MAGNETIC FIELD DEPENDENCE OF THERMONUCLEAR ALPHA PARTICLES TRANSPORT QUANTITIES

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In this work, the equilibrium distribution function of fusion generated alpha particles in a magnetized plasma is found by solving the Fokker-Planck (F-P) equation taking into account the effects of the magnetic field and of temperature and density gradients. Then, we use this distribution to calculate transport quantities as heat and particle fluxes for the case of an azimuthal magnetic field configuration. Finally, we derive from it the alpha particles contribution to the dielectric function for a plasma in a uniform magnetic field.

1. Steady state distribution function

Being generated isotropically, the zero order distribution function of the alpha particles is taken to be isotropic in velocity space [1,2] $f_0 = f_0(v)$ and the F-P equation can be written as [3]

$$\frac{df_0}{dt} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(Mf_0 + \frac{MT}{m_\alpha v^2}\frac{\partial f_0}{\partial v}\right) + S(v) - \frac{f_0}{\tau} \tag{1}$$

when $T_e = T_i = T$. Being $M = C_e\phi_1(x_e) + C_i\phi_1(x_i)$, $\phi_1(x) = \phi(x) - x\partial\phi/\partial x$ with ϕ the error function and

$$C_\beta = (\frac{Z_\alpha Z_\beta e^2}{\varepsilon_0})^2 \frac{n_\beta \ln\Lambda}{4\pi m_\alpha m_\beta} \tag{2}$$

The index α standing for alpha particles and β for ions and electrons, $x_\beta = b_\beta v$ with $b_\beta = \sqrt{m_\beta/(2T_\beta)}$ and τ is the time of confinement of alphas in the plasma. The source term is [3]

$$S(x) = \frac{\sqrt{\gamma}\dot{n}_\alpha b_e^3 \tau_{\alpha e}}{4\pi^{\frac{3}{2}} x_\alpha x}\left(e^{-\gamma(x-x_\alpha)^2} - e^{-\gamma(x+x_\alpha)^2}\right) \tag{3}$$

where the dimensionless velocity is $x \equiv x_e$, $x_\alpha = b_e v_\alpha$, $v_\alpha = \sqrt{2E_\alpha/m_\alpha}$, with E_α being the alpha particles mean initial energy, $\gamma = m_i/m_e$, $\dot{n}_\alpha$ the alphas production rate and $\tau_{\alpha e}$ the alphas heat exchange time $\tau_{\alpha e} = m_\alpha \tau_e/(2m_e)$, τ_e being the electron collision time.
With boundary conditions [4] $f_0(v, t = 0) = 0$, $f_0(v \gg v_\alpha, t) = 0$ and $\partial f_0/\partial v = 0$ at $v = 0$, the numerical solution of Eq. (1) (Fig. 1) shows that the equilibrium is reached in a finite time. Thus we have found the new general solution for the equilibrium (i.e. with $df_0/dt = 0$)

$$f_0 = \frac{m_\alpha e^{-\frac{m_\alpha v^2}{2T}}}{T}\int \frac{ve^{\frac{m_\alpha v^2}{2T}}}{M}\left(\int S(v')v'^2dv' - \frac{n_0(v)}{\tau}\right)dv \tag{4}$$

$$\int S(v')v'^2dv' = \frac{\sqrt{\gamma}\dot{n}_\alpha\tau_{\alpha e}}{4\pi^{\frac{3}{2}}v_\alpha}[\frac{\left(e^{-\gamma b_e^2(v-v_\alpha)^2} - e^{-\gamma b_e^2(v+v_\alpha)^2}\right)}{2\gamma} + \frac{i\sqrt{\pi}b_e v_\alpha}{2\sqrt{\gamma}}\left(\phi(i\sqrt{\gamma}b_e(v-v_\alpha)) - \phi(i\sqrt{\gamma}b_e(v+v_\alpha))\right)] + const. \tag{5}$$

where the constant of integration is determined from the second boundary condition to be the negative of the indefinite integral evaluated at $v \gg v_\alpha$. The analytic dimensionless distribution function $F_0 = f_0/(\dot{n}_\alpha 10\tau_{\alpha e} b_e^3)$ is plotted in Fig. 2 for comparison with the numerical solution (Fig. 1) at the equilibrium time $t/(10\tau_{\alpha e}) \approx 1$, . For $\tau \to \infty$, i.e. without loss, Eq. (4) gives the exact solution of the steady state F-P equation. For finite τ, Eq. (4) is a good approximation for large τ and with n_0 calculated from the exact solution $f_{0\tau\to\infty}$

