ON THE ANOMALOUS RESISTIVITY IN THE CURRENT–CARRYING CORONA OF Z - PINCH

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The goal of this paper is to build a scenario of resistivity in a rarefied current-carrying corona of the fast Z-pinch which is typical of most regimes of the pinch dynamics. In particular, randomly inhomogeneous ("fractal") corona is of interest due to very strong effect of this inhomogeneity onto the resistance. Such random density fluctuations may result from the self-consistent MHD dynamics, especially in the regime of enhanced stability (see, e.g., [1-3]).

In our recent paper [4] we have considered some features of the regime of anomalous resistivity in P.O.S. In particular, our statement was, that in most pulsed plasmas ion-acoustic mechanism of anomalous resistivity seemed to be predominating one. Of course, some competing instabilities (low-hybrid, modified two-stream) can also join the game. In fact, they can predominate in the resistive mechanism only under the following condition: $\omega_{Be} \geq \omega_{pe}$ or, what is the same, $B^2 \geq 4\pi nmc^2$ which is not typical of the pulsed plasmas up-to-date.

The whole hierarchy of the anomalous resistivity includes, generally speaking, linear excitation of waves [5], quasilinear stage [6], nonlinear saturation [7] and anomalous transport [8]. However, on the level of very precise estimates it is not less known that such a hierarchy works as a whole only if the electric field is not so strong: $E << (m/M)(4\pi nT)^{1/2}$. If, as in most pulsed plasmas, the opposite inequality is true, quasilinear effects are immaterial. This essentially nonlinear regime is just the case of the Kadomtsev's spectrum [8] and Sagdeev's formula for conductivity [9] we have used in [4].

All the resistive plasma systems may be separated in two classes: a) with a given electric field inside the gap, and b) with a net current completely determined by the outer circuit. The Z-pinches represent just the second case, hence, in such a system the current flow velocity cannot be restricted by the threshold of the instability. If this velocity exceeds essentially the threshold, the quasilinear (Cherenkov) effects do take place but cannot determine the effective collisional frequency. At first sight, it means that only essentially nonlinear regime could be realized in the dynamics of anomalous resistivity of pulsed plasmas. However, it was noticed as early as in [7] that effect of the inhomogeneous plasma density $n(\mathbf{r})$ could effectively compete with the typical nonlinearity, i.e., nonlinear wave-particle scattering.

The basic system of equations ("optical" or Hamiltonian) describing the trajectory of the ion-acoustic quantum in the phase space is the following one:

$$\frac{\partial \mathbf{r}}{\partial t}=\frac{\partial \omega}{\partial \mathbf{k}},\quad \frac{\partial \mathbf{k}}{\partial t}=-\frac{\partial \omega}{\partial \mathbf{r}}. \tag{1}$$

For simplicity, we will demonstrate all the basic effects on the 1-D model, i.e., $n=n(x)$, $\mathbf{j}||\mathbf{E}||\mathrm{Ox}$. We will take into account only the spatial inhomogeneity of plasma density as the ion-acoustic waves are much less sensitive to the effect of inhomogeneous temperature. Eqs. (1) result in the following integrals of motion:

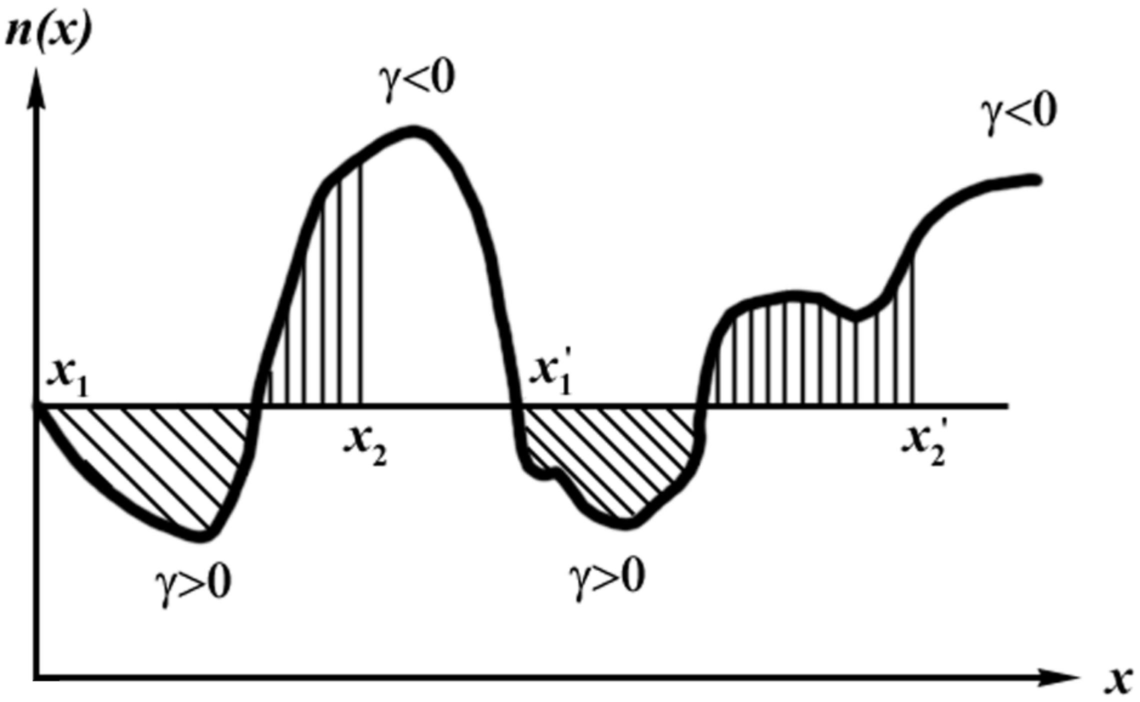
$$k^{-2}+r_{De}^{-2}=\mathrm{inv},\ r_{De}\propto n^{-1},\ \frac{dn}{dx}>0\Rightarrow k\to k_{\min},\ n\to 0\Rightarrow k\to\infty, \tag{2}$$

