

NON-LINEAR EVOLUTION OF THE INHOMOGENEOUS BEAM PLASMA INSTABILITY

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Abstract

Electron momentum anisotropy, generated for example by the braking of plasma waves, is a very important source of kinetic energy which can be converted into magnetic energy by the development of the electromagnetic beam-plasma instability. Here we investigate the evolution of this instability excited by two initially interpenetrating counter-streaming relativistic electron beams localized in a layer comparable to the electron skin depth. This instability plays a key role in the generation of the quasi-static magnetic wake field observed in the PIC simulations of the interaction of an ultra-short, ultra-intense laser pulse with an underdense plasma.

1. Introduction

Fast electron beams can be easily generated in a plasma by the breaking of "finite amplitude" plasma waves. Then, in order to maintain quasi-neutrality, the plasma reacts immediately by producing slow, dense return currents such that the total net current is zero. However, in these conditions the system becomes unstable since any initial transverse displacement is reinforced due to the fact that two opposite directed currents repeal each other. This is the physical mechanism of the current filamentation instability (hereafter CF instability) [1]-[11], which converts part of the anisotropic kinetic energy of the electron beams into magnetic energy. The resulting magnetic field grows exponentially in time in the direction perpendicular to the wavevector of the perturbation and to that of the electron beams and has a null real part of the frequency, i.e. it grows without propagating. This instability is similar to the Weibel instability [12] in the case of anisotropic temperature distribution.

In the opposite limit, i.e. when the the initial perturbation is strictly parallel to the electron beams, the beam-plasma instability (hereafter BP instability) generates an electrostatic field with a non zero real part of the frequency. No magnetic field is generated in this case.

In general, the wavevector of the perturbation is not purely transverse or parallel to the electron streams, and the resulting mode is a "mixture" of the CF and BP instabilities, known as the electromagnetic beam-plasma instability (hereafter EMBP instability). In particular, when going towards the relativistic regime, the most unstable mode is more and more aligned to the electron beams (i.e. $k_x \simeq 0$), since the CF instability dominates with respect to the BP instability. On the other hand, the important difference between the CF and the EMBP instability is that the magnetic field resulting from the development of the EMBP instability propagates with a frequency which depends on the angle between the electron beams and the wavevector. For a detailed discussion of the dispersion relation of the EMBP instability see Ref. [11].

In this paper we discuss the evolution of the inhomogeneous EMBP instability in the strong relativistic limit excited by two initially counter-propagating electron beams localized in a thin layer of typical dimension slightly larger than the electron skin depth. This case is relevant for laser plasma applications since the EMBP instability is thought to be the driving physical

mechanism of the observed quasi-static magnetic field in the wake of the laser pulse [13, 14].

In the next Section we introduce the governing equations. In Section 3 the numerical results are presented. Finally, conclusions are drawn in Section 4.

2. The equations

We use the two-fluid electron equations where the ions are assumed to be at rest providing a uniform neutralising background. This is consistent with the typical time of the development of the EMBP instability which, for velocities comparable to the speed of the light, is of the order of the inverse of the electron plasma frequency. The dimensionless equations read:

$$\frac{\partial n_a}{\partial t} = \nabla \cdot \mathbf{j}_a, \quad (1)$$

$$\frac{\partial \mathbf{p}_a}{\partial t} + \mathbf{v}_a \cdot \nabla \mathbf{p}_a = -(\mathbf{E} + \mathbf{v}_a \times \mathbf{B}), \quad (2)$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \sum_a \mathbf{j}_a, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (3)$$

$$\mathbf{v}_a = \frac{\mathbf{p}_a}{(1 + p_a^2)^{1/2}}, \quad \mathbf{j}_a = -n_a \mathbf{v}_a, \quad a = 1, 2.$$

These equations are normalized by using a characteristic density \bar{n} , the speed of light c and the electron plasma frequency $\bar{\omega}_p = (4\pi\bar{n}e^2/m)^{1/2}$. Notice that the normalized electron skin depth d_e is equal to one.

