THE INFLUENCE OF HIGH-ENERGY PARTICLES ON THE TRANSPORT PROCESSES OF A MULTI-COMPONENT PLASMA FOR FOUR MHD TIMESCALES

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1. Introduction

In a recent paper [1], a multiple timescale approach has been applied to the Fokker-Planck equation and to Maxwell´s equations to derive the plasma transport equations for a strongly magnetized and weakly collisional two-component plasma for four different timescales. In this paper the multiple timescale approach is extended to a three-component plasma including high-energy particles, where the parameter range of a typical fusion plasma is assumed.

2. The multiple timescale approach within kinetic theory and Maxwell´s equations

The starting point of the investigations is a kinetic description of a multi-species plasma by the Fokker-Planck equation to which the multiple timescale approach [1] is applied. The considered kinetic timescales are the Larmor time $\tau_{j0} = \Omega_j^{-1}$, the transit time $\tau_{j1} = \omega_j^{-1}$, the inverse collision rate $\tau_{j2} = \nu_j^{-1}$, and the transport timescale τ_{j3}, which satisfy for a typical fusion plasma for each species the ordering: $\tau_{j0} = \Omega_j^{-1} << \tau_{j1} = \omega_j^{-1} << \tau_{j2} = \nu_j^{-1} << \tau_{j3}$. With the help of the expansion parameter $\delta_j = \omega_j / \Omega_j$, the dimensionless Fokker-Planck equation reads

$$\frac{\partial f_j}{\partial t} + \delta_j \vec{V} \cdot \vec{\nabla} f_j + \sigma(q_j)\left(a_j \vec{E} + \vec{V} \times \vec{B}\right) \cdot \frac{\partial f_j}{d\vec{V}} = \Lambda_j \delta_j^2 C_j + \delta_j^2 (S_j - L_j), \qquad (1)$$

where C_j is the collision integral and S_j and L_j are the source and collisionless loss terms of species *j*, respectively. The dimensionless physical quantities are obtained by normalizing with respect to some characteristic values. For the electrons and ions stationary conditions are assumed, it means that the phase space averaged source and loss term cancel each other. Concerning the energetic particles, the phase space averaged source term and the total loss term have to cancel each other, thus expressing the conservation of the total number of particles. The dimensionless factors Λ_j in front of the collision operators, defined in Ref.[1], are of order of unity.

Next, Maxwell´s equations are brought into a dimensionless form, where the inverse ion gyrofrequency Ω_i^{-1} and the Alfvén velocity v_A are applied for the time and velocity normalization, respectively. For the self-consistent electric and magnetic fields, created by both the electron and ion motion, the geometric mean δ_i and δ_e is employed as expansion parameter $\delta = \sqrt{\delta_i \delta_e}$, leading to

$$\vec{j} = curl\,\vec{B} - \delta \frac{\partial \vec{E}}{\partial t}, \quad div\,\vec{B} = 0, \quad \frac{\partial \vec{B}}{\partial t} = -\delta\, curl\,\vec{E}, \quad q = \delta^3 div\,\vec{E}. \qquad (2)$$

The Fokker-Planck and Maxwell´s equations fully describe the dynamic processes in terms of the distribution function.

3. The zero- and first-order equations

For timescales much shorter than the collisional ones, the interaction between the particles is taken into account in the collisionless Fokker-Planck (Vlasov) equation through the average electric and magnetic fields. The selection of a particular distribution from the infinity of possible solutions of Eq.(1) is outside the scope of the Vlasov equation. This choice would involve the history of the plasma or the known features of the underlying plasma model. For example, if the system has existed for many collision times, the Maxwellian distribution is an appropriate choice. Another example is a system prepared by injecting a monoenergetic beam of particles. In this case the distribution function may be significantly different from a Maxwellian one.

From the zero-order Fokker-Planck equation and the zero-order Maxwell´s equations, one concludes that the zeroth order distribution functions are independent of t_{j0}, i.e. $\partial f_{j0}/\partial t_{j0} = 0$, and that in the zeroth-order there is no influence of the energetic particles, thus leading to the same results as for the two-component plasma in [1] i.e., to the zero-order Ohm´s law of ideal MHD:

$$\vec{E}_0 + \frac{1}{c}\vec{u}_0 \times \vec{B}_0 = 0, \tag{3}$$

where $\vec{u}_0$ is now the zero-order fluid velocity.

