

STOCHASTIC INSTABILITY OF THE MODIFIED DECAY

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The wave-to-particle and weak nonlinear wave-to-wave interactions determine fundamental properties of plasma, electron beams, etc. The regular dynamics of these processes has been extensively studied. It is well established now that nearly all dynamical systems which describe plasmas phenomena and based on wave-to-particle and wave-to-wave type of coupling, in their extended phase space (a space of all canonical variables and parameters) have regions of chaotic behavior.

As it was shown in [1] the dynamics of wave-to-wave coupling will be chaotic when parameter

$$Ch = 2G / \Omega - 1 > 0, \quad (1)$$

where G is the increment of the three wave instability, which plays the role of the width of nonlinear resonance, Ω is the minimal frequency of the coupling waves, which corresponds to the distance between resonances of different waves.

In the present report a decay of the HF (high-frequency) electromagnetic wave which propagated in the uniform, unbounded plasma on the HF electromagnetic and LF (low-frequency) Langmuir waves is investigated. It is shown that the modified decay always stochastic unstable and that the criterion $Ch > 0$ well describes the shape of the stochastic region in the area of the control parameters.

Let us consider the decay of the HF electromagnetic wave (i) with frequency ω_i , wave vector \vec{k}_i and amplitude \vec{E}_i , which propagated in the uniform, unbounded plasma on the HF (s) electromagnetic wave $(\omega_s, \vec{k}_s, \vec{E}_s)$ and LF Langmuir wave $(\omega_{pe}, \vec{k}_p, \phi)$. In order to describe this process we started from Maxwell's equations for electromagnetic fields and hydrodynamic equations for plasma electrons. We neglected the movement of ions, assuming that background ions serves for compensation of electron charge. The averaging over the time t_o ($t_{slw} \gg t_o \gg t_{fst}$, $t_{slw} \propto 1/\omega_{pe}$, $t_{fst} \propto 1/\omega_i$ - periods of slow and fast variables, respectively) leads to the following system of coupled equations:

$$i \cdot \frac{d\varepsilon_i}{d\tau} = \varepsilon_s \rho \exp(i\Delta\tau), \quad i \cdot \frac{d\varepsilon_s}{d\tau} = \varepsilon_i \rho^* \exp(-i\Delta\tau), \quad \frac{d^2\rho}{d\tau^2} + \Omega^2\rho = -\varepsilon_i \varepsilon_s^* \exp(-i\Delta\tau), \quad (2)$$

$$\text{where } \varepsilon_i = \frac{E_i}{E_{io}}, \quad \varepsilon_s = \frac{E_s}{E_{io}} \sqrt{\frac{\omega_s}{\omega_i}}, \quad \rho = \frac{\delta n_p}{n_{eo}} \left[\frac{\omega_{pe}^2 \cos(\alpha) \pi m n_{eo}}{2k_p^2 \sqrt{\omega_i / \omega_s} E_{io}^2} \right]^{1/3}, \quad \Omega^2 = \left[\frac{8m^2 \omega_{pe} \omega_i \omega_s^2}{e^2 E_{io}^2 \cos^2(\alpha) k_p^2} \right]^{2/3},$$

$$\tau = t \left[E_{io}^2 \cos^2(\alpha) \frac{e^2 k_p^2 \omega_{pe}^2}{8m^2 \omega_i \omega_s^2} \right]^{1/3}, \quad \Delta = (\omega_i - \omega_s) \left[\frac{8m^2 \omega_i \omega_s^2}{e^2 E_{io}^2 \cos^2(\alpha) k_p^2 \omega_{pe}^2} \right]^{1/3}, \quad \vec{E}_i(t) \text{ and } \vec{E}_s(t) -$$

slowly varying in time amplitudes of the HF pumping and scattered waves, respectively,

δn_p - slowly varying in time LF plasma density, $\cos(\alpha)$ - angle between \vec{E}_i and \vec{E}_s ,

$E_{io} = E_i(t=0)$, $\omega_{pe}^2 = \frac{4\pi e^2 n_{eo}}{m}$ - plasma frequency. In order to obtain the set of equations (2)

we assume that there is a spatial synchronism between coupling waves: $\vec{k}_i - \vec{k}_s = \vec{k}_p$.

