SELF-SIMILAR EXPANSION OF PLASMA BUNCH INTO VACUUM

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Plasma expansion into vacuum has received significant attention of physicists over the last decades. Following the pioneer work by A.V. Gurevich [1] most of the studies have been based on the model of semi-infinite collisionless plasma. The analysis of the expansion of an initially confined plasma bunch requires another approach, because the process in this case is accompanied by considerable cooling of electrons. An appropriate solution to this problem has been obtained only recently. Numerical [2,3] and analytical [4-6] investigations have shown that the expansion of a collisionless plasma bunch can be described by a self-similar solution. Such an exact self-similar solution of the set of three-dimensional Vlasov equations for both electrons and ions is presented below. This solution has been found within a quasi-neutral approach. It is valid for an arbitrary value of the electron-to-ion mass ratio, arbitrary relation between electron and ion thermal energies, and for a large variety of electron velocity distribution functions and initial spatial plasma density distributions, including an anisotropic one. It is also demonstrated below that the approximation of cold ions and adiabatic description of electron motion in the charge separation electric field make it possible to account for the interparticle collisions as well as other processes responsible for the isotropy of the electron distribution function.

The physical model assumes that in a space with dimensionality s (s = 1, 2, 3) there is a bunch of collisionless plasma whose ion and electron distribution functions are described by the Vlasov kinetic equations:

$$\frac{\partial f_{i,e}}{\partial t} + (v, \nabla_r) f_{i,e} - \left(\frac{q_{i,e}}{m_{i,e}} \nabla_r \varphi, \nabla_v \right) f_{i,e} = 0\,, \quad f_{i,e} = f_{i,e}(v,r,t), \qquad (1), (2)$$

where $q_{i,e}$ and $m_{i,e}$ are ion and electron charge and mass respectively ($q_i \equiv Ze$, $q_e \equiv -e$, $m_i \equiv M$, $m_e \equiv m$), and $\varphi(r,t)$ is potential of charge separation electric field.

The quasi-neutral approximation implies that

$$n_e(r,t) = Zn_i(r,t); \quad n_{e,i}(r,t) \equiv \int f_{e,i}(v,r,t) d^s v\,; \qquad r = (x_1,...,x_s). \qquad (3)$$

The integrals of the system (1)-(3) are the equality of electron and ion average velocities:

$$u_e(r,t) = u_i(r,t); \quad u_{e,i}(r,t) \equiv \int v f_{e,i}(v,r,t) d^s v \Big/ \int f_{e,i}(v,r,t) d^s v , \tag{4}$$

and conservation of full kinetic energy of the particle motions along each direction x_k:

$$W_k \equiv \iint m v_k^2 f_e(v,r,,t) d^s v d^s r + \iint M v_k^2 f_i(v,r,,t) d^s v d^s r = const, \qquad k = 1,\dots,s. \tag{5}$$

They allow us to obtain universal law of quasi-neutral plasma expansion:

$$l_k^2(t) \equiv \frac{\int x_k^2 f_{e,i}(v,r,,t) d^s r}{\int f_{e,i}(v,r,,t) d^s r} = \left(l_k(0) + \eta_k(0) t\right)^2 + c_k^2 t^2; \quad \eta_k \equiv dl_k/dt \tag{6}$$

$$c_k^2 + (\eta_k(0))^2 \equiv W_k \Big/ \left(\iint m f_e(v,r,,t) d^s v d^s r + \iint M f_i(v,r,,t) d^s v d^s r \right) = const. \tag{7}$$

An exact self-similar solution to (1)-(3) can be presented in the following form:

$$f_e = F(G_1,\dots,G_{\acute{\imath}}), \qquad G_k = \left(\frac{x_k}{l_k(t)}\right)^2 + \left(\frac{v_k - u_k}{V_k(t)}\right)^2, \tag{8}$$

$$f_i = \frac{1}{Z} F(Q_1,\dots,Q_s) \prod_{k=1}^{s} \frac{U_k}{V_k}, \qquad Q_k = \left(\frac{x_k}{l_k(t)}\right)^2 + \left(\frac{v_k - u_k}{U_k(t)}\right)^2, \tag{9}$$

$$\begin{Bmatrix} V_k(t) \\ U_k(t) \end{Bmatrix} = \left(\frac{\iint (v_k - u_k)^2 f_{\{e,i\}}(v,r,t) d^s v d^s r}{\iint f\{e,i\}(v,r,t) d^s v d^s r} \right)^{1/2} = \begin{Bmatrix} V_k(0) \\ U_k(0) \end{Bmatrix} \times \frac{l_k(0)}{l_k(t)} , \tag{10}$$

$$e\varphi = -\frac{mM}{2(M + Zm)} \sum_{k=1}^{s} \left(V_k^2(0) - U_k^2(0)\right) \frac{l_k^2(0)}{l_k^4(t)} x_k^2, \qquad c_k^2 = \frac{Z m V_k^2(0) + M U_k^2(0)}{M + Zm}. \tag{11}$$

According to the obtained results, the ion and electron thermal energies are gradually converted to the energies of their collective motion as the plasma expands. The particles cool down in an adiabatic manner (10). This is a specific peculiarity of the plasma bunch expansion, independent of the value of Zm/M. What is more, the adiabatic law holds for both sorts of particles, although particles of only one sort oscillate in the potential φ while the other particles are ejected from the plasma by the electric field.

