DYNAMICS OF INTENSE AND FINITE LENGTH ION BEAM PROPAGATING IN PLASMA CHANNEL

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Abstract

Time evolution of a propagating ion beam in a prepared plasma channel is considered by solving the Vlasov-Maxwell equations. The solution obtained by usual perturbation method contains secularity and it is not valid for a long enough time to apply it for a fusion energy driver. We improved the solution by operator method. And it is applied for two concrete beam models and the deformations of the propagating ion beam profile are shown. From these numerical results, a hollow beam is prevented from diverging at the beam head.

1. Introduction

In order to utilize an intense ion beam for the inertial confinement fusion, it is necessary to propagate the ion beam a distance of about 10 meters from ion source to target with good focusing and stabilizing. The propagating beam tends to diverge due to the Coulomb repulsive force. To suppress this repulsive force, a high density plasma is prepared along the way of ion beam called plasma channel. Due to the interactions between the ion beam and the background plasma various kinds of instabilities are occurred, and those were investigated by many authors [1],[2],[3],[4]. But those analyses were using infinite length beams, so the motions of the propagating ions near the beam head were not investigated. In order to analyze the motion of the ion beam near the beam head, we consider a finite length ion beam.

2. Return Current

When the finite length ion beam propagates through the prepared plasma channel, the return current is produced by the electromagnetic fields which are induced by the ion beam. The plasma response to the ion beam is calculated by Vlasov-Maxwell equations,

$$\begin{aligned}
\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} &= -\frac{1}{\varepsilon_0}(\rho_b + \rho_p), \\
\nabla^2\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} &= -\mu_0(\boldsymbol{J}_b + \boldsymbol{J}_p), \\
\nabla\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} &= 0,
\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{E} &= -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t}, \\
\boldsymbol{B} &= \nabla \times \boldsymbol{A}, \\
\frac{\partial f_1}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_1}{\partial \boldsymbol{r}} - [\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}] \cdot \frac{\partial f_0}{\partial \boldsymbol{v}} &= -\nu_{ei} f_1,
\end{aligned}
$$

where ρ_b is the charge density of ion beam, $\boldsymbol{J}_b$ is the current density of ion beam, ρ_p is the charge density of plasma response, $\boldsymbol{J}_p$ is the current density of return current and ν_{ei} is electron-ion collision frequency. We prepare the high density plasma channel $n_p \gg n_b$, and it justifies the linearization of the Vlasov equation with respect to an electron distribution function $f_p = f_0 + f_1$(unperturbed distribution function f_0 is assumed as the Maxwell distribution). In our model the ion beam is cylindrically symmetric, finite length 2ℓ and constant velocity v_b, and the channel plasma is composed by electrons and immobile, neutralizing, background ions and also cylindrically symmetric.

By solving these equations, the return current is calculated and only z-component is shown below. And it shows that the ion beam charge is almost completely neutralized by the background plasma, while current density is not neutralized completely. Although the radial component of the ion beam current is null, the return current has a radial component. These result in the deformation of the ion beam.

3. Time Evolution of Ion Beam

We now consider the time evolution of the finite length ion beam propagating in the plasma channel. In earlier section we considered the return current as a response to the finite length ion beam propagation. This return current is formed after a transient time, and during that time the ion beam must be deformed by the electromagnetic field induced by the net charge and current. Considering the mass ratio, electrons respond to the field much faster than ions. So while we consider the time evolution of the ion beam, we can assume that the electrons respond so quickly that the transient time is negligible. By this assumption, the return current and electromagnetic fields are formed so that they are consistent with the ion beam instantly. As a result, the ion beam will be deformed by these electromagnetic fields induced by both the ion beam and the return current obtained above. The time evolution of the ion beam is discussed by using the Vlasov equation for the distribution function of the beam ions f_b. The collisional effect on the time evolution of the distribution function is negligible because the ion beam is propagating with high velocity and the Coulomb collisional effect is very small. We write Vlasov equation with unperturbed and perturbed Liouville operator.

$$
\begin{aligned}
\frac{\partial f_b(\boldsymbol{r}, \boldsymbol{v}, t)}{\partial t} &= iL_0 f_b(\boldsymbol{r}, \boldsymbol{v}, t) + iL_1(t) f_b(\boldsymbol{r}, \boldsymbol{v}, t), \\
iL_0 &\equiv -v_b \frac{\partial}{\partial z},
\end{aligned}
$$

$$iL_1(t) \equiv -\frac{e}{M}(\boldsymbol{E}(t) + \boldsymbol{v} \times \boldsymbol{B}(t)) \cdot \frac{\partial}{\partial \boldsymbol{v}} - (\boldsymbol{v} - v_b \boldsymbol{e}) \cdot \frac{\partial}{\partial \boldsymbol{r}}.$$

where, e and M denote charge and mass of ion, respectively. The unperturbed operator iL_0 expresses the effect of propagation of the beam with the velocity v_b along z axis, and the perturbed operator $iL_1(t)$ denotes the effect of deformation of the ion beam distribution due to the electromagnetic interaction. By paying attention that iL_0 and $iL_1(t)$ do not commute each other, we obtain the formal solution as follows,

$$f_b(\boldsymbol{r}, \boldsymbol{v}, t) = \exp(iL_0 t)\left[f_b(0) + \int_0^t \exp(-iL_0 t') iL_1(t') f_b(t') dt'\right],$$

where $f_b(0)$ is an initial beam ions distribution function. This is an integral equation and is solved by the successive iteration method. In this case, however, attention must be paid that the integrand $iL_1(t)$ depends on the distribution function of the ion beam $f_b(t)$, therefore it is a nonlinear integral equation. At each step of the iteration the equation must be linearized simultaneously. We calculate the electric and magnetic field $\boldsymbol{E}$ and $\boldsymbol{B}$ of the perturbed operator $iL_1(t)$ from the Maxwell equations by using the ion beam distribution function of the n-th approximation $f_b^{(n)}(t)$ in the n-th step iteration. And we sum up the infinite numbers of terms in order to avoid secularity. As a result, we obtain time evolution of the ion beam distribution function in terms of initial distribution function.