Equations (1)-(3) are integrated in the (x, y) plane of dimension $0 \leq x \leq 2\pi$, $-40 \leq y \leq 40$. We use periodic boundary conditions in the direction of the electron streams, the x -axis, and free-slip boundary conditions, $\partial/\partial y = 0$, in the transverse direction, the y -axis. The initial counter-propagating and interpenetrating electron beams are modelled by

$$\mathbf{v}_{0,1} = v_{0,1} \cosh^{-2}(y/l) \mathbf{e}_x, \quad \mathbf{v}_{0,2} = -\mathbf{v}_{0,1} n_{0,1}/n_{0,2}, \quad (4)$$

where the subscript zero refers to zero order quantities. Here $l = 4$ is the typical width of the beams (we recall that $d_e = 1$), $n_{0,1} + n_{0,2} = 1$ and the initial total net current is zero, $n_{0,1}v_{0,1} + n_{0,2}v_{0,2} = 0$. Furthermore, we assume that the initial densities of the electron beams are homogeneous.

We limit our analysis to a one-dimensional magnetic field transverse to the z -axis, $\mathbf{B} = (B_z)$, while the electric field $\mathbf{E} = (E_x, E_y)$ and the electron momenta $\mathbf{p}_a = (p_{0,a} + p_{a,x}, p_{a,y})$ lie in the (x, y) plane. The two initial beams are perturbed by a "small" magnetic perturbation,

$$B_z = \epsilon R(x) \sin(y) e^{-y^2/(2\sigma^2)}, \quad (5)$$

where $\epsilon = 10^{-3}$ is the amplitude of the perturbation, $\sigma = 7.07$ the typical transverse width and $R(x)$ represents a random spatial noise in the interval $[-1, 1]$.

3. Numerical results

In Fig. 1 we show the electric field, the magnetic field and the total density, $n = n_1 + n_2$, resulting from a numerical simulation with $v_{0,1} = 0.995$, $v_{0,2} = -0.199$, $n_{0,1} = 0.167$ and $n_{0,2} = 0.833$. This case corresponds to the non-symmetric strong relativistic limit in which, as discussed in [11], the CF instability dominates with respect to the BP instability. In fact, even