On the linear stage of the decay when $|\varepsilon_i| = \text{const}$ we can obtain from (2) the dispersion relation:

$$(\omega^2 - \Omega^2)(\omega + \Delta) = 1 \quad (3)$$

and following expressions for maximum values of the growth rates:

$$G = \text{Im} \omega = 1 / \sqrt{2\Omega}, \quad \Omega^2 \gg 1; \quad G = \text{Im} \omega = \sqrt{3} / 2, \quad \Omega^2 \ll 1.$$

In the first case, when the amplitude of the pumping wave is small, parameter $Ch < 0$, ($\Delta \propto \Omega$) and dynamics of the decay according to (1) must be regular. At large amplitudes of the incident wave $Ch > 0$, ($\Delta \propto 0$) and the decay must be chaotic. Note that the region of parameters where $K \gg 1$ is related to the modified decay.

The contour plot of the parameter Ch as function of Ω and Δ , obtained by means of numerical solution of the dispersion equation (3), we show in Fig. 1 (left). Fig. 1 (central) represents the maximum Lyapunov's index S (measure of the divergence of initially neighboring trajectories) obtained from numerical solution of differential equations (2). Magnitudes of the Ch and S in Fig. 1 are represented with the help of colours, as it is usually done on geographical maps. The correspondence between colours in Fig. 1 (central) and value of S is represented in Fig. 1 (right). Comparing Fig. 1 (left) and Fig. 1 (central), it is easy to see, that in the region of parameters, where $Ch > 0$, $S > 0$ too and dynamics of the modified decay is chaotic. In the region, where $Ch < 0$ - $S \approx 0$ and dynamics of the decay is regular. It is visible from the Figs. 1 (left and central) that criterion $Ch > 0$ well describes the shape of the chaotic region in the area of the control parameters Ω and Δ .

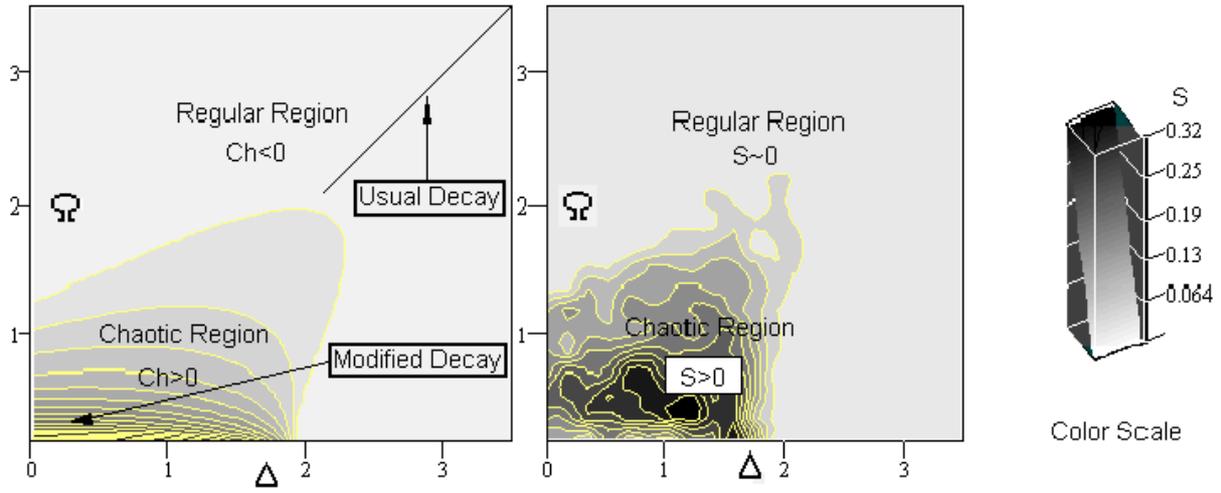


Fig. 1.