$$\begin{aligned}
f_b(t) &= en_b R(r'(t), z'(t)) \delta(v_r - v_r'(t)) \delta(v_z - v_z'(t)), \\
r'(t) &= \left| r - \frac{et^2}{2M}(E_r - v_b B_\theta) \right|, \\
z'(t) &= z - v_b t - \frac{et^2}{2M} E_z, \\
v_r'(t) &= \frac{1}{1 + (\frac{et}{M} B_\theta)^2} \frac{et}{M} \left(E_r - v_b B_\theta - \frac{et}{M} E_z B_\theta \right), \\
v_z'(t) &= \frac{1}{1 + (\frac{et}{M} B_\theta)^2} \left(v_b + \frac{et}{M} E_z + (\frac{et}{M})^2 E_r B_\theta \right), \\
f_b(0) &= en_b R(\boldsymbol{r}) \delta(\boldsymbol{v} - v_e \boldsymbol{e})
\end{aligned}$$

where $R(\boldsymbol{r})$ is initial distribution function of the ion beam with respect to coordinate variable. This solution is not valid for infinite time because we used approximation in the operator c alculus, but it is valid for a long enough time to apply this for the fusion driver.

4. Concrete models

We applied our solution for two interesting cases, the Gaussian model and a hollow model. In both cases, the ion beams are strongly deformed only in a narrow adjacent near the beam head and beam tail. And most part of the ion beams do not change drastically. At the beam head strong electric field acts on the ion beam and at the beam tail both electric and magnetic field act on the ion beam. The radial profile of the electric field is proportional to $-\frac{\partial \rho_b}{\partial r}$ and that of the magnetic field is $-\frac{1}{r}\{\int_0^r rR(r)dr\}$. Therefore, the force acting on the ion beam is proportional to $-\frac{\partial \rho_b}{\partial r}$ near the beam head. Consequently, the beam whose radial profile is a decreasing function of r as the Gaussian model tends to expand radially , but the beam whose radial profile is an increasing function of r as the hollow model tends to pinch radially at the beam head as is shown in Fig. 1.

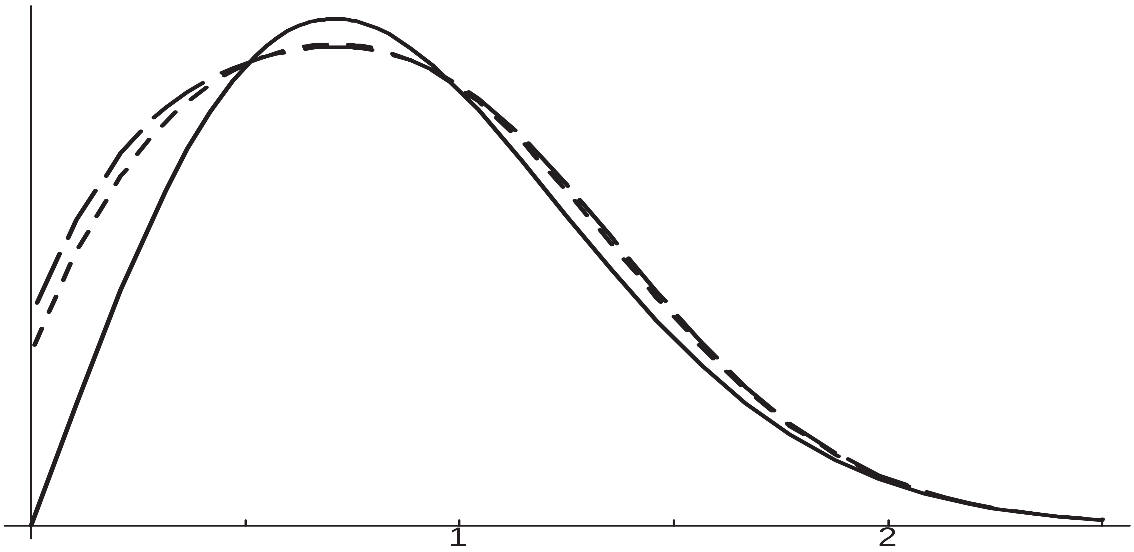

Figure 1. Radial density profile of ion beam vs. radial coodinate r. The deformation of the ion beam current density at the beam head. Solid line corresponds to $\Omega_{\mathrm{p}}\mathrm{t} = 0$, dotted line to $\Omega_{\mathrm{p}}\mathrm{t} = 3000$ and wide dotted line to $\Omega_{\mathrm{p}}\mathrm{t} = 4300$ (10 m propagation).

Using the obtained analytical solution, we can see the time evolution of the ion beam distribution function. And it is clear that by numerical analysis the hollow beam is suitable for the fusion energy driver because it is prevented from diverging as it proceeds.

References

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