PHASE MIXING OF NONLINEAR PLASMA OSCILLATIONS IN AN ARBITRARY MASS RATIO COLD PLASMA

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The physics of the damping of nonlinear cold plasma oscillations is a topic of considerable interest, both from fundamental and application point of view. Fundamentally, it is the simplest nonlinear collective irreversible phenomenon observed in a plasma. From an application point of view, it has wide applications to a number of problems of current interest, such as particle acceleration by wakefields and beat waves created by intense lasers or particle beams, the fast ignitor concept in inertial fusion where relativistically intense coupled electromagnetic - plasma wave modes propagate deep into overdense plasmas to create a 'hot spark' and a number of other astrophysical / laboratory / device based plasma experiments where intense plasma oscillations are generated. The conventional thinking about the physics of this interaction is well illustrated by the sheet model of Dawson [1] ,by the Lagrangian based calculation of Konyukov [2,3] and by the stream function method of Kalman [4]. The exact solution which shows a stable coherent oscillation if $\frac{keE}{m\omega_p^2} < 0.5$ and wave breaking if $\frac{keE}{m\omega_p^2} > 0.5$ is however valid only if the background positive species are infinitely massive $(\frac{m_-}{m_+} \equiv \Delta \to 0)$ and are homogeneously distributed. If the background is inhomogeneous, then as was shown by Dawson [1], cold plasma oscillations phase mix away in a time scale $t \sim \frac{\pi}{2(d\omega_p/dx)X}$, at arbitrarily low amplitudes. For a sinusoidal distribution of background species, such a phenomenon in the form of mode coupling of a long wavelength mode to short wavelength modes was observed by Kaw et. al. [5]. They found the time scale in which energy goes from long wavelength mode to short wavelength mode is $t \sim \frac{2}{\epsilon\omega_{p0}}$, where '$\epsilon$' is the amplitude of the inhomogeneity. The exact solution for the cold plasma oscillations in a fixed sinusoidal background was given by Infeld et. al. [6] who described phase mixing in terms of electron density burst.

In this paper we show that the phenomenon of phase mixing will also occur in a homogeneous plasma at arbitrarily low amplitudes, provided the background positive species are allowed to move and reorganize themselves ($\Delta \neq 0$). This is because the background species respond to ponderomotive forces either directly or through low frequency self - consistent fields and thereby acquire inhomogenities in space. Such an effect has been observed in electron positron plasmas $(\Delta = 1)$ by Stewart [7]. Similarly, the well known Zakharov equations for Langmuir turbulence in a warm plasma, take account of the interaction between electron plasma oscillations and a self-consistently generated slowly changing background; in this case one observes formation of stationary envelope solitons in 1-D and the phenomenon of Langmuir collapse in higher dimensions [8]. In a cold plasma, stationary states cannot form even in 1-D because thermal effects are inadequate to balance the ponderomotive forces. As a result, a secularly increasing density inhomogeneity forms in the background; this inhomogeneity causes different parts of the plasma oscillation to oscillate at different frequencies [1,5,6] and leads to intense phase mixing of the plasma oscillations.

Thus it is physically expected that if the background species is allowed to move and get redistributed into inhomogeneous clumps of density, the phase mixing damping of cold plasma oscillations should come in at arbitrarily low amplitudes and is not restricted to waves with $\frac{keE}{m\omega_p^2} > 0.5$. In this paper we carry out particle simulations for elucidating the physics of phase mixing damping of nonlinear cold plasma oscillations in an arbtrary mass ratio plasma (Δ arbitrary). We also present a perturbation - theoretic analysis to give a quantitative estimate of the phase mixing time for moderate amplitude oscillations and compare it with simulation.

First we write down the cold plasma equations for both positive and negative species with arbitrary value of Δ in dimensionless form.