$$n_0(v) = \int_0^v d^3v' f_{0\tau\to\infty}(v') \tag{6}$$

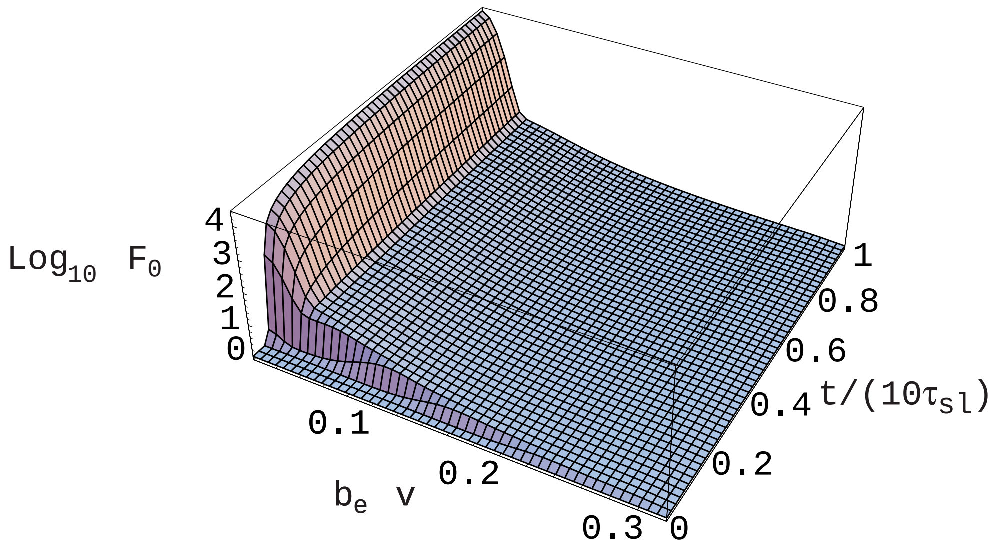

Figure 1. Numerical calculation of the time dependent alpha particle distribution function.

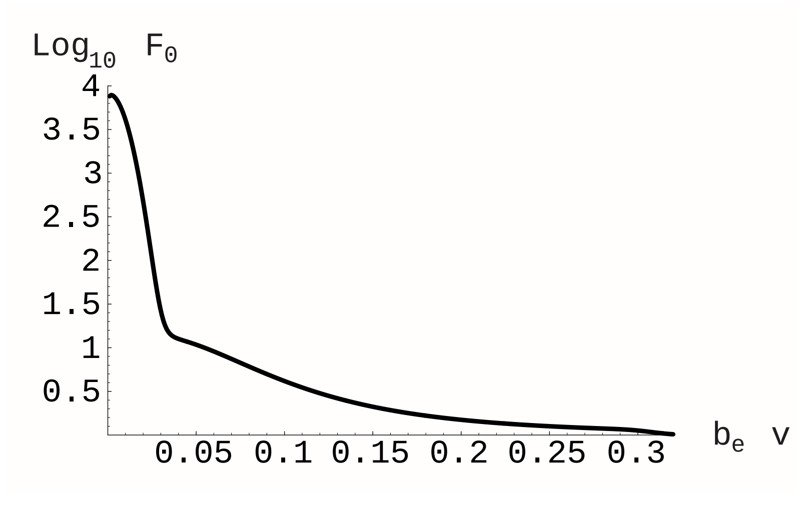

Figure 2. Analytical steady state alpha particle distribution function.

