepsilon_0}\left[n_i(x) - n_e^p(x) - n_e^s(x)\right], \qquad (2)$$

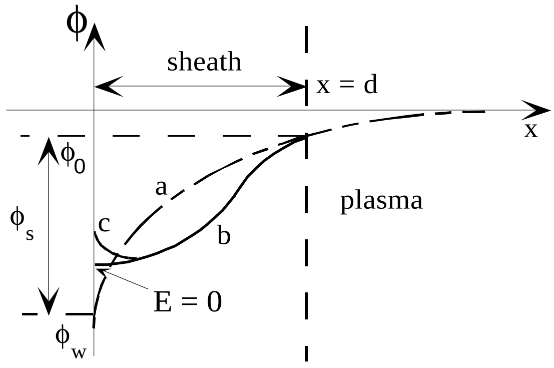


***Fig. 1*** *Model for the plasma sheath*

where $n_i$ and $n_e^p$ are the densities of plasma ions and electrons, and $n_e^s$ is the density of emission electrons from the material surface.

The plasma electrons are assumed to obey the Boltzman relation,

$$n_e^p(x) = n_{se}^p exp\left[e\,(\phi-\phi_0)\,/T_e\right], \tag{3}$$

where $n_{se}^p$ is the density of plasma electrons at the sheath edge $x = d$.

The cold ions are assumed to fall freely to the material surface. The ion density is thus given by continuity equation and energy conservation,

$$n_i(x) = n_{se}\left[1\text{-}2e\,(\phi - \phi_0)/(M^2T_e)\right]^{-1/2}, \tag{4}$$

where $n_{se}$ is the density of electron at the sheath edge , which is given by,

$$n_{se} = n_{se}^p + n_{se}^s\ , \tag{5}$$

here $n_{se}^s$ is the emission electrons density. $M$ is the Mach number, defined as follows :

$$M = V_0\,/C_s = C_s\,(T_e\,/m_i)^{-1/2}, \tag{6}$$

Now we consider the emission current $j_{em}^-$ based on the secondary electron emission, arising from the electron impact:

$$j_{em}^- = en_e^s(x)V_e^s(x) = \gamma\, j_p^-\ , \tag{7}$$

where $\gamma$ is the ratio of the emission electron current to the primary electron current. $V_e^s(x)$ is the velocity of emission electrons in the sheath. The emitted electrons emitted from the material surface are assumed to have negligible velocity. We thus have $V_e^s(x) = \sqrt{2e\,(\phi - \phi_w)/m_e}$. On the other hand, the plasma electron current density to the material surface $j_p^-$ is given by

$$j_p^- = en_{se}^p\sqrt{T_e\,/(2\pi m_e)}\ exp(e\phi_s\,/T_e), \tag{8}$$

where $\phi_s = \phi_w - \phi_0$.

Using Eqs. (2) - (8) and introducing non-dimensional variables:

$$\Phi = e\,(\phi - \phi_0)/T_e\ ,\ \xi = x\,/\lambda_D\ ,\ G = \gamma\left(-\,4\pi\Phi_s\right)^{-1/2} exp\left(\Phi_s\right), \tag{9}$$

where the Debye length $\lambda_D = \sqrt{\varepsilon_0 T_e\,/\,(n_{se}e^2)}$ .

Poisson's equation becomes

$$\frac{d^2\Phi}{d\xi^2} = \frac{\exp\,(\Phi)}{1+G} + \frac{G}{1+G}\left(1\text{-}\ \frac{\Phi}{\Phi_s}\right)^{-1/2} - \left(1\text{-}\ 2\Phi\,/M^2\right)^{-1/2}. \tag{10}$$

As Eq. (10) is multiplied by $d\Phi\,/d\xi$ and integrated from $\infty$ to $x$, it gives :

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 = \frac{\exp(\Phi\,)\text{-}1}{1+G} - \frac{2\Phi_s G}{1+G}\left[\left(1\text{-}\ \frac{\Phi}{\Phi_s}\right)^{1/2}\text{-}1\right] + M^2\left[\left(1\text{-}\ 2\Phi/M^2\right)^{1/2}\ \text{-}1\right]. \tag{11}$$

Around the sheath edge $\left|\Delta\Phi\right| << 1$, Eq. (11) can be expanded in the form

$$\left(\frac{d\Phi}{d\xi}\right)^2 = \left[\left(1+0.5G\,/\Phi_s\right)(1+G)^{-1}\text{-}\ \frac{1}{M^2}\right]\left(\Delta\,\Phi\right)^2$$

to give a new Bohm Criterion :

$$M^2 \geq \left(1+0.5G\,/\Phi_s\right)^{-1}(1+G)\ . \tag{12}$$

Without the secondary emission electrons where $\gamma = 0$, Eq. (12) becomes $M \geq 1$, which coincides the conventional Bohm Criterion.

