THEORY OF CROSS-POLARIZATION SCATTERING FROM MAGNETIC FLUCTUATIONS IN THE UPPER HYBRID RESONANCE

E.Z. Gusakov

Ioffe Institute, St.Petersburg, Russia

Development of the magnetic turbulence diagnostics for tokamak experiments is important because of possible interconnection between magnetic fluctuations and transport. The cross-polarization scattering (CPS) diagnostics utilizing microwave probing perpendicular to the tokamak magnetic field is a feasible candidate for measuring the magnetic turbulence level in the hot plasma core because the density fluctuations do not contribute to the CPS signal in this experimental geometry [1,2].

The present paper is devoted to the theoretical analysis of the scheme in which the CPS takes place in the vicinity of the Upper Hybrid Resonance (UHR) of the probing or scattered microwave [3]. It is shown that unlike the laser scattering case [1] in the electron cyclotron frequency range the nonlinear pondermotive force and current nonlinearities are comparable to the Lorenz force nonlinearity and should be taken into account when calculating the nonlinear current leading to the CPS effect. The CPS signal for x $\rightarrow$ 0 and 0 $\rightarrow$ x forward scattering processes is calculated for the Gaussian probing beam taking into account refraction and diffraction effects and linear conversion of the X-mode in the UHR.

The CPS signal according to [4] is given by

$$A_{cps} = \frac{1}{4}\int \vec{j}_s \vec{E}_s^{\,+} d^3 r \tag{1}$$

where $\left|A_{cps}\right|^2$ is a scattered power in the receiving antenna; $\vec{E}_s^{\,+}$ is the electric field distribution radiated by receiving antenna at unite power in the case of opposite magnetic field. The general expression for the nonlinear current in the MHD approximation is given by

$$\vec{j}_s = eN\vec{v}_s + en_i\vec{v}_\Omega + en_\Omega\vec{v}_i \tag{2}$$

where subscripts "i", "s" and "Ω" correspond to incident, scattered waves and fluctuations; v_s is the electron velocity at the scattered wave frequency $\omega_s = \omega_i + \Omega$, determined by the nonlinearities of the motion equations and N is unperturbed plasma density. In the case of incident X-mode and scattered O-mode propagating across magnetic field the only component of (2) contributing to the CPS signal is the component parallel to it

$$j_{sz} = eNv_{sz} + en_i v_{\Omega z} \tag{3}$$

where the last term have not vanished because the X-mode possesses the electrostatic component in the UHR. Taking into account the electron dynamics parallel to the magnetic field one readily obtains the expression for the CPS current in the form

$$j_{sz} = \frac{eN}{\omega_i}\left(\vec{k}_i + \vec{k}_\Omega\right)\vec{v}_i v_{\Omega z} + \frac{i\omega_{Pe}^2 m}{4\pi\omega_s}\frac{v_{ix}B_{\Omega y} - v_{iy}B_{\Omega x}}{c} \tag{4}$$

demonstrating that the CPS effect could be produced not only by magnetic field, but also by parallel electron velocity fluctuations. Using the Ampere law we obtain from (4)

$$j_{sz} = i\frac{\vec{k}_s\vec{v}_i}{\omega_s}\frac{c}{4\pi}\left[\vec{k}_\Omega \times \vec{B}_\Omega\right]_z + \frac{i\omega_{Pe}^2}{4\pi\omega_i}\frac{\left[\vec{v}_i \times \vec{B}_\Omega\right]_z}{c} \tag{5}$$

in which the Bragg condition $k_s = k_i + k_\Omega$ was used. It is easy to show that, unlike conclusion of [5], the first term is not negligible in the ECR frequency range and is dominant in the UHR, where $k_{\Omega x}$ - component in the inhomogenity direction could be very large.

The main component of O-mode field - E_{sz}^+ in the paraxial approximation and slab geometry is given by

$$E_{sz}^+ = \sqrt{\frac{\omega_s}{ck_0}}\int\frac{dk_{sy}dk_{sz}}{(2\pi)^2}\exp i\left\{-\int_0^x k_0 dx' + \frac{ck_{sy}^2}{\omega}\int_0^x O_y dx' + \frac{ck_{sz}^2}{\omega}\int_0^x O_z dx' + ik_{sy}y + ik_{sz}z\right\}f\left(k_{sy}, k_{sz}\right) \tag{6}$$

where $f\left(k_y, k_z\right) = \frac{4\pi\rho}{c}\exp\left\{-\frac{\rho^2}{c}\left(k_y^2 + k_z^2\right)\right\}$ is the O-mode antenna pattern; $k_0 = \frac{\omega_s}{c}\sqrt{1 - \frac{\omega_{Pe}^2(x)}{\omega_s^2}}$; $x = a$ is the position of O-mode antenna at the low field side; $O_y = \frac{\omega_s}{2k_0 c}$, $O_Z = \frac{k_0 c}{2\omega_s}$.

