

FAST HEAT PULSE PROPAGATION IN HOT PLASMAS

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1. Introduction

Among the many unresolved aspects of heat transport in magnetically confined plasmas, and namely in tokamaks, there is the new evidence of ultra fast propagation of heat perturbations, on time scales much shorter than allowed by acceptable diffusion rates [1,2]. In contrast with early experiments of small heat pulse propagation recently a wealth of new accurate experimental observations has proved that the processes involved can be non diffusive because for instance ΔT does not decrease as r^{-2} but presents even a change of sign of the heat disturbances generated at the plasma edge and nearly instantaneously measured in the central region [1,2]. Explanations of the phenomena in terms of heat diffusion equations embodying a variety of heat conductivity models have failed to explain the observations, leaving ground to physically unclear concepts as "non local transport" where it is meant that the heat diffusivity depends explicitly on time $\chi = \chi(\vec{x}, t)$, allowing an "instantaneous" response of the heat flux in points radially far apart [1]. In alternative to this unphysical assumption we consider here the role of non-linear dependence of χ on temperature and its gradient and interpret the *facts* concerning the *seemingly* "non local" *heat propagation* in terms of a property [3] of the non-linear diffusion equations generally employed in the treatment of one or two-fluid plasma heat transport, in the paradigmatic form

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[\chi \left(T, \frac{\partial T}{\partial x} \right) \frac{\partial T}{\partial x} \right] \quad (1)$$

The important point is that an equation of type (1) has special solutions which are *not* of diffusive type, but of propagating wave type, of argument $(x-vt)$ with a finite velocity v which depends on the initial width (or typical wavelength) of the perturbation. A heat diffusivity of the type $\chi_{\text{eff}} = \chi_{\beta}(T)^{\delta-\beta} (\nabla T)^{\beta}$ is considered, where $\chi_{\beta} = \chi_0 L^{\beta}$, L is the scale length of the temperature perturbation β and δ are appropriate exponents, and χ_0 is the thermal diffusivity for $\beta = 0$. If L is "large" compared with the plasma minor radius a , the $\text{grad}T$ contribution is negligible and the scaling is akin to Coppi Mazzucato Gruber (CMG) ($\beta = 0$, $\delta = -1$), otherwise it is more like Zhang Mahajan (ZhMa) ($\beta = 1$, $\delta = 1$) [3].

2. Propagating solutions of nonlinear heat transport equations

The geometry considered is a one dimensional plasma slab in the closed interval $0 \leq x \leq a$. Equation (1) can have a solution of the propagating type $T(x, t) = f(z)$ with $z = x - vt$. The velocity v is assumed to be a constant, and Eq. (1) takes the form :

$$\frac{df}{dz} = -\frac{\chi_\beta}{v} \frac{\partial}{\partial z} \left[\frac{\beta+1}{\delta+1} \left(\frac{d}{dz} f^{\frac{\delta+1}{\beta+1}} \right) \right]^{\beta+1} \quad (2)$$

with the "dispersion relation"

$$k = \left(\frac{v}{\chi_\beta} \right)^{1/(\beta+1)} \left(\frac{\delta+1}{\beta+1} \right) \quad (3)$$

Changing dependent variable to $y = f^{\frac{\delta+1}{\beta+1}}$ and integrating twice with respect to z one obtains

$$k(z - z_0) = \int_{y_0}^y \frac{dy}{\left[y_0^{(\delta+1)/(\beta+1)} - y^{(\delta+1)/(\beta+1)} \right]^{1/(\beta+1)}} \quad (4)$$

Equation (4) gives the solution in inverse form, expressing $z = x - vt$ as a function of $T = y^{(\delta+1)/(\beta+1)}$. The exact formula (4) may be considered to include a uniform and constant background T_0 and a heat wave $\tilde{T}(x - vt)$. However, in a linear approximation a mildly non uniform background can be considered so that $T = \tilde{T} + T_0 - \alpha x$ provided $\alpha \ll \left| \frac{\partial \tilde{T}}{\partial x} \right|$. An

interesting property of the solutions is given by the dispersion relation (3) relating the wave number $k = 2\pi/\lambda_v$ (or L^{-1}) to the velocity pulse v , which plays the role of the phase velocity, or front velocity for shock-like solutions. It is apparent that the velocity $v = \frac{\chi_0}{L} \left(\frac{\beta+1}{\delta+1} \right)^{\beta+1}$ is not vanishing for any value of β and $L > 0$. Heat transport times associated with v can be much shorter than diffusive times $\tau_\chi \approx a^2/\chi$. For large k , i.e. pulses compact in space, the possibility of a fast propagation of pulses of high amplitude ($|\tilde{T}| \approx |T_0 - \alpha x|$) exists as inherent properties of the diffusion operator. For propagating solutions of oscillatory type one may relate the phase velocity $v_{ph} = \omega/k = v$ to the "group velocity" $v_g = (\partial k / \partial \omega)^{-1}$ and find from the relation (3) that $v_g = (2 + \beta)v$ (*nota bene*, independent of the diffusion coefficient and of δ).