$$\partial_t \delta n_d + \partial_x [v + \frac{V\delta n_d + v\delta n_s}{2}] = 0 \tag{1}$$

$$\partial_t \delta n_s + \partial_x [V + \frac{V\delta n_s + v\delta n_d}{2}] = 0 \tag{2}$$

$$\partial_t V + \partial_x (\frac{V^2 + v^2}{4}) = -(1-\Delta)E \tag{3}$$

$$\partial_t v + \partial_x (\frac{Vv}{2}) = (1+\Delta)E \tag{4}$$

$$\partial_x E = \delta n_d \tag{5}$$

where $V = v_+ + v_-$, $v = v_+ - v_-$, $\delta n_d = \delta n_+ - \delta n_- = n_+ - n_-$ and $\delta n_s = \delta n_+ + \delta_- = n_+ + n_- - 2$. Scalings of the various quantities are: $n_\pm \to n_\pm / n_0$, $x \to kx$, $t \to \omega_{p-} t$, $v_\pm \to v_\pm / \omega_{p-} k^{-1}$, $E \to E/(4\pi n_0 e k^{-1})$, with $\omega_{p-}^2 = 4\pi n_0 e^2/m_-$ and $\Delta = m_-/m_+$. Linearly δn_d and v couple to each other giving space charge generation and describing the cold plasma oscillation. Similarly δn_s and V corresponds to the quasineutral 'ion - acoustic' type perturbations which linearly give a zero - frequency mode because the plasma is cold.

Using $n_-(x,0) = 1 + A\cos kx$, $n_+(x,0) = 1$ and $v_\pm(x,0) = 0$, as initial conditions, the solutions of the linearised equations show coherent oscillations at the plasma frequency ω_p. Both the species oscillate with the same frequency which is independent of position. In the 2nd order, the solutions are expressed as:

$$\delta n_d^{(2)} = -A^2 \cos 2kx [\frac{1-\Delta}{1+\Delta}(\frac{1}{2} + \frac{1}{4}\omega_p t \sin\omega_p t + \frac{1}{2}\cos 2\omega_p t - \cos\omega_p t) - \frac{1}{4}\omega_p t \sin\omega_p t] \tag{6}$$

$$\delta n_s^{(2)} = \frac{A^2}{2}\cos 2kx [\frac{\Delta}{1+\Delta}t^2 - \frac{\Delta(1-\Delta)}{(1+\Delta)^2}\omega_p t \sin\omega_p t - \frac{3}{8}(1-\cos 2\omega_p t) - (\frac{1-\Delta}{1+\Delta})^2 (2\cos\omega_p t - \frac{5}{8}\cos 2\omega_p t - \frac{11}{8})] \tag{7}$$

$$V^{(2)} = -\frac{A^2}{2k}\sin 2kx [\frac{\Delta}{1+\Delta}t + \frac{\omega_p}{2}(\frac{1-\Delta}{1+\Delta})^2 (\frac{1-3\Delta}{2(1-\Delta)}\omega_p t \cos\omega_p t + \frac{7-5\Delta}{2(1-\Delta)}\sin\omega_p t - \frac{5}{4}\sin 2\omega_p t) - \frac{1}{8}\omega_p \sin 2\omega_p t] \tag{8}$$

$$v^{(2)} = -\frac{A^2\omega_p}{8k}\sin 2kx[\sin\omega_p t - \omega_p t\cos\omega_p t - \frac{1-\Delta}{1+\Delta}(2\sin 2\omega_p t - 3\sin\omega_p t - \omega_p t\cos\omega_p t)] \quad (9)$$

The 2nd order solutions clearly exhibit generation of 2nd harmonics in space and time as well as bunching of plasma particles in space. Both of these features are also evident in the solution of Kaw et. al. [5] and Infeld et. al. [6]; but in contrast to their work, where the background ion density was kept fixed in time, here the density of the plasma particles self - consistently changes with time as $\sim t^2$. Because of variation of plasma density with time, the phase mixing of an initial coherent oscillation happens much faster in this case. To make an estimate of the phase mixing time, consider the charge density equation (δn_d in this case) which, correct upto third order stands as

$$\partial_{tt}\delta n_d + \omega_p^2[1 + \frac{A^2t^2\Delta}{4\omega_p^2}\cos 2kx]\delta n_d \approx 0 \quad (10)$$

Using the initial conditions $\delta n_d = A\cos kx$ and $\partial_t \delta n_d = 0$ the WKB solution of the above equation is

$$\delta n_d \approx A\cos kx \sum_{n=-\infty}^{n=\infty} \cos(\omega_p t + \frac{n\pi}{2} - 2nkx) J_n(\frac{A^2t^3\Delta}{24\sqrt{1+\Delta}}) \quad (11)$$

The above expression clearly shows that the energy which was initially in the primary wave at mode k goes into higher and higher harmonics as time progresses. This can be interpreted as damping of the primary wave due to mode coupling to higher and higher modes. Microscopically, as the plasma particles oscillate at the local plasma frequency, they gradually go out of phase and eventually the initial coherence is lost. The time scale in which the initial coherence is lost (or the phase mixing time) can be seen from equation (11) as $\omega_{p-}t_{mix} \sim [A^2\Delta/(24\sqrt{1+\Delta})]^{-1/3}$. It shows that only for the ideal case $\Delta = 0.0$ (infinitely massive ions), phase mixing time is infinity, i.e. the initial coherence is maintained indefinitely [2,3].