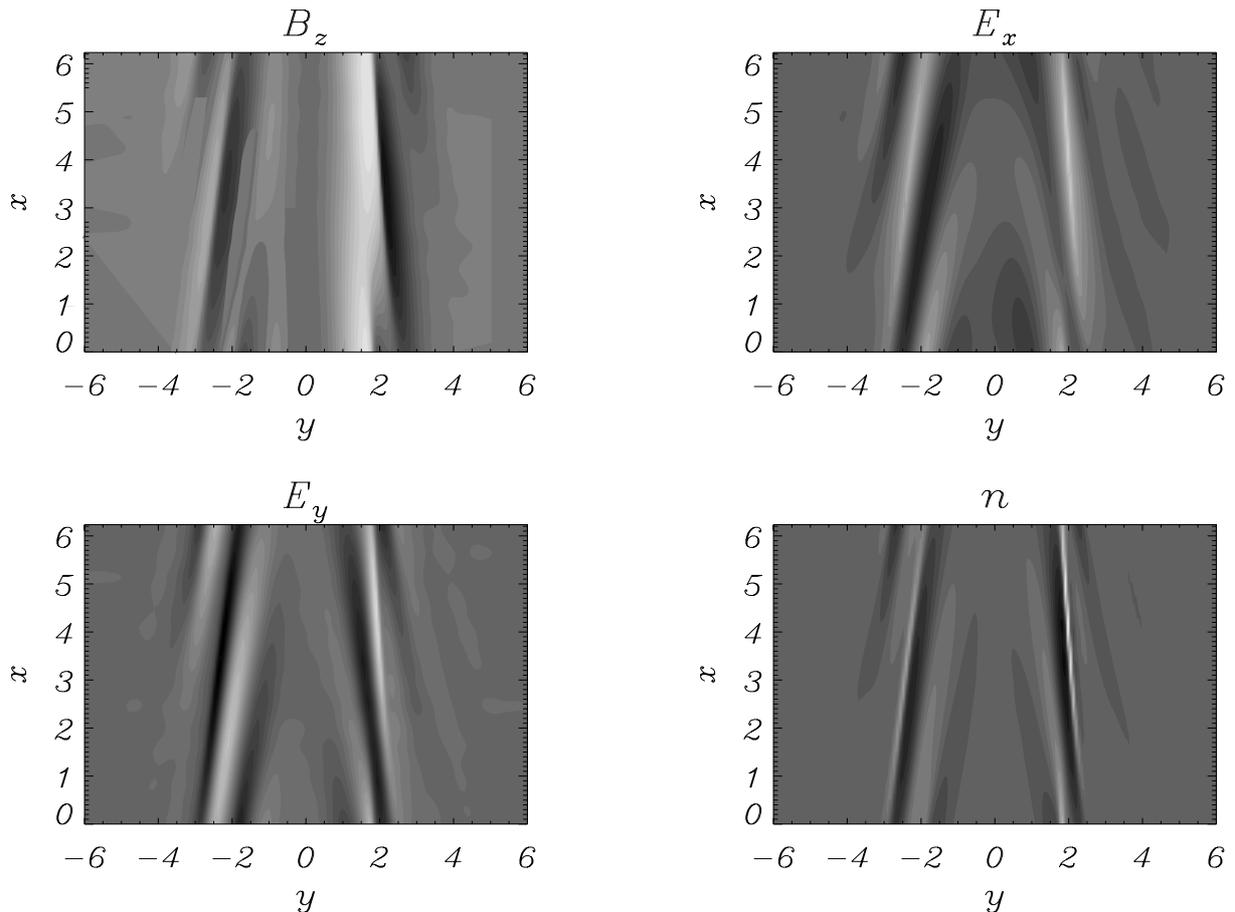


Figure 1. The shaded isocontours of the magnetic field, the electric field E_x and E_y and of the total density, $n = n_1 + n_2$ at $t = 27$. The parameters of the run are: $v_{0,1} = 0.995$, $v_{0,2} = -0.199$, $n_{0,1} = 0.167$ and $n_{0,2} = 0.833$.

if at the initial time the perturbation has a random distribution along the beam direction, the magnetic and electric field, as well as the electron densities resulting from the development of the EMBP instability, are practically homogeneous along the x axis. The most relevant feature shown by Fig. 1 is that all the physical quantities, B_z , E_x , E_y , n are concentrated inside two narrow layers symmetric with respect to the $y = 0$ axis. This characteristic structure of the inhomogeneous relativistic EMBP mode has a twofold nature. First of all, during the linear phase, the perturbation is pinched into two distinct resonant layers (much larger than those of Fig. 1), regardless of the characteristic spatial scales of the initial perturbation. Then, as soon as nonlinear interactions become important, a strong transverse breaking take place due to the formation of singularities in a finite time [9, 10]. As a result, the two initially resonant layers become thinner and thinner, and the characteristic transverse length scale becomes comparable or even smaller than the typical kinetic length scales. At this point, the fluid approximation becomes meaningless, and the simulation must be stopped. On the other hand, in the stream direction, nonlinear effects have no significant consequence on the structure of the resonant mode developed in the linear phase. Finally, we note that in each layer the magnetic field has a dipolar structure in the direction transverse to the initial electron beams.

4. Conclusions

Magnetic field generation is a fundamental process in plasma physics and astrophysics since

these fields play a very important role in the plasma dynamics and are an important energy source which can be rapidly released on fast time scales by some mechanism, as for example magnetic reconnection. Recently, the important role of the magnetic fields has been shown in laser-plasma numerical simulations where the magnetic field is generated in the wake of the laser pulse by some mechanism induced by the presence of counter-streaming electron beams.

In this paper, we have discussed the evolution of the EMBP instability in the strong relativistic regime driven by two inhomogeneous electron currents. We have used a relativistic two-fluid approach, which gives the possibility to study the dynamics of such system before the strong nonlinear interactions generates scales comparable to the characteristic kinetic scales. Within the fluid approach, we have shown the characteristic dipolar magnetic structure produced by the EMBP instability which is associated to a central "fast" current and two "slow" return currents, in good agreement with the results observed in laser-plasma simulations [15, 16]. However it should be observed that the occurrence of singularities leading to very small spatial scales cannot be fully described with the fluid approximation adopted in this paper. A kinetic description that properly describes this phase is for the moment available only for the case of a homogeneous plasma [9]. An extension of the present work to the full nonlinear kinetic regime stage after the formation of the singularities is already in progress by numerical integration of the Vlasov-Maxwell equations.

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