3. Discussion of particular solutions

Physical insight is helped by analysis of few particular idealized solutions. For $\beta=1$ $\delta=0$ and $\chi_1 = \chi_0 L$, $T = \tilde{T} + T_0$ the solution of eq.(2) is:

$$T / (\tilde{T}_{t=0} + T_0) = \cos^2 \left(\frac{1}{2} \sqrt{|v/\chi_1|} (x - x_0 + vt) \right) \quad (5)$$

T_0 represents the background temperature and \tilde{T} a travelling pulse. For $t>0$ and $x > x_0$ the perturbation propagates, framed within the characteristics $z_i = x_0 - x - vt$ and $z_f = x_0 - x - vt + \phi$ (see Figs.1 and 2 for $\phi=\pi/2$). The expression $v = (8\pi\chi_0/\lambda_v)$ shows that *the shorter the pulse wavelength the faster is the speed of propagation* v . Another example with similar properties, but for a negative pulse, is obtained with $\beta=1$ $\delta=1$, in the form $T = T_0 - (v/4\chi_1)(x - vt)^2$ (as shown in Fig.3). In this way a variety of somewhat artificial particular cases can be constructed whose value is essentially that of allowing estimates of the pulse propagation time scale. Indeed comparison of order of magnitude with published experimental data is encouraging as shown in Table 1. Furthermore it can and will be shown (elsewhere) that the use of the dispersion relation (3) in conjunction with the equations, obtainable from (1), for the amplitude, width and velocity of the temperature perturbation can explain a change of sign of the propagating amplitude, as observed in experiments. As a simple connection of the model solution with measurements of heat pulse propagation for realistic parameter values let us consider data from recent RTP experiments [3]. (small radius $a=0.16m$). The value experimentally found in Ref.2 for the propagation time is $\Delta t = 600\mu s$ and $L \sim 2cm$ and $\chi_0 \approx 1.3 m^2/s$. With our notation the model gives for $\beta = \delta = 1$, $\chi_{eff} = \chi_0 L |\nabla T|$ a propagation speed $v = 4\chi_0/L$. The propagation time over a distance a is then $\Delta t = \frac{a}{v} = aL/4\chi_0$. For the parameters L and χ_0 we consider the following values: $L=0.01,0.02,0.03$ m; $\chi_0 = 1.0, 1.5, 2.0$ m^2/s in the range estimated from experiments ². Table 1 shows that order of magnitude estimates of Δt obtained from our model agree with observations for reasonable uncertainties in the measurements as well as in the modeling.

$L \backslash \chi_0$	1.0	1.5	2.0
0.01	400	270	200
0.02	800	530	400
0.03	1200	800	600

Table 1. Propagation times Δt in μs (L in m and χ_0 in m^2/s)

References

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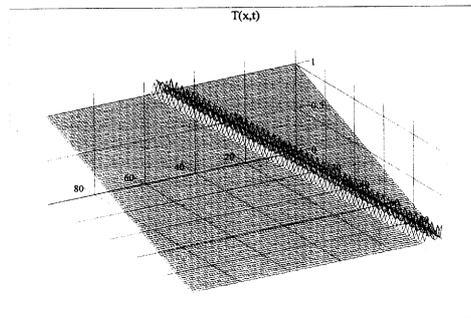
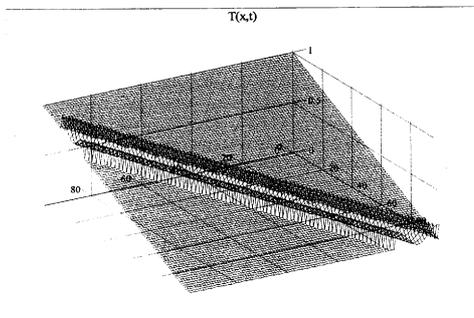
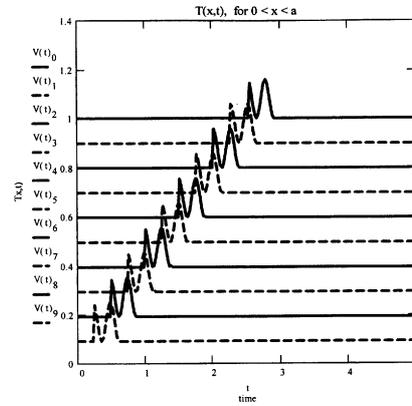
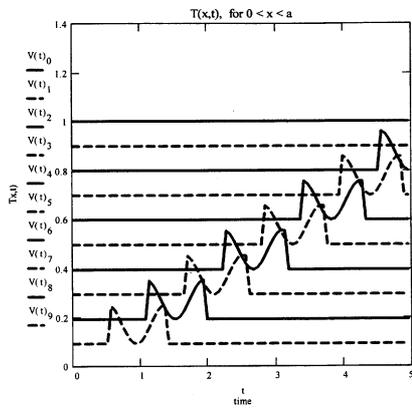


Fig. 1. a) (upper) Heat pulse propagation of type (5) bracketed between the two characteristics $z_i = x_0 - x - vt + \pi/2$ $z_f = x_0 - x - vt$, with: $\chi_0 = 0.1$, $\lambda_v = 2$; $a = 10$; $v = 3.948$; $k = 0.1$
 1-b) (lower) Propagation on a mildly non uniform background

Fig. 2. Same as Fig.1 with parameters: $\chi_0 = 0.1, \lambda_v = 1.5$; $a = 10$; $v = 7.018$; $k = 0.1$

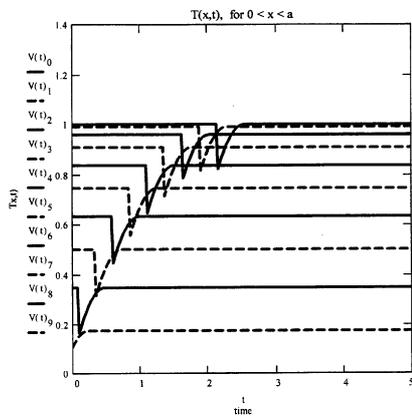


Fig. 3. Propagation of a negative pulse with parameters of Fig.2.