

CNOIDAL MODES OSCILLATIONS AS A GENERALIZATION OF ION ACOUSTIC WAVES AND SOLITON

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1. Introduction

In nondissipative plasma with weak nonlinearity and dispersion, the propagation of ion - acoustic waves and solitons are described by the normalized Korteweg de Vries (KdV) equation [1]:

$$\partial_{\tau} \Psi + \Psi \partial_{\xi} \Psi + \frac{1}{\theta^2} \partial_{\xi \xi \xi} \Psi = 0 \quad (1)$$

where Ψ is the perturbed - to - unperturbed plasma density ratio, $\delta n/n_0$, or the wave potential normalized by the electron temperature $e\Phi/kT_e$, $\xi = x\lambda_D^{-1}$ is the spatial coordinate normalized by the electron Debye length λ_D , $\tau = \omega_{p_i} t$ is the time normalized by the ion plasma frequency ω_{p_i} and θ the ratio between the dispersion and nonlinear coefficient. The equation (1) expresses the fact that nonlinear effects, $(\Psi \partial_{\xi})$, and dispersion ones, $(\theta^{-2} \partial_{\xi \xi \xi})$, are responsible for the normalized time dependence of the perturbed function.

In this paper new solutions of the KdV equation connected with the experimental studies are discussed. In the second section the cnoidal solution and dispersion relation are obtained. Ion-acoustic and soliton waves result as degeneration of cnoidal modes oscillations. Experimental results are describe in the third section. The conclusions of the present paper are summarized in section forth.

2. Mathematical model

By using the method given in [2], the perturbative solutions of Eq. (1) are of the form:

$$\psi = \psi_0 + A \text{cn}^2(u; s), \quad u = \alpha(\xi - c\tau) \quad (2)$$

with

$$c = \frac{u - c_s}{c_s} = \psi_0 + \frac{A}{3}(2 - s^{-2}), \quad \alpha = \frac{\theta}{s} \left(\frac{A}{12} \right)^{1/2}, \quad \psi_0 = \text{const.} \quad (3)$$

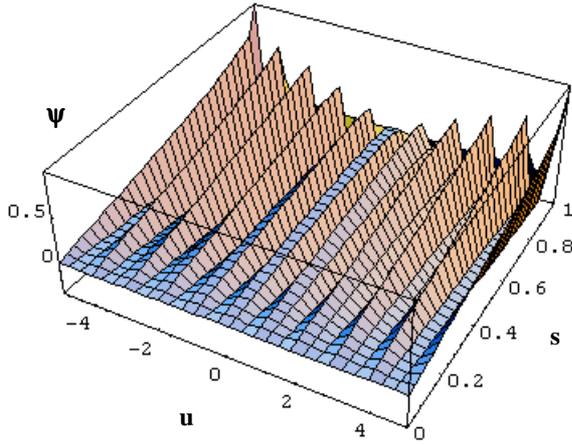


Fig. 1. Cnoidal waves.

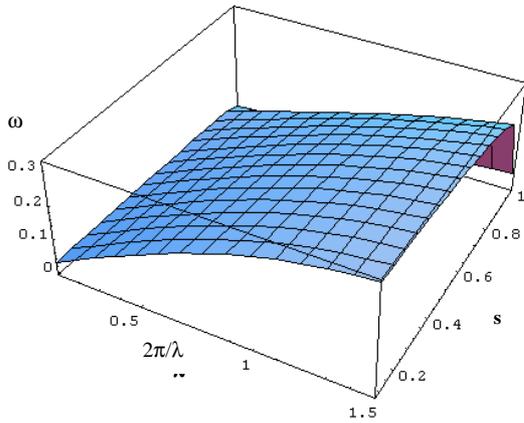


Fig. 2. Normalized dispersion relation.

In these relations A is the normalized amplitude, c the normalized perturbation speed, c_s the ion-acoustic speed and cn the Jacobian elliptic function with modulus s [3]. The result (2) shows that the modes of the plasma oscillations are cnoidal (Fig. 1). The dispersive relation corresponding to cnoidal modes oscillations comes out of the periodicity of the solution [2]. It has the expression:

$$2K(s)v = \frac{1}{\sqrt{\frac{1}{sn^2(2K/\lambda; s)} - 1 + \frac{E(s)}{K(s)}}}, \quad (4)$$

where $\omega=2K(s)v$ is the normalized wave frequency, λ the normalized wave length, sn the Jacobian elliptic function, $K(s)$ the complete integral of the first kind and $E(s)$ the complete integral of the second kind [3]. In Fig. 2 the dispersion relation (4) is plotted. For small values of θ the dispersive effects appear before the non -

linear ones, while for great θ values the situation inverts. Moreover through the variation of the module s , one can discuss the whole range of the wave modes according to relation (3) between s , θ and A . But the following degenerations are possible to be analyzed in view of experimental evidence:

