INTERFACE LOCALIZED MODES (ILMs)

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Abstract

The theory of resistive free boundary modes localized at the plasma-vacuum interface in a plane slab equilibrium is improved and extended. Such modes are unstable whenever the current density and the magnetic field are not perpendicular to each other in the interface. The instability occurs for a wide range of angles between the nodal lines and the magnetic field lines in the interface and does not depend on a mode resonant surface. For sufficiently small wave length the unstable eigenmodes become independent of the resistivity and their growth rates diverge like the inverse square root of the wave length.

1. The model

The equilibrium depends only on one Cartesian coordinate x, which is chosen such that the plasma occupies the interval $-x_p < x < 0$, and the plasma-vacuum interface is at $x = 0$. The mass density ρ and the resistivity η are constant.

The equations of incompressible resistive MHD are linearized about the equilibrium and the perturbations are assumed proportional to $exp(\sigma t + i\mathbf{k}\cdot\mathbf{x})$, where σ is the growth rate, $\mathbf{x}$ is the position vector, and $\mathbf{k}$ is the wave vector, with components only in the two ignorable directions. The problem is made nondimensional in terms of the plasma diameter x_p, the magnitude B_0 of the magnetic field in the interface, the Alfven speed $u_A = B_0/(\mu_0\rho)^{\frac{1}{2}}$, the Alfven transit time $\tau = x_p/u_A$, and the quantity $\eta_0 = x_p u_A$, as units of length, magnetic field, velocity, time, and resistivity.

After integrating the equations in the vacuum, and then eliminating all other perturbing quantities, one is left with a set of equations for the two quantities $u = -iu_x$ and $b = b_x$, where $\mathbf{u}$ and $\mathbf{b}$ are the perturbing plasma velocity and magnetic field. For $-1 \leq x \leq 0$, one obtains the ordinary differential equations

$$\eta(b'' - k^2 b) - (\sigma b + Fu) = 0, \tag{1}$$

$$\sigma\eta(u'' - k^2 u) - F(\sigma b + Fu) + \eta F'' b = 0, \tag{2}$$

where $k = |\mathbf{k}|$, and $F = \mathbf{k} {\cdot} \mathbf{B}$. At $x = 0$ the boundary conditions are

$$\sigma[b' + k\ coth(kx_V)b] + F'u = o, \tag{3}$$

$$\sigma^2 u' + F'Fu + \sigma F'b = 0, \tag{4}$$

where x_V is the vacuum diameter. At $x = -1$ there are two independent boundary conditions which we do not specify. One thus has an eigenvalue problem with the growth rate σ as an eigenvalue parameter.

2. Localized eigenfunctions

It is assumed that for $\eta \ll 1,\ k \gg 1$ there are solutions of eqs. (1), (2) which are localized near the interface $x = 0$. Then in leading order the coefficients of eqs. (1) and (2) can be considered to be constant.

$$F = F_0,\ F' = F_0',\ F'' = F_0''. \tag{5}$$

In addition, it is assumed that the interface is non-resonant $(F_0 \neq 0)$. The solution of eqs. (1), (2) which rapidly tends to zero for $x < 0$ can then be written in the form

$$\begin{pmatrix} u \\ b \end{pmatrix} = \begin{pmatrix} -\sigma \\ F_0 \end{pmatrix} C_1\ exp(kx) + \begin{pmatrix} F_0 \\ \sigma \end{pmatrix} C_2\ exp(\phi x) \tag{6}$$

where

$$\sigma\eta(\phi^2 - k^2) = F_0^2 + \sigma^2. \tag{7}$$

A more rigorous derivation can be found in [1]. The boundary conditions (3) and (4) represent two linear homogeneous equations for the two unknowns C_1 and C_2, whose determinant must vanish.

$$(2F_0^2 + \sigma^2)\sigma^2\phi + F_0'F_0(2F_0^2 + \sigma^2) + \sigma^4 k = 0. \tag{8}$$

Eliminating ϕ from eqs. (7) and (8) one finds

$$\begin{array}{c} \eta F_0'F_0(2F_0^2 + 3\sigma^2)[F_0'F_0(2F_0^2 + 3\sigma^2) + 2\sigma^4 k] \\ -\sigma^3(F_0^2 + \sigma^2)[(2F_0^2 + \sigma^2)^2 + 4\eta k^2 F_0^2 \sigma] = 0, \end{array} \tag{9}$$

the dispersion relation for ILMs. Once a root has been found, the corresponding ϕ is computed from eq.(7), where eq.(8) determins the sign. If

$$Re(\phi) > 0 \tag{10}$$

then the root is an eigenvalue.

3. Interface localized instabilities

There is only one root of eq.(9) which is real and positive and meets condition (10) and hence is the only root of interest. This root is $o(k)$, which implies that eqs. (8) and (9) reduce to

$$\sigma^2\phi + F_0'F_0 = 0, \tag{11}$$

$$\sigma^3(\eta k^2\sigma + F_0^2) - \eta(F_0'F_0)^2 = 0. \tag{12}$$

Since $\phi^2 > 0$ if $\sigma > 0$ condition (10) is equivalent to $F_0'F_0 < 0$. The stability criterion for a given wave vector $\mathbf{k}$ is thus $F_0'F_0 \geq 0$. The marginal wave vectors are perpendicular to the magnetic field vector $\mathbf{B_0}$ or parallel to the current density vector $\mathbf{J_0}$, and no unstable wave vectors exist if $\mathbf{J_0}\cdot\mathbf{B_0} = 0$, which is thus the overall stability criterion. If the quantity $\eta^2|F_0'|k^3/|F_0|^3$ is large then the solution of equation (12) is

$$\sigma \cong (F_0'^2F_0^2/k^2)^{1/4} \sim O(k^{1/2}). \tag{13}$$

4. Summary and discussion

ILMs depend on equilibrium quantities in the interface, but nowhere else. In particular, they are independent of the boundary conditions on the other side of the plasma slab.

ILMs are almost always unstable, i.e., they are stable only in the special case of zero shear in the interface. Instability occurs , in general, for a finite range of angles between the wave vector and the magnetic field in the interface. ILMs thus need not be localized at a resonant surface, and hence are not related to tearing modes.

We do not try to answer the question whether the present calculation predicts an observable effect, or merely shows that the underlying model is unapplicable. Instead, we remark that our results are certainly unapplicable if the localization length fails to be much larger than any of the lengths that are tacitly assumed to vanish, such as gyroradius, Debye length, mean free path, or the distance over which the resistivity changes (i.e., the thickness of the actual boundary layer between plasma and vacuum). These restrictions put lower bounds upon the wave length, and hence upper bounds upon the growth rate that one may expect to observe.

Reference

[1] D. Lortz and G. Spies: Phys. Plasmas **2**, 1255 (1998)