For the first order approximation we write $f = f_0 + f_1$ and neglect collision effects on f_1, thus F-P equation for f gives

$$\mathbf{v} \cdot \nabla_r f_0 + \omega_{c\alpha} \mathbf{v} \cdot \hat{b} \times \frac{\partial f_1}{\partial \mathbf{v}} = 0 \tag{7}$$

with solution [5] $f_1 = \mathbf{v} \cdot \hat{b} \times \nabla_r f_0 / \omega_{c\alpha}$, where $\hat{b} = \mathbf{B}/B$. For the following order one writes $f = f_0 + f_1 + f_2$ taking into account collisions on f_2 and obtains the following equation to the second order

$$\omega_{c\alpha}\mathbf{v}\times\hat{b}\cdot\frac{\partial f_2}{\partial \mathbf{v}} = C(f_1) \tag{8}$$

For high density plasmas as in a magnetized target fusion experiment (MTF) [6] , the magnetic field B is strong (MGauss) and the plasma density is high enough to give a Larmor radius much longer than the Debye length in such a way that the collision operator C does not depend on B and the solution to Eq. (8) is $f_2 = -C(\mathbf{v}\cdot\nabla_r f_0)/\omega_{c\alpha}^2$. In the case of a uniform density current of electrons generating an azimuthal magnetic field, the collision operator may be written in spherical coordinates as

$$C(f) = \frac{1}{v^2}\frac{\partial}{\partial v}(v^2 I_{\|}) + \frac{1}{v\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta I_{\perp}) \tag{9}$$

in terms of $I_{\|}$ and $I_{\perp}$ [7], components parallel and perpendicular to the velocity, functions of the friction vector $\mathbf{a}$ and diffusion tensor $\bar{\bar{\mathbf{b}}}$ [8]. And f_2 becomes

$$f_2 = \frac{1}{\omega_{c\alpha}^2 v^2}[\sin\theta\frac{\partial}{\partial v}\left(v^2\left[a_\beta v\bigtriangledown_\rho f_0 - d_{\|\beta}\frac{\partial}{\partial v}(v\bigtriangledown_\rho f_0)\right]\right) + \cos\theta\times$$
$$\frac{\partial}{\partial v}\left(v^2\left[a_\beta v\bigtriangledown_z f_0 - d_{\|\beta}\frac{\partial}{\partial v}(v\bigtriangledown_z f_0)\right]\right)] - d_{\perp\beta}\bigtriangledown_\rho f_0 cos\theta + d_{\perp\beta}\bigtriangledown_z f_0 sin2\theta \tag{10}$$

where $a_\beta = -C_\beta\phi_1(x_\beta)/v^2$, $d_{\|\beta} = m_e C_\beta\phi_1(x_\beta)/(2m_\alpha x_\beta^2 v)$ and

$$d_{\perp\beta} = \frac{m_e C_\beta}{2m_\alpha v}\left[\phi(x_\beta) - \frac{\phi_1(x_\beta)}{2x_\beta^2}\right] \tag{11}$$

2. Transport quantities

As the analytic calculation of the transport quantities is very involved we approximate the source by $S(v) = \dot{n}_\alpha\delta(v - v_\alpha)/(4\pi v_\alpha^2)$, and take $\phi_1(x) \approx px^3/(1+px^3)$, $p = 4/(3\sqrt{\pi})$. Then, the dimensionless expression for the zero order distribution F_0, can be approximated by [9]

$$F_0(x) \approx \left(\frac{1}{2\pi^{3/2}} - \frac{\eta}{p\pi^{3/2}x} + \frac{i\eta^{3/2}e^{-\eta x^2}\phi(i\sqrt{\eta}x)}{p\pi}\right)\Theta(x_\alpha - x) \tag{12}$$

where $\eta = m_\alpha/m_e$. Neglecting gradients along the z-axis and collisions with ions the flux of alphas

$$\mathbf{j} = q_\alpha\int d^3v\mathbf{v}f(v) \tag{13}$$

is calculated to be

$$\mathbf{j} = \frac{2e\sqrt{2\pi T}\dot{n}_\alpha v_\alpha^2}{C_e\omega_{c\alpha}}\left[{}_2F_2(1,1;-\tfrac{3}{2},2;-E_\alpha/T) - 1\right]\bigtriangledown_\rho T\hat{k} \tag{14}$$

and the energy flux

$$\mathbf{q} = \int d^3v\frac{m_\alpha v^2}{2}\mathbf{v}f(v) \tag{15}$$

is

$$\mathbf{q} = \frac{3\sqrt{\pi T}\dot{n}_\alpha v_\alpha^2}{32\sqrt{2}m_e^{3/2}\omega_{c\alpha}}\left[10T({}_2F_1(1,1;-\tfrac{5}{2};-E_\alpha/T) - 1) - 2E_\alpha\right]\bigtriangledown_\rho T\hat{k} \tag{16}$$

There are two outstanding characteristics of these fluxes. One is that both are in the current direction. The other is that the second order approximation f_2 does not contribute to them (i.e. they present a $1/B$ dependence).