The electron velocity at the incident wave frequency is given by

$$v_{ix} = \frac{e\omega_i}{m\left(\omega_i^2 - \omega_{ce}^2\right)}\left(iE_{ix} - \frac{\omega_{ce}}{\omega}E_{iy}\right);\; v_{iy} = \frac{e\omega_i}{m\left(\omega_i^2 - \omega_{ce}^2\right)}\left(\frac{\omega_{ce}}{\omega}E_{ix} + iE_{iy}\right) \tag{7}$$

where as the X-mode field components are according to [4,6] given by:

$$
\begin{aligned}
E_{ix} &= \int \frac{dk_{iy}dk_{iz}}{(2\pi)^2} D \int_0^{\infty} \exp\left\{ i\left[\frac{\kappa t^3}{3} - \frac{1}{t} + \xi t \right] \right\} dt \\
E_{iy} &= \int \frac{dk_{iy}dk_{iz}}{(2\pi)^2} \frac{D}{\ell k_c^2} \int_0^{\infty} \left[k_{iy} - i \left| \frac{\omega}{\omega_{ce}} \right| \frac{1}{t\ell} \right] \exp\left\{ i\left[\frac{\kappa t^3}{3} - \frac{1}{t} + \xi t \right] \right\} dt
\end{aligned} \qquad (8)
$$

where $\kappa = \ell^4 \ell_T^2 k_c^6$, $k_c = \frac{\omega_{ce}}{c}$, $\ell_T = \frac{3v_{Te}^2}{3\omega_{ce}^2 - \omega_{pe}^2}$, $\ell = \left(\frac{d\varepsilon_\perp}{dx} \right)\Big|_{X = X_{UHR}}^{-1}$,

$\varepsilon_\perp = 1 - \frac{\omega_{Pe}^2}{\omega^2 - \omega_{ce}^2} + i\varepsilon''$, $\xi = (x - x_{UHR} + i\varepsilon''\ell)\ell k_c^2$,

$D = \frac{(\ell k_c)^{3/2}}{\sqrt{\pi}} \sqrt{\frac{|\omega_{ce}|}{\omega}} \exp\left[i\phi_{ex} - i\frac{3\pi}{4} \right] f(k_{iy}, k_{iz}) \sqrt{P_i}$,

$\phi_{ex} = \int_{-a}^{x_{UHR}} \left(k_{ex} - \frac{c}{\omega} k_{iy}^2 \chi_y - \frac{c}{\omega} k_{iz}^2 \chi_z \right) dx'$, $k_{ex} = \sqrt{\frac{\varepsilon^2 - g^2}{\varepsilon}}$, $\chi_y = \frac{\omega_i}{2k_{ex}c}$,

$\chi_z = 1 + \frac{(\omega_i^2 - \omega_{Pe}^2)}{2(\varepsilon^2 - g^2)} \frac{\omega_{ce}^2 \omega_{Pe}^2}{\omega_i^4 (\omega_i^2 - \omega_{ce}^2)} \frac{ck_{ex}}{\omega_i}$, $g = \frac{|\omega_{ce}|}{\omega_i} \frac{\omega_{Pe}^2}{(\omega_i^2 - \omega_{ce}^2)}$, k_{ex} is the X-mode wavenumber and $x = -a$ - position of the probing antenna at the high magnetic field side. Magnetic field fluctuations could be represented in the form

$$
\frac{\vec{B}_\Omega}{B_0} = \int \frac{d^3\vec{q}}{(2\pi)^3} \vec{b}_{\vec{q},\Omega} e^{-i\vec{q}\vec{r}} \qquad (9)
$$

Substituting Eqs.(6)-(9) into (1) and carrying out necessary integration one can obtain the expression for the CPS signal given by

$$
\begin{aligned}
P_{S\Omega} &= \frac{\omega_{ce}^4}{\omega_{Pe}^4} \frac{\pi \ell k_c P_i e^{-\tau}}{\sqrt{[1 + 4(X_y^{ex} - X_y^0)^2][1 + 4(X_z^{ex} - X_z^0)^2]}} \int \frac{d^3\vec{q}}{(2\pi)^3} S(\vec{q}) \\
&\exp\left\{ -2\varepsilon''(q_x - k_c)\ell - \frac{q_y^2\rho^2}{4} \frac{1 + 8\left[(X_y^0)^2 + (X_y^{ex})^2 \right]}{1 + 4\left[X_y^{ex} - X_y^0 \right]^2} - \frac{q_z^2\rho^2}{4} \frac{1 + 8\left[(X_z^0)^2 + (X_z^{ex})^2 \right]}{1 + 4\left[X_z^{ex} - X_z^0 \right]^2} \right\}
\end{aligned} \qquad (10)
$$