The solution for the first order distribution function f_{j1} has to be obtained from the first order Vlasov equation. In order to prevent a secular growth of f_{j1}, it must be required that

$$< \frac{\partial f_{j0}}{\partial t_{j1}} + \vec{V} \cdot \vec{\nabla} f_{j0} >= 0, \tag{4}$$

where <...> denotes time-averaging over the zero order timescale t_{j0}. Therefore, on the first timescale the distribution function f_{j0} is governed by the drift-kinetic equation and depends on the constants of motion. Thus, on the Alfvén timescale the inclusion of the energetic particles has no influence on the single-fluid ideal MHD equations and on the first-order Ohm´s law. If for all physical quantities the expansion up to first-order is taken into account, then one arrives at the familiar form of Ohm´s law, valid on the Alfvén timescale

$$\nabla P_e + en\left(\vec{E} + \frac{1}{c}\vec{u} \times \vec{B}\right) - \frac{1}{c}\vec{j} \times \vec{B} = 0, \tag{5}$$

where, for a fusion plasma near reactor conditions, the gradient of the electron pressure and the Hall term must not be neglected [1].

4. The second-order equations

In [2] it is shown, that the zero-order distribution function for electrons and ions approaches a local drifted Maxwellian (provided that the confinement time is much larger than the collision time). The case for the energetic particles is different and their distribution function, f_{h0}, can be obtained from the solution of the second order equation with a Fokker-Planck collision operator of the form

$$-\frac{\partial f_{h0}}{\partial t_{h2}} + \Lambda_h C_h(f_{h0}, f_{j0}) + S_{h0} - L_{h0} = \frac{\partial f_{h1}}{\partial t_{h1}} + \frac{\partial f_{h2}}{\partial t_{h0}} + \vec{V} \cdot \vec{\nabla} f_{h1}. \tag{6}$$

The left-hand side of this equation is only a function of t_{h2}. In order to prevent a secular growth of f_{h1}, f_{h2}, it must be required that $< \frac{\partial f_{h1}}{\partial t_{h1}} + \frac{\partial f_{h2}}{\partial t_{h0}} + \vec{V} \cdot \vec{\nabla} f_{h1} >= 0$. On the t_{h2}-timescale, the only particle flows are due to collisions. In analogy to the above employed procedure, one obtains for f_{h0} the following equation

$$\frac{\partial f_{h0}}{\partial t_{h2}} = << \Lambda_h C_h (f_{h0}, f_{j0}) + S_{h0} - L_{h0} >>, \tag{7}$$

where <<...>> denotes time-averaging over the zeroth and first order timescales t_{h0}, t_{h1}.

5. The solution of the kinetic equation for the highly energetic particles.

The kinetic equation for the highly energetic particles which collide with the Maxwellian electron and ion background, may be written in the following simple form

$$\frac{\partial f_{h0}}{\partial t_{h2}} = \frac{V_0^3}{\tau_s V^2} \frac{\partial}{\partial V} \left(\frac{V_0^2 a(V)}{2V} \frac{\partial f_{h0}}{\partial V} + b(V) f_{h0} \right) + S_{h0} - L_{h0} = 0, \tag{8}$$

together with the boundary conditions $f_{h0}(V=0) \neq 0,\ f_{h0}(V \to \infty) = 0$, where $a(V)$ and $b(V)$ describe slowing down and parallel diffusion of the energetic particles in the velocity space:

$$a(V) = \frac{m_h}{E_h} \left(\frac{T_e}{m_e} a_e + \frac{T_i}{m_i} a_i \right),\ \ b(V) = m_h \left(\frac{b_e}{m_e} + \frac{b_i}{m_i} \right),\ \ a_j(V) = b_j(V) = \frac{4}{\sqrt{\pi}} \int_0^{V/V_j} s^2 \exp(-s^2) ds.$$

Here, pitch-angle scattering is not included, since we are mainly interested in the distribution function of fast particles confined in the central part of the plasma with small radial excursions, where pitch angle scattering should play a minor role.

For the source and loss term the following assumptions are made

$$S_h = \frac{S_0}{4\pi V_0^2} \delta(V - V_0),\ \ L_h = L_0 V^\alpha H(V_0 - V), \tag{9}$$

where α is some constant, which allows the investigation of different loss spectra, and *H(V)* is the Heaviside step-function. If one neglects the velocity diffusion, then the steady-state solution for the energetic particle zeroth-order distribution has the form

$$f_{h0}(\vec{x}, \vec{V}; t_2) = \frac{\tau_s S_0}{4\pi V_0^3} \frac{V_c^3}{V^3 + V_c^3} \left(\frac{V}{V_0} \right)^{\alpha+3} H(V_0 - V), \tag{10}$$

which coincides for α=-3 with the well known thermonuclear distribution function [3]. Here V_c is the critical velocity at which the contribution of electrons and ions to the slowing down becomes equal, $V_c^3 = \frac{3\sqrt{\pi}}{4} \frac{m_e}{m_i} V_{the}^3$. If one takes into account the diffusion term then one arrives at the solution

$$f_{h0} = C_h \exp(-m_h v^2 / 2T_j) + \frac{\tau_s S_0}{4\pi V_0^3} \frac{V_c^3}{V^3 + V_c^3} \left(\frac{V}{V_0} \right)^{\alpha+3} H(V_0 - V), \tag{11}$$

which is obviously more general, than the one of Eq. (10). Moreover, it explicitly indicates that the zeroth-order energetic particle distribution consist of a Maxwellian and a non-Maxwellian part.

