TWO DIMENSIONAL PROPAGATION OF INTENSE LASER PULSES IN PLASMAS

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1. Introduction

The physics of self-trapping of light in a plasma by nonlinear electron dynamics is a problem of considerable interest in laser fusion[1, 2], particle acceleration by laser wake fields[3], light boring through dense materials[4, 5] etc. Two simple models have emerged as interesting candidates for explaining the observed phenomena - one for propagation in overdense warm plasmas in laser fusion and the other for problems relevant to propagation in cold underdense plasmas from the point of view of particle acceleration by laser wake fields. The basic difference in the two models arises from the nature of the nonlinear terms and the role played by the electron and ion fluids. An important question pertains to the existence of stationary pulse like states of the trapped light - often called light bubbles - that can propagate in the plasma. While such pulse like solutions have been definitively established in model one dimensional equations, their existence in two or three dimensions is a relatively open question. In this paper we address this question and carry out a detailed numerical investigation of two dimensional pulse propagation in both these regimes. Using a novel nonlinear eigenvalue search procedure we demonstrate the existence of two dimensional soliton like solutions for two model nonlinear equations pertinent to the two regimes described above.

2. Pulse propagation in warm overdense plasmas

Light trapping in overdense plasmas arises through nonlinear development of the filamentation instability[1]. The primary nonlinear effect in this case is that associated with plasma evacuation from regions occupied by intense light through ponderomotive force effects; propagation of light through regions with modulated density then leads to further focussing of the light and increase of its intensity. The nonlinear development of light filaments is typically on an ion time scale and involves motion of the ions. Further, when $\omega \leq \omega_p$, the light filaments are likely to be unstable to a trapping instability which closes the ends along the axis with overdense plasmas. Since the finally trapped bubble is stationary, the electrons and ions satisfy a Boltzmann like distribution $n_e \approx n_i \approx n_0 exp[-e^2 \mid E \mid^2 /2m\omega^2(T_e + T_i)]$. Use of this density in the wave equation for the electromagnetic fields then gives a nonlinear propagation equation which describes the trapped light bubbles. Solutions for such an equation with radially localized azimuthal fields was first found by Kaw *et al.* [6]. As stated above, these solutions are however subject to modulational instability in the z direction and the light wave can break up into small pieces along the direction of propagation. Using semi-analytic arguments and on heuristic grounds, Kaw *et al.* [6] conjectured that such a modulational instability could saturate and lead to a final stationary state which would be in the form of a three dimensional soliton that was localized in all directions. However they did not prove this conjecture either analytically or through numerical demonstrations. In this work we have carried out a direct numerical investigation of their generalised nonlinear propagation equation and have found stationary pulse solutions. The nonlinear equation we have solved[6] is,

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\right]\phi + \left[1 - \sigma - exp(-\phi^2)\right]\phi = 0 \tag{1}$$

where $\sigma = (\omega_p^2 - \omega^2)/\omega_p^2$, ϕ is an appropriately normalized field amplitude and other notations are standard. Eq. (1) constitutes a nonlinear eigenvalue equation where well behaved pulse like solutions (soliton states) exist for different values of the parameter σ. The numerical search for these solutions is carried out in an iterative fashion with each iterate having to satisfy the normalisation condition of $\int_{-\infty}^{\infty} dz \int_0^{\infty} rdr[1 - \sigma - exp(-\phi^2)]\phi = 0$. Fig. 1 illustrates the lowest eigenstate with an eigenvalue of $\sigma = 0.793$. Fig. 2 illustrates the next higher eigenstate (with more radial structure). These solutions confirm the conjecture of Kaw *et al.* [6] and demonstrate the existence of higher dimensional pulse like stationary solutions for large amplitude electromagnetic waves propagating in a warm plasma.

3. Pulse propagation in cold underdense plasmas

We next consider the problem of propagation of large amplitude electromagnetic waves in an underdense plasma ($\omega \gg \omega_p$). The directed electron response in this case is assumed to be so intense that electron temperature effects are unimportant. At these amplitudes the basic nonlinearity in a cold plasma model is that associated with the relativistic electron motions and the consequent coupling of the light waves to plasma waves (through forward Raman scatter effects). Since the light bubbles propagate nearly with the speed of light, ion motions can be ignored. Solutions of this kind in the one dimensional propagation case have been obtained in [3] for arbitrary wave amplitudes. For the small amplitude case, a detailed WKB theory has also been worked out by Kuehl *et al.* [7] which explains most of the salient features of the pulse solutions including the nature of the discrete spectrum of eigenvalues. In this paper we consider a two dimensional generalisation for small amplitudes[8] and carry out a systematic numerical search for the existence of trapped light bubble solutions that are confined both axially and radially. Our model equations are[8]