$$\frac{W_{\mathbf{k}}}{\omega_{\mathbf{k}}}\equiv N(\mathbf{k},\mathbf{r})=N(\mathbf{k}_0,\mathbf{r}_0)\cdot\exp\left[\int_{\mathbf{k}_0,\mathbf{r}_0}^{\mathbf{k},\mathbf{r}}\frac{2\gamma_{\mathbf{k}}}{|\partial\omega/\partial\mathbf{k}|}d\mathbf{l}\right] \tag{3}$$

where $\gamma_e\cong\sqrt{\frac{\pi}{8}}(\mathbf{ku}-\omega)\left(\frac{\omega}{kv_{Te}}\right)^3\left(\frac{M}{m}\right)$, $\gamma_i\cong-\pi^2\omega\cdot\left(\frac{\omega}{k}\right)^3 f_i\left(\frac{\omega}{k}\right)$, $\mathbf{u}=\mathbf{j}/ne$ is the current flow velocity and $W_{\mathbf{k}}$ is the energy density of the noises. One of the important consequences of (2,3) is that no reflection of an ion-acoustic plasmon may occur in the 1-D problem but it can disappear due to the ion Landau damping conditioned by the essential decrease of the density. The spatial density of momentum varies in time according to Eqs. (1):

$$\frac{\partial}{\partial t}\frac{\partial \mathbf{p}_e}{\partial V}=-\frac{\partial}{\partial t}\frac{\partial \mathbf{p}_i}{\partial V}=\sum_{\mathbf{k}}N_{\mathbf{k}}\frac{\partial \mathbf{k}}{\partial t}=\sum_{\omega}N_{\omega}\frac{\partial \mathbf{k}(\omega,\mathbf{r})}{\partial t}=-\sum_{\omega}N_{\omega}\nabla\omega \tag{4}$$

Let us consider the random spatial inhomogeneity consisting of the density wells and hills with x_2-x_1 being the typical space scale of the region occupied by the noises in a partial well.



Balance of the electron momentum results in

$$\int_{x_1}^{x_2} ne\mathbf{E}dx = \mathbf{j}\frac{m}{e}\int_{x_1}^{x_2} \nu_{eff}(x)dx \tag{5}$$

which allows to present the following estimate of ν_{eff}:

$$j\frac{m}{e}\int_{x_1}^{x_2} \nu_{eff}(x)dx = \int_{x_1}^{x_2} dx \int d\omega N_{\omega}\frac{\partial\omega}{\partial x} \cong \int_{x_1}^{x_2} dx \frac{W(x)}{\omega}\frac{\partial\omega}{\partial x} \tag{6}$$

Besides, let us note that two different estimates of ν_{eff} may be introduced, viz, ν_B typical of the field diffusion and ν_R which determines the averaged resistance of Z–pinch. Then Eqs. (2 – 6) may be transformed in the following estimate:

$$\nu_B(x_2 - x_1) = \nu_R \cdot L = \beta\frac{\langle n\rangle^{5/2} e^4}{mj\sqrt{T}}\left(\frac{1}{n(x_1)} - \frac{1}{n(x_2)} + \varepsilon\frac{\omega}{\nu_{Te}}\cdot\int_{x_1}^{x_2}\frac{dx}{n(x)}\left(\frac{j}{e\omega}\frac{k(x)}{n(x)} - 1\right)* \right. \tag{7}$$

$$\left. *\exp\left[\kappa\int_{x_1}^{x}\left(\frac{j}{e\omega}\frac{k(\xi)}{n(\xi)} - 1\right)d\xi\right]\right)$$

where $\beta, \varepsilon \leq 1$, $\kappa \cong (m/M)^{1/2} r_D^{-1}$, L is the "quasiperiod" of spatial inhomogeneity.