Now we present results from a 1-D particle-in-cell simulation which confirms our scaling of phase mixing time. For numerical simulation, we have used a 1-dimensional model with periodic boundary conditions and have followed 5120 electrons and as many positively charged particles (the plasma taken as a whole is neutral) in their own self - consistent fields. The particles are initially at rest and the system is set into motion by giving a density perturbation of the form $n_- = 1 + \delta\cos kx$ to the electrons. In the simulation, we follow the time development of various modes of charge density (δn_d). Fig. (1) shows the variation of $\mid \delta n_d \mid_{n=1}$ for $\Delta = 0.1$ and $A = 0.1$. It is clear from the figure that our approximate expression (11) captures the early evolution of the plasma quite well. Fig. 2 shows the variation of $\omega_p t_{mix}$ with Δ for a fixed $A = 0.1$ (curve (1)) and with A for a fixed $\Delta = 0.01$ (curve (2)). These curves clearly confirm our formula for phase mixing time.

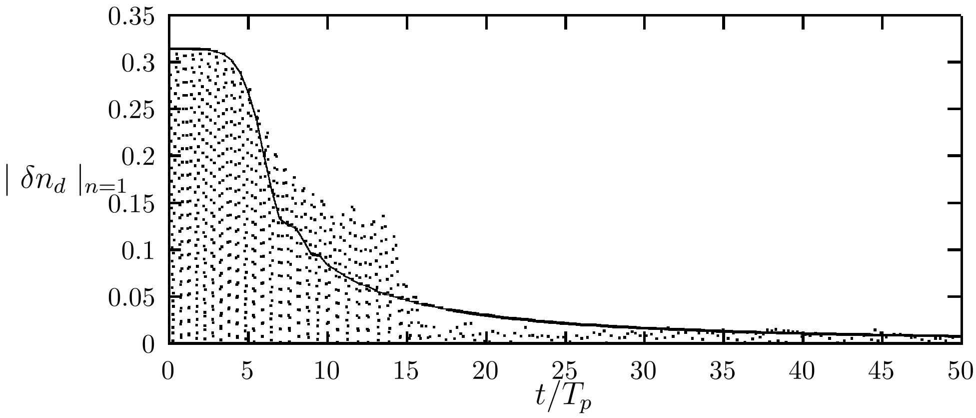

Fig. 1.$\mid \delta n_d \mid_{n=1}$ vs. t/T_P for Δ = 0.1 and A = 0.1

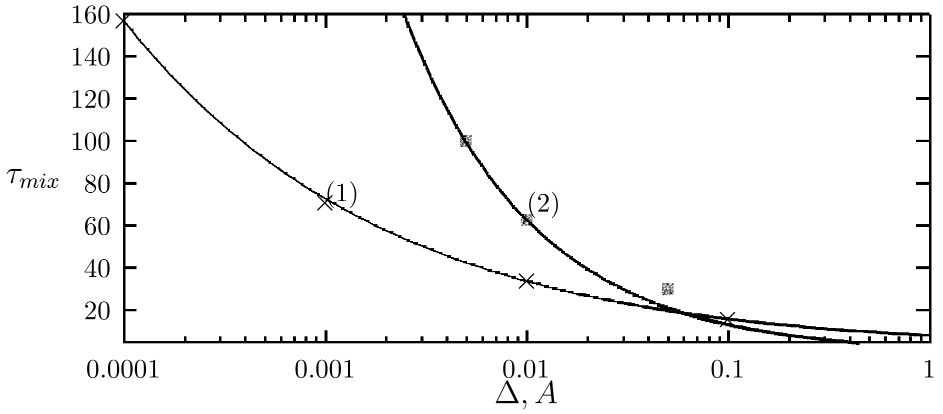

Fig. 2.τ_{mix} vs. Δ and A

In conclusion, we have shown that phase mixing damping of plasma oscillations arises in a homogeneous plasma with finite mass ratio, for arbitrarily low amplitudes of the oscillation.

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