The set of equations (2) has been solved numerically for the different values of the Δ and Ω . Time-dependent dynamics of amplitude of pumping wave and its power spectra we show on Figs. 2 - 5. Fig. 2 show the temporal evolution of magnitude of the HF pumping wave amplitude $|\varepsilon_i|$ at $\Delta = \Omega = 12.5$. This is usual decay ($\omega_i - \omega_s = \omega_{pe}$, $\vec{k}_i - \vec{k}_s = \vec{k}_p$). In this case the increment $\text{Im}\omega = 1/\sqrt{2\Omega}$, $Ch < 0$ and the temporal dynamics of the decay is regular. The Lyapunov's index is zero in this case. In Fig. 3 in a logarithmic scale the power spectra of pumping wave is represented. Its form corresponds to the regular periodic process. Fig. 4 show temporal evolution of magnitude of the HF pumping wave amplitude $|\varepsilon_i|$ at $\Delta = 0.1; \Omega = 0.3$. This is the case of the modified decay, at which the increment $\text{Im}\omega \approx \sqrt{3}/2 > \Omega$ and $Ch > 0$. The temporal dynamics of the decay in this case is chaotic. The Lyapunov's index in this case goes to the constant value $S \approx 0.22$. In Fig. 5 in a logarithmic scale the power spectra of pumping wave is represented. Its form corresponds to the quite complicated, in fact chaotic process.

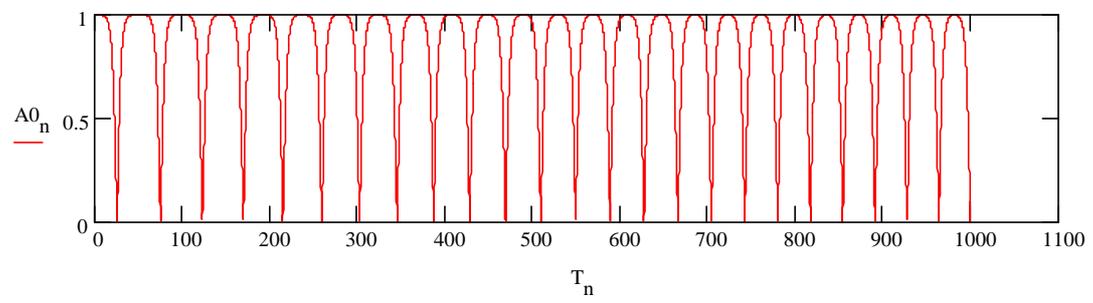


Fig.2. Temporal evolution of the HF pumping wave amplitude $|\varepsilon_i(t)|, \Delta = 12.5, \Omega = 12.5$.

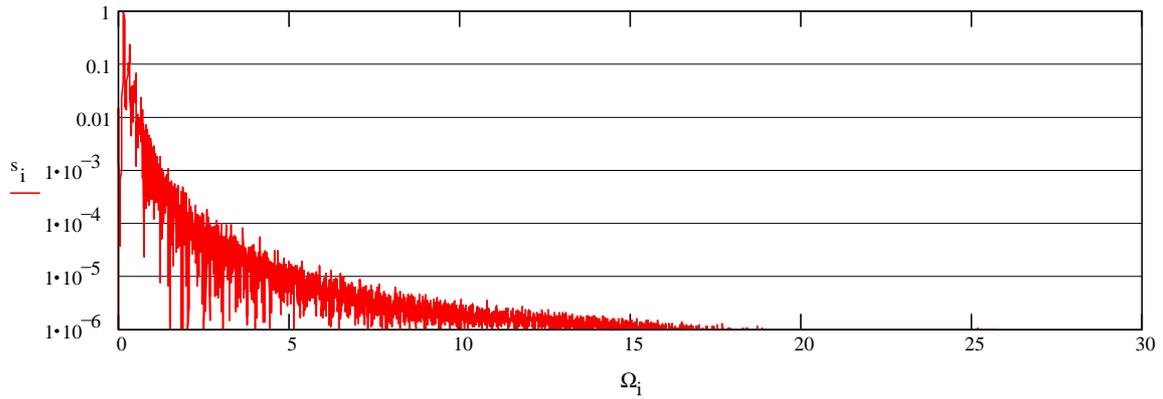


Fig.3. Power spectra of HF electromagnetic wave $\varepsilon_i(t), \Delta = 12.5, \Omega = 12.5$.