After that, we will have to deal with rather few parameters in the subsequent estimates, i.e., $n_{\min}$, $a = x_2 - x_1$ and, may be, $L > a$. All the trajectories of quasiparticles have to obey the condition $k^{-2} + r_D^{-1} = \mathrm{inv}$. By using the approach of quasi–parabolic profile of the well, we can establish the relation between the "noisy" interval $[x_1, x_2]$ and the typical space scale a or L:

$$n_1 = \frac{j}{ec_s} = n_0\left(1 + \frac{(\Delta x)^2}{a^2}\right) \Rightarrow (x_2 - x_1) \sim 2(x_1' - x_1) \sim 4\Delta x \sim 4a\sqrt{\frac{j}{n_0 ec_s} - 1}\,.$$

Introducing the dimensionless variables $(x) \equiv (x_2 - x_1)/a$, $\Delta/a \to \Delta = \sqrt{j/(n_0 ec_s) - 1}$, one can obtain the final result as follows:

$$\nu_R = (x)\nu_B = \beta\frac{\langle n\rangle^{5/2} e^4}{n_0 amj\sqrt{T}}\cdot\left\{\frac{(x)^2 - 2(x)\Delta}{(1+\Delta^2)\left(1 + [(x)-\Delta]^2\right)} + \right.$$

$$+\varepsilon\sqrt{\frac{m}{M}}\frac{a}{r_D(0)}\int_{-\Delta}^{(x)-\Delta}\frac{dx}{1+x^2}\left[\sqrt{\frac{1+\Delta^2}{1+x^2}}\sqrt{\frac{\Delta^2+\delta}{x^2+\delta}} - 1\right]* \tag{8}$$

$$\left. *\exp\left[\varsigma\sqrt{\frac{m}{M}}\frac{a}{r_D(0)}\int_{-\Delta}^{x} d\xi\left(\sqrt{\frac{1+\Delta^2}{1+\xi^2}}\sqrt{\frac{\Delta^2+\delta}{\xi^2+\delta}} - 1\right)\right]\right\}, \quad \beta, \varepsilon, \varsigma \sim 1,$$

where $(x) = \sqrt{1+\Delta^2}\sqrt{\Delta^2+\delta}\int_{-\Delta}^{(x)-\Delta}(1+\xi^2)^{-1/2}(\xi^2+\delta)^{-1/2}d\xi$, $\delta \cong T_i/T_e < 1$.

In the essentially nonlinear regime [8,9] with a given current the following estimate of the density of noises is true: $(W/nT)_{nl} \cong j/nev_{Te}$. In our case,

$$\left(\frac{W}{nT}\right)_{\max} << \frac{\exp\Gamma_0}{N_D}, \; \Gamma_0 = \frac{a}{r_D}\sqrt{\frac{m}{M}}\int_{-\Delta}^{\Delta}\left(\sqrt{\frac{1+\Delta^2}{1+\xi^2}}\sqrt{\frac{\Delta^2+\delta}{\xi^2+\delta}}-1\right)d\xi \, .$$

Thus, inhomogeneity determines the resistivity if

$$\left(\frac{W}{nT}\right)_{\max} << \left(\frac{W}{nT}\right)_{nl} \Rightarrow \Gamma_0 < \ln\left(\Delta^2 N_D\right), \; \Delta^2 \sim \frac{j}{n_0 e c_s}, \; N_D = \frac{1}{6\sqrt{\pi}}\frac{T_e^{3/2}}{n^{1/2}e^3} \, .$$

After all, the following inequality provides the applicability of our model:

$$\frac{a}{r_D}\sqrt{\frac{m}{M}}\Delta\sqrt{1+\Delta^2}\int_{-\Delta}^{\Delta}\frac{d\xi}{\sqrt{1+\xi^2}\sqrt{\xi^2+\delta}} < \ln\left(\Delta^2 n r_D{}^3\right) . \tag{9}$$

Roughly, Eq. (9) may be rewritten as

$$\frac{a}{r_D}\frac{j}{nev_{Te}} < \frac{\Lambda}{\ln T_e / T_i}$$

where Λ is the Coulomb logarithm. This inequality is often true, hence, our scenario seems to be realistic in many high-current plasma systems.

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