where $X_\alpha^0 = \frac{c}{\omega\rho^2} \int_{-a}^{x_{UH}} O_\alpha dx'$, $X_\alpha^{ex} = \frac{c}{\omega\rho^2} \int_{a}^{x_{UH}} \chi_\alpha dx'$, $\langle b_{\vec{q}\Omega} b_{\vec{q}'\Omega'}^* \rangle = |b_{\vec{q}\Omega}|^2 (2\pi)^4 \delta(\vec{q} - \vec{q}') \delta(\Omega - \Omega')$,

$S(\vec{q}) = |b_{y\vec{q},\Omega}|^2 \left(\frac{q_x}{k_c} - \frac{\omega_{pe}^2}{k_c{}^2 c^2} \right)^2 + |b_{x\vec{q},\Omega}|^2 \frac{\omega_{pe}^4}{\omega_{ce}^2 \omega_i^2}$, τ is optical thickness of the ECR layer for X-mode. Significant feature of the CPS signal (10) for $q_x > k_c$ is amplification factor q_x^2, related to the fluctuations of parallel electron velocity.

The expression for the O to X-mode cross-polarization scattering signal could be also obtained from (1) and (2) in the same form (10).

In the case of slightly oblique propagation of X and O modes the electron density fluctuations will also contribute to the CPS signal. To estimate this interference effect we should take into account the nonlinear current $\vec{j}_n = en_\Omega \vec{v}_i$ and calculate the product, not neglecting the small current component parallel to the magnetic field and scattered field components perpendicular to it. It is easy to show that

$$j_{nz} = i\frac{e^2 n_\Omega}{m\omega_i} E_{iz} \tag{11}$$

where according to [6] in the vicinity of UHR $E_{iz} = D\frac{k_z}{\ell k_c^2}\int_0^\infty \exp\left\{i\left(\frac{\kappa t^3}{3} - \frac{1}{t} + \xi t\right)\right\}\frac{dt}{t}$.

The perpendicular component of $\vec{E}_s{}^+$ is given by

$$E_{sx}^+ = \frac{ck_z\omega_{ce}}{\omega^2}E_{sz}^+ \; ; \; E_{sy}^+ = -i\frac{ck_z}{\omega}E_{sz}^+ \tag{12}$$

The final result for the CPS signal produced by density fluctuations is given by (10), where the factor *S(q)* is taken in the form

$$S(\vec{q}) = \frac{\omega_{pe}^8}{\omega_{ce}^6\omega_i^2}\left|\delta n_{\vec{q},\Omega}\right|^2\left[\frac{k_{iz}}{q_x + k_c} - \frac{(q_z - k_{iz})k_c}{(q_x - k_c)^2}\right]^2 \tag{13}$$

where $\left\langle n_{\vec{q}\Omega} n_{\vec{q}\Omega}^* / N^2 \right\rangle = (2\pi)^4 \left|\delta n_{\vec{q}\Omega}\right|^2 \delta(\Omega - \Omega')\delta(\vec{q} - \vec{q}')$. The essential consequence of this expression is the suppression of the spurious CPS signal in the UHR for large q_x. It could be interpreted in terms of approaching the transverse propagation by all ray trajectories in the UHR.

Acknowledgements

The paper was supported by RFBR grants 98-02-18348; 96-15-96367 and by the grant of FOM-Institute for Plasma Physics.

References

[1] Lehner T., Gresillon D. et al.: *Proc. 12th EPS Conf. on Controlled Fusion and Plasma Physics*, Budapest, 1985, part II, p.664.

[2] Zou X.L. et al.: *Proc. 20th Conf. on Controlled Fusion and Plasma Physics*, Lisbon, 1993, v.III, p.1091.

[3] Donne A.J.H., Oomens A.A.M.: *Proc. 9th Joint Workshop on EC Emission and EC Heating*, Borgeru Springs, 1995, p.645.

[4] Novik K.M. and Piliya A.D.: Plasma Phys. Control. Fusion **36**, 357 (1994).

[5] L.Vahala, G.Vahala and N.Bretz: Phys.Fluids B **4**, 619 (1992).

[6] Bruesehaber B., Gusakov E.Z. et al.: Plasma Phys. Control. Fusion **36**, 997 (1994).