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} + (\phi - \frac{\partial^2 \phi}{\partial y^2} - \Lambda)R = 0 \tag{2}$$

$$\frac{\partial^2\phi}{\partial x^2} + \epsilon^2\phi = \epsilon^2 R^2 \tag{3}$$

where ϵ^2 represents the ratio of plasma and the light wave frequency, R denotes the field amplitude, ϕ stands for the scalar potential and the x,y variables are defined in the moving frame of the traveling wave. All variables are expressed in appropriate normalized units. The above set of coupled equations are once again solved by using a nonlinear eigenvalue search approach. Eq. (2) is treated as a Schrodinger like equation (with an initial assumed profile for ϕ) and solved as a linear problem for the eigenfunction R and eigenvalue Λ. For the given eigenfunction R, Eq. (3) is next solved to determine the next iterate of ϕ. In seeking convergence of this iterative process it is also necessary to satisfy two normalization conditions at each iterative step. They determine the absolute amplitude of R and the relative scaling of the R and ϕ fields. These conditions can be derived from the two equations above by suitable integration over all space and using vanishing boundary conditions for R, ϕ and their derivatives at large distances. The two conditions are, $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy(R^2 - \phi) = 0.$ and $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy(\phi - \partial^2\phi/\partial y^2 - \Lambda)R = 0.$ With the help of this novel iterative procedure it was possible to achieve a convergence and a typical converged set of solutions for ϕ and R are shown in Fig. 3 and Fig. 4 respectively. These results correspond to the symmetrical solution for R, corresponding to a value of $\Lambda = 4.66$. It is interesting to note that the converged ϕ for this particular case, however, is not radially symmetric.

4. Conclusion

In conclusion, we have used a novel nonlinear eigenvalue solver to demonstrate the existence of two dimensional cylindrically trapped solutions of light bubbles in warm overdense and cold underdense plasmas. The former are stationary in the laboratory frame while the latter propagate with the speed of light. Further work is required for generalization of the cold plasma case to arbitrary amplitudes and to investigate the stability of such solutions to 3d perturbations.

References

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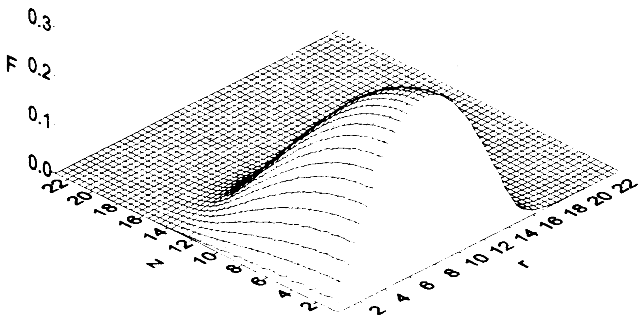

Figure 1. Lowest eigenfunction solution of Eq. (1).

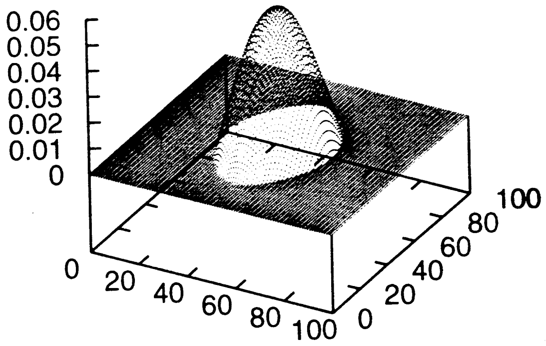

Figure 3. Surface plot of scalar potential.

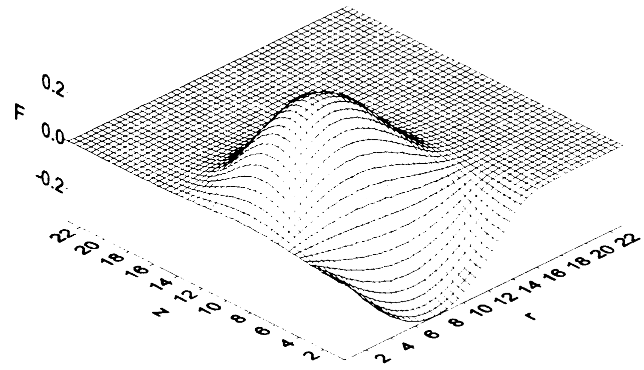

Figure 2. Second eigenfunction solution of Eq. (1).

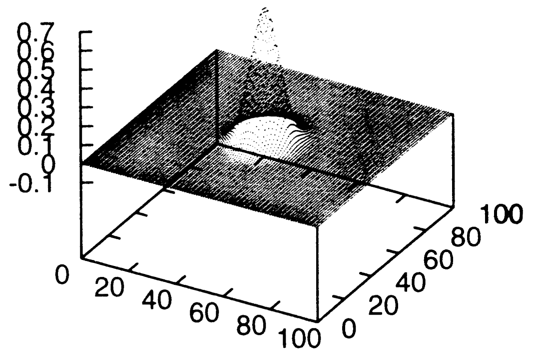

Figure 4. Surface plot of field amplitude.