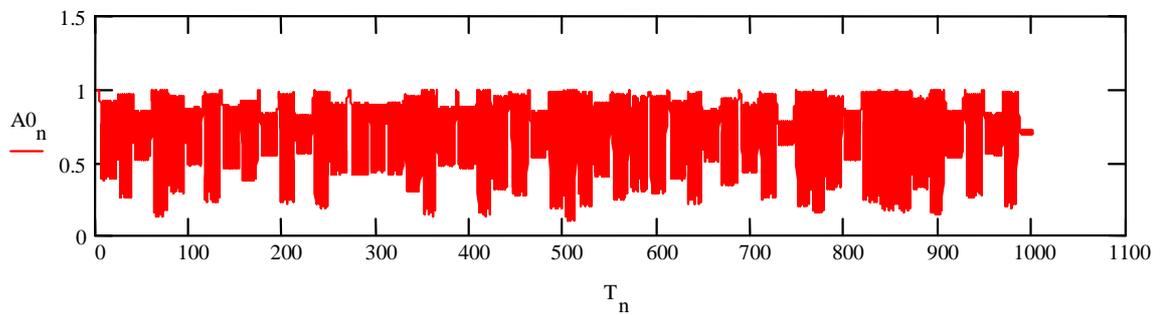


Fig.4. Temporal evolution of the HF pumping wave amplitude $|\varepsilon_i(t)|, \Delta = 0.1, \Omega = 0.3$.

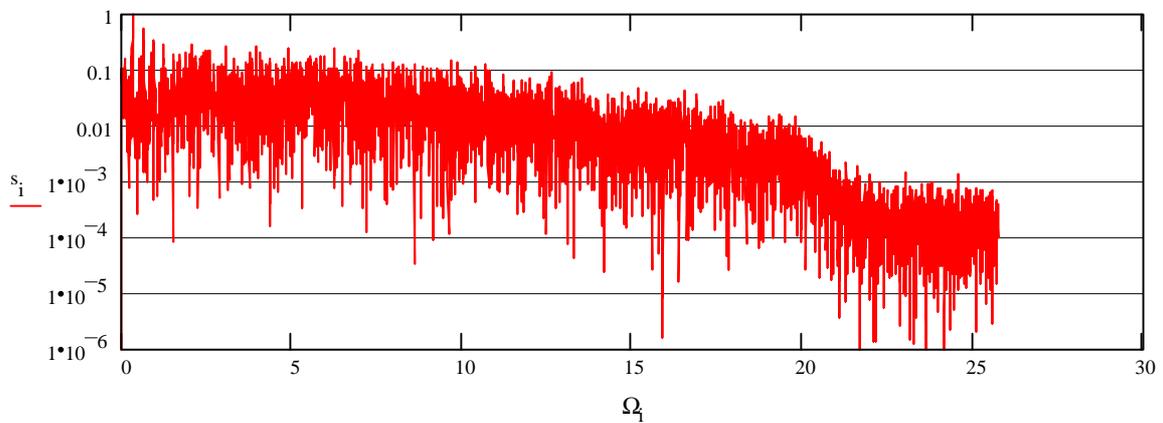


Fig.5. Power spectra of HF electromagnetic wave $\varepsilon_i(t), \Delta = 0.1, \Omega = 0.3$.

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References

- [1] V.A. Buts, O.V. Manuilenko, K.N. Stepanov and A.P. Tolstoluzhskii: Fizika Plasmy **20**, 794 (1994)