The obtained solution of Eqs. (1)-(3) only allows us to investigate expansion of a plasma with similar electron and ion distribution functions (8), (9). The self-similarity of the solution restricts possible initial spatial distributions of plasma density for a given velocity distribution. Nevertheless, due to the fact that the obtained solution contains such arbitrary independent parameters as initial scale lengths of plasma density and average velocities of electron and ion thermal motion, it can be applied to many cases of plasma expansion.

The presented exact self-similar solution makes it possible to study plasma bunch acceleration by external potential forces acting on electrons. If the potential of these forces

$\varphi_{\{ext\}}$ is described by a quadratic form of coordinates, equations for the time dependencies of plasma density scale lengths can be easily obtained. They show that for the proper sign of the Laplacian of the external electric potential the plasma can be confined in certain region of space. The motion of the center of mass of the plasma bunch is indentical to the motion of a particle with effective mass $(m + M/Z)$in the potential $\varphi_{\{ext\}}$.

The obtained solution (8)-(11) coincides with the solution found using a hydrodynamic equation for ion motion [6] if $U_k(0) = 0$. The adiabatic approach presented in [4,5] means that the spatial plasma density distribution corresponds to the equilibrium state of electrons in an instantaneous electric potential slowly varying in time and the electron movement is governed by the adiabatic invariant. This approximation can be obtained from the above solution in the case $Zm/M << 1$, $U_k(0)/V_k(0) << \sqrt{Zm/M}$:

$$\langle f_e \rangle \approx F(G_1,...,G_s), \qquad G_k = \frac{x_k^2}{l_k^2(t)} + \frac{v_k^2}{V_k^2(t)}, \tag{12}$$

$$e\varphi = -\frac{m}{2}\sum_{k=1}^{s} V_k^2(0) l_k^2(0) x_k^2 / l^4(t), \qquad c_k^2 = \frac{Zm}{M} V_k^2(0). \tag{13}$$

The solution (12), (13) only describes the process of quasi-neutral plasma expansion during a limited time interval when the characteristic electron velocities remain higher than the velocities of the accelerated ions: $u_k << V_k$.

It follows from the solution obtained that the process of expansion of a plasma with cold ions is governed by the initial electron velocity distribution function. Specifically, if the latter is an isotropic one, i. e. $V_k(0) = V_j(0)$ for $k \neq j$, the spatial density distribution function becomes symmetric as plasma expands even if initially it was not such. At the same time, if $l_k(0) \neq l_j(0)$ for $k \neq j$, electron velocity distribution function becomes essentially anisotropic, i. e. $V_k(t) \neq V_j(t)$. Such an asymmetry in the velocity distribution is not typical one for a real situation. There are many processes responsible for the isotropy of the electron distribution function, for example, interparticle collisions. The adiabatic approximation allows us to investigate the process of plasma expansion in this case. For the electron distribution function the following relationships can be obtained instead of (12), (13):

$$\langle f_e \rangle = F\left(\frac{v^2}{V_T^2(t)} + \sum_{k=1}^{s} \left(\frac{x_k}{l_k(t)} \right)^2 \right), \qquad V_T(t) = V_T(0) \prod_{k=1}^{s} \left(\frac{l_k(0)}{l_k(t)} \right)^{1/s}, \tag{14}$$

$$e\varphi(r,t) = -\frac{m}{2}\sum_{k=1}^{s} \left(\frac{V_T(t)}{l_k(t)} \right)^2 x_k^2 , \qquad c^2 = \frac{Zm}{M} V_T^2(0). \tag{15}$$

where the functions $l_k(t)$ are solution to the set of differential equations:

$$d^2 l_k / dt^2 - c^2 l_k^{-1} \prod_{j=1}^{s} \left(l_j(0) / l_j(t) \right)^{2/s} = 0\,. \qquad (16)$$

The system (16) has two integrals. For initially immobile plasma bunch they are:

$$\sum_{k=1}^{s} (\eta_k)^2 = sc^2 \left(1 - \prod_{k=1}^{s} \left(\frac{l_k(0)}{l_k(t)} \right)^{2/s} \right), \qquad \sum_{k=1}^{s} (l_k)^2 = \sum_{k=1}^{s} (l_k(0))^2 + sc^2 t^2\,. \qquad (17)$$

Analytical and numerical investigations of Eqs. (16), (17) show that in contrast to the collisionless case, the process of collision plasma expansion depends considerably on the relation between initial scale lengths of the plasma in different directions. Specifically, a preferential acceleration of ions is found to occur in the direction of the minimum initial plasma scale length.

Acknowledgements

The work is supported by Russian Foundation for Basic Research, Grant No 98-02-17052.

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