3. Dielectric function

In a simple geometry, with a uniform magnetic field and using f to first order, the contribution of the alphas to the dielectric function [10]

$$\varepsilon(\mathbf{k},\omega) = \frac{4\pi^2 q_\alpha^2}{m_\alpha \varepsilon_0 k^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} v_\perp dv_\perp \left(\frac{n\omega_{c\alpha}}{v_\perp} \frac{\partial f}{\partial v_\perp} + k_\parallel \frac{\partial f}{\partial v_\parallel} \right) \frac{J_n^2(\zeta)}{k_\parallel v_\parallel + n\omega_{c\alpha} - \omega} \tag{17}$$

with $\zeta = k_\perp v_\perp / \omega_{c\alpha}$ is calculated with the approximation of Eq.(12) for F_0 and an asymptotic expansion of ϕ; good enough for $\sqrt{\eta}x > 1$, i.e. for non thermalized alphas.

$$\begin{aligned}
\varepsilon(\mathbf{k},\omega) = \frac{16\pi^2 e^2}{p m_\alpha \varepsilon_0 b_e^2 k^2 k_\parallel} \sum_{n=-\infty}^{\infty} & (Z_1(z) \dot{n}_\alpha \tau_{\alpha e} b_e^3 \eta^{3/2} k_\parallel e^{-\frac{a^2}{2\eta}} I_n\left(\tfrac{a^2}{2\eta}\right) \\
& - Z_2(z) \dot{n}_\alpha \tau_{\alpha e} b^3 \eta^{3/2} e^{-\frac{a^2}{2\eta}} \omega_{c\alpha} n I_n\left(\tfrac{a^2}{2\eta}\right) + \\
& -\frac{n}{b_e}(C_1 \left[\frac{\eta^{1-n} a^{2n}}{2^{2n+1}\Gamma(n+1)} {}_2F_2\left(n+\tfrac{1}{2}, n+\tfrac{1}{2}; n+1, 2n+1; -a^2/\eta\right) Z_2(z) \right] + \\
& C_2 T [\frac{\eta^{2-n} a^{2n}}{2^{2n}\Gamma(n+1)} {}_2F_2\left(n+\tfrac{1}{2}, n+\tfrac{1}{2}; n+1, 2n+1; -a^2/\eta\right) Z_1(z) + \\
& - \frac{\eta^{1-n} a^{2n} \Gamma(n+\frac{3}{2})}{2^{2n}\Gamma^2(n+1)} {}_2F_2\left(n+\tfrac{1}{2}, n+\tfrac{3}{2}; n+1, 2n+1; -a^2/\eta\right) Z_2(z)]) \frac{\nabla n_e}{n_e}
\end{aligned} \tag{18}$$

for a plasma with uniform pressure ($\nabla n_\beta / n_\beta = -\nabla T / T$), where

$$C_1 = b_e^3 \dot{n}_\alpha \tau_{\alpha e} - T(\dot{n}_\alpha b_e^3 \frac{\partial \tau_{\alpha e}}{\partial T} + \tau_{\alpha e} b_e^3 \frac{\partial \dot{n}_\alpha}{\partial T} + 3\dot{n}_\alpha \tau_{\alpha e} b_e^2 \frac{\partial b_e}{\partial T}) \tag{19}$$

and $C_2 = \tau_{\alpha e} \dot{n}_\alpha b_e^3 / 2T$, $a = k_\perp / (\omega_{c\alpha} b_e)$, $z = (\omega - n\omega_{c\alpha}) b_e / k_\parallel$, $Z_1 = \frac{z}{\sqrt{\pi\eta}} - z(i + z - i\phi(i\sqrt{\eta}z))e^{-\eta z^2}$, $Z_2 = (2i - i\phi(i\sqrt{\eta}z))e^{-\eta z^2}$ and in Bessel and hypergeometric functions n stands for $\mid n \mid$.

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