The emission current Eq. (7) is written by using Eqs. (5) and (9):

$$j_{em}^- = G\,(1+G)^{-1}n_{se}e\sqrt{\text{-}2\Phi_s T_e\,/m_e}. \tag{13}$$

## 3. Space-charge limited emission current

As the electron emission becomes strong, a sufficient electron cloud exists in front of the material surface, which realizes the zero electric field at the critical emission coefficient $\gamma_c$. When $\gamma$ is increased above $\gamma_c$, a potential minimum, so called virtual cathode, (see "c" in Fig. 1) appears. The emitted electrons whose kinetic energy is greater than the magnitude of the potential barrier may arrive and pass through the sheath edge. If we assume that emission electrons are cold, space-charge limited emission current can be given by a zero electric field condition on the material surface.

Substitute $\Phi = \Phi_s$ and $d\Phi / d\xi = 0$ into Eq. (11), and $M$ is given by Eq. (12) in the case of equal. A solution of $G$ in Eq. (11) is obtained in the form

$$G = [-\beta_1+(\beta_1^2 - 4\beta_0\beta_2)^{1/2}]/(2\beta_2), \quad (14)$$

$$\beta_0 = -4\Phi_s^2 - 2\Phi_s(F^2-2F), \ \beta_1 = 4(2F-1)\Phi_s^2 + 8F\Phi_s - F^2,$$

$$\beta_2 = 4\Phi_s^2 - 8\Phi_s^3, \ F = \left[exp(\Phi_s) - 1\right].$$

The space-charge limited emission current is given by Eqs. (13) and (14) which does not depend on $\gamma$. This is shown in Fig. 2 by thick solid line as a normalized form, and is compared with the modified CL equation $j_{\mathrm{CL}}$ for several $k$ values. Certainly both values agree each other for $k$ = 2.2 around $\Phi_s$ ~ -1 corresponding to a floating voltage under the space-charge limited condition. Except this point, the modified CL equation is found to overestimate the emission current.

This new formula for the space-charge limited current is also valid for the thermoelectron emissions from the hot material surface. When the emission current is less than the space-charge limited current, it is given by Richardson-Dushman's formula

$$\bar{j}_{em} = AT^2 exp\left[-e\phi_{WF}/(kT)\right], \quad (15)$$

where $A$ is Dushman's constant, $T$ is the surface temperature and $\phi_{\mathrm{WF}}$ is the work function of the material surface.

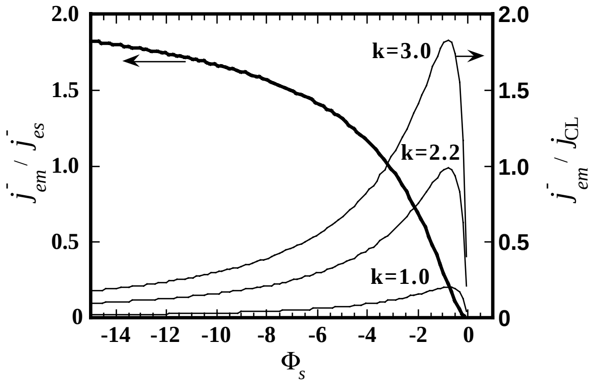


***Fig. 2*** *Space charge limited current as a function of sheath voltage. It is compared with modified Child- Langmuir expressions with several sheath thickness,where* $\bar{j}_{es} = 0.25en_{se}\sqrt{8T_e/(\pi m_e)}$ *is the random electron current density.*

## 4. Simulated experiments on space-charge limited current

The experimental setup is shown schematically in Fig. 3. $LaB_6$ ( 40 × 3.7 × 1.8 mm) bar is fixed in a carbon holder, which can be externally heated and is biased. In the case of negative biasing without heating, the current into the target plate is the plasma ion current. When the target plate was heated, the current increases with an increase in the surface temperature $T$ due to the thermoelectron emission from the material surface. The magnitude of the current change should give the thermoelectron emission current. It should be noted that it is important to have an uniform surface temperature distribution in the electron emission region for the precise measurement of the space charge limited current. In our experiment, only 1cm length region around the center of the $LaB_6$ bar was exposed to the plasma, and other part is covered by BN plate.