- i) for $s \rightarrow 0$, situation in which the dispersive effects are dominant with respect to nonlinear ones, (2) reduces to the ion acoustic wave packet. With the identifications $\alpha' = k$, $4\theta^2 = \beta$, $k \frac{v}{c_s} = \omega$, from relations (3) and by multiplication with k we obtain the known results [4];
- ii) for $s \rightarrow 1$, situation in which the dispersive and nonlinear effects become asymptotic comparable, relation (2) reduces to soliton packet;

iii) for $s \equiv 1$, situation in which we restrict to the conditions where the dispersive and non-linear effects are comparable, (2) with $\psi_0'' = 0$ reduces to the soliton:

$$\psi'' = A'' \operatorname{sech}^2[\alpha''(\xi - c''\tau)] \quad (5)$$

and (3) to

$$c'' = \frac{A''}{3}; \quad \alpha'' = \theta \left(\frac{A''}{12} \right)^{\frac{1}{2}}; \quad A'' = A(s=1); \quad \alpha'' = \alpha(s=1); \quad c'' = c(s=1) . \quad (6)$$

Thus : 1) the soliton speed c'' relative to the uniform state ψ_0'' is proportional (in this case by a factor 1/3) to the amplitude, 2) the width $D = (\alpha'')^{-1}$ of the solitary wave is inversely proportional to the square root of the wave amplitude and therefore taller solitary waves are narrower in width and travel faster than shorter ones, 3) the amplitude A'' is independent of the uniform state ψ_0'' , the limit of $\psi''(\xi, \tau)$ as $|\tau| \rightarrow \infty$.

3. Experimental results

The experimental results were obtained in the DP-plasma device presented in [4,5]. The $s \rightarrow 0$ approximation may be applied when the ion-acoustic wave is excited by a bi-potential structure and small amplitude of the exciting signal [4]. In this case nonlinear effects are negligible and dispersion can be dominant. Physical results are presented in Fig. 3, where experimental points fit very well with theoretical dispersion relation (full curve) obtained for s up to about 0.91. This results correspond also to that presented in Fig. 2 which shows that dispersion relation is very little dependent on s up to about 0.96.

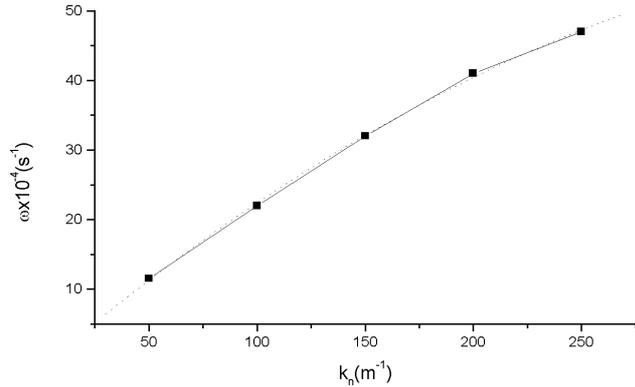


Fig. 3. The experimental (full curve) and theoretical (dot curve) dispersion relation.

For $s \rightarrow 1$ approximation a DP-mode method of the ion-acoustic soliton was used. The exciting signal was a short pulse of rather large amplitude applied to the source chamber with respect to the target one. In general, the soliton speed is considered to be a linear function of the soliton amplitude, but taking into account relation (3), a possible nonlinear dependence of the soliton speed might be expected. This result is presented in Fig. 4 where soliton amplitude

multiplied by time propagation for a fix position of the soliton detector shows a clear increasing with amplitude of the soliton. Similar results are also presented in [7].

4. Conclusions

The main results of the work are the following:

- the perturbative solutions of the KdV equation have been made in the shape of the elliptical functions cn^2 of modulus s ; these solutions have been associated to the cnoidal modes of oscillations of the plasma. The relation of the cnoidal modes' dispersion has been found.
- the ion - acoustic waves and the solitons have been obtained as representing degeneration of the cnoidal modes for $s \rightarrow 0$ respectively $s \equiv 1$;
- the dispersion experimental curve was obtained through the excitation of the ion - acoustic wave by a plane bipolar potential structure, and it was compared with that provided by the theoretical model taking into account the significance of the elliptical function's modulus (see the degeneration of oscillation cnoidal modules), the value $s = 0.91$ indicates the fact that inside the examined plasma the dispersive effects are dominant comparing with nonlinear one.

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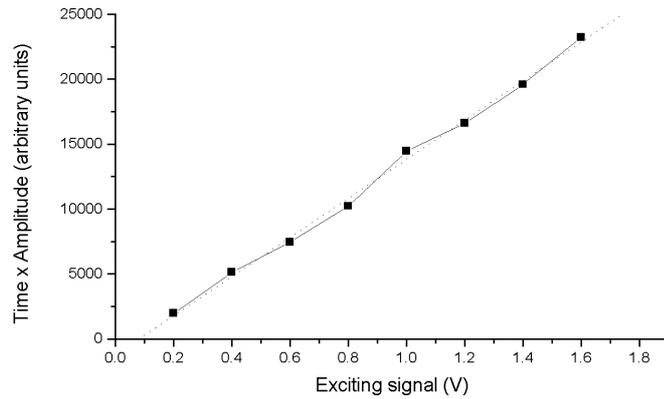


Fig. 4. Time x amplitude dependence function of exciting signal.