6. Velocity moments of the collision operators

The zero-order moment, defined by $<v^0>_j := \int C_{j0} d^3v$, vanishes on account of the velocity divergence of the collisional operator, hence expressing particle conservation.

The performance of *the first-order velocity moment*, defined by $<\vec{v}>_j := \int \vec{v}\, C_{j0} d^3v$ first leads to the friction forces between the species of the background plasma in the form

$$\vec{\tilde{R}}_{ei} = \delta_e m_e \bar{n}_e \Lambda_e \omega_e v_{te} <\vec{v}>_e = \tilde{\rho}_{e0} \tilde{\nu}_{e0} (\vec{\tilde{u}}_{i0} - \vec{\tilde{u}}_{e0}) = -\vec{\tilde{R}}_{ie}. \tag{12}$$

The friction forces between the energetic particles and the background plasma are then evaluated as:

$$\vec{\tilde{R}}_{he} = -\tilde{\rho}_{h0} \tilde{\nu}_s \vec{\tilde{u}}_{e0} = -\vec{\tilde{R}}_{eh}, \quad \vec{\tilde{R}}_{hi} = -\tilde{\rho}_{h0} \tilde{\nu}_s \vec{\tilde{u}}_{i0} \frac{v_c^3}{v_0^3} = -\vec{\tilde{R}}_{ih}. \tag{13}$$

From *the second-order velocity moment*, defined by $<v^2/2>_j := \int v^2/2\, C_{j0} d^3v$, one obtains the expression for the collisional energy exchange in dimensional form:

$$\begin{aligned}
\tilde{E}_{ei} &= \frac{3}{m_i} \tilde{\rho}_{e0} \tilde{\nu}_{e0} (\tilde{T}_{i0} - \tilde{T}_{e0}) + \vec{\tilde{u}}_{e0} \vec{\tilde{R}}_{ei}, \quad \tilde{E}_{hi} = 3\tilde{\rho}_{h0} \tilde{\nu}_s v_0^2 G_i, \\
\tilde{E}_{ie} &= \frac{3}{m_i} \tilde{\rho}_{e0} \tilde{\nu}_{e0} (\tilde{T}_{e0} - \tilde{T}_{i0}) + \vec{\tilde{u}}_{i0} \vec{\tilde{R}}_{ie}, \quad \tilde{E}_{he} = 3\tilde{\rho}_{h0} \tilde{\nu}_s v_0^2 G_e,
\end{aligned} \tag{14}$$

with

$$G_e = \frac{1}{\ln(1 + 1/v_c^3)} \int_0^1 \frac{v^4 dv}{v^3 + v_c^3}, \quad G_i = \frac{1}{\ln(1 + 1/v_c^3)} \int_0^1 \frac{v v_c^3 dv}{v^3 + v_c^3}.$$

From the definition for the particle heating, there follow the expressions

$$\tilde{Q}_i = \frac{3}{m_i} \tilde{\rho}_{e0} \tilde{\nu}_{e0} (\tilde{T}_{e0} - \tilde{T}_{i0}) + \tilde{E}_{hi}, \quad \tilde{Q}_e = -\frac{3}{m_i} \tilde{\rho}_{e0} \tilde{\nu}_{e0} (\tilde{T}_{e0} - \tilde{T}_{i0}) + \tilde{E}_{he} - \vec{\tilde{R}}_{ei} (\vec{\tilde{u}}_{e0} - \vec{\tilde{u}}_{i0}). \tag{15}$$

7. Conclusions

The Fokker-Planck equations and the corresponding transport equations have been obtained for all three species. It is shown that the solution of the zero-order equations results in ideal Ohm´s law. The solution of the first-order equations leads under the assumption of a weakly collisional plasma to the ideal MHD equations, where no influence of the high-energy particles is found. On the MHD-collision timescale the corresponding transport equations are derived, showing the interaction of the high-energy particles with the electrons and the ions. The steady-state zeroth-order distribution function of the charged fusion products is found to consist of a Maxwellian and a non-Maxwellian part and turns out to be strongly dependent on the loss spectra. Analytical expressions for the corresponding friction forces and the particle heating are derived.

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