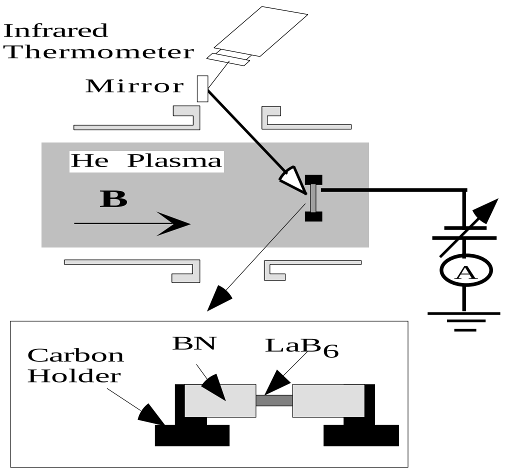


*Fig. 3* *Schematic drawing of the experimental setup*

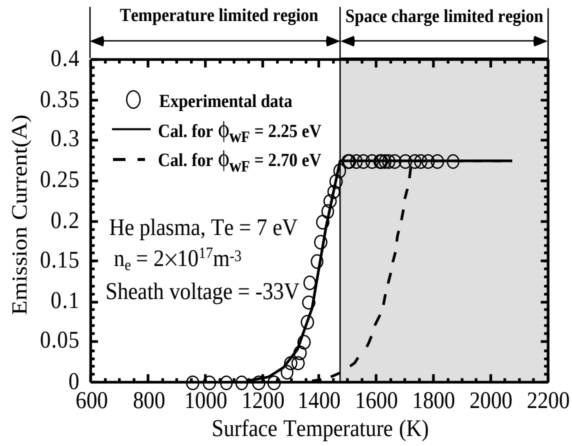


***Fig. 4*** *Comparison of observed emission current from $LaB_6$ through plasma sheath with theoretical formula.*

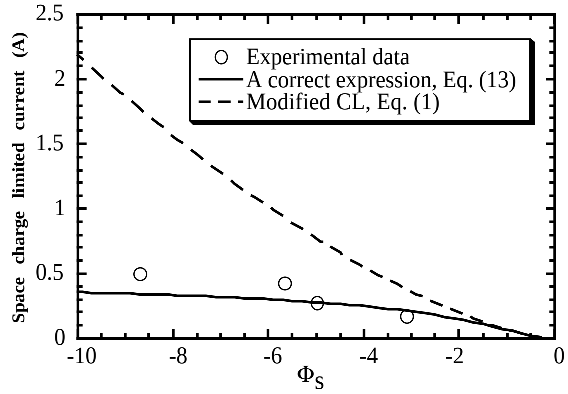


*Fig. 5 Experimental results on the space-charge limited current, compared to new formula based on Eqs. (13) and (14) and modified CL equation with k=2.2.*

The characteristics of thermoelectron emission from $LaB_6$ through plasma sheath was measured under the helium gas discharge with the plasma density of $2.0\times 10^{17} m^{-3}$ and the electron temperature of 7 eV. The experimental result shows that the emission current increases as increasing the surface temperature $T$, and saturates at $T$=1500 K, as shown in Fig. 4. Solid line shows the calculated result by Eqs. (13), (14) and (15) with $\phi_{WF}$ = 2.25 eV and $A$ = 2.9 $\times 10^5$ $Am^{-2}K^{-2}$. A good agreement is obtained. Dashed line shows the calculation with $\phi_{WF}$ = 2.70 eV which is a recommended value in a handbook. The difference of $\phi_{WF}$ may come from the material surface condition, facing to the plasma. On the other hand, the value of space-charge limited current calculated by new formula is in a good quantitative agreement with experimental date. Figure 5 shows the dependence of the space-charged limited current on the sheath voltage to compare with the new formula and the modified CL equation with $k$ = 2.2. It is clearly found that the space-charge limited currents given by the new formula are much closer to the experimental results than those given by Eq. (1).

## 5. Summary

A precise expression under space charge limited condition is given for arbitrary sheath voltage, which quantitatively agrees with the experimental data.

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## Reference

[1] G.D. Hobbs and J.A. Wesson: Plasma Phys. **9**, 85 (1967)
[2] S. Takamura, M.Y. Ye, T. Kuwabara and N. Ohno: Phys. Plasmas, **5**, 2151(1998)