HELICITY PINCH EQUILIBRIUM WITH RADIATION PROCESSES

Petr Kulhánek and **Martin Žáček**

Department of Physics, FEE CTU, Technická 2, 166 27, Prague 6
(e-mail: kulhanek@fel.cvut.cz, zacek@fel.cvut.cz*)*

Helicity pinch

Pinches are one of the most common structures in both space and laboratory plasmas [6]. The pinches (magnetically confined plasma) have either cylindrical or planar geometry (sheet pinch). In „long living" pinches the helicity structure is usually observed. In the laboratory pinch the spirals suddenly occur in the final steps of the evolution. The current density and magnetic field have both axial and azimuthal components. Similar structure can be established in the sheet pinch as well. Field aligned currents generate internal magnetic field, which has the same role as the azimuthal field in the cylindrical pinch. The helical structures seem to be relatively stabile and common feature of the plasma behavior. The aim of this paper is to manifest some phenomena closely related to the spiral plasma structures.

Helicity

Helicity can be defined for the vector field $\mathbf{K}$ by the relation [1]

$$H \equiv \mathbf{K} \cdot \operatorname{rot} \mathbf{K} . \tag{1}$$

Helicity is zero for all fields with rot $\mathbf{K} = 0$. For vortices with circular streamlines the helicity is zero as well. The fields with helical streamlines have nonzero helicity proportional to cos β. (β - pitch angle). For the pinch structure, the following helicity definitions may be meaningful (vector potential, magnetic field, current density, electric field and velocity field):

$$
\begin{aligned}
H_A &= \mathbf{A} \cdot \operatorname{rot} \mathbf{A} = \mathbf{A} \cdot \mathbf{B} , \\
H_B &= \mathbf{B} \cdot \operatorname{rot} \mathbf{B} = \mu_0 \, \mathbf{j} \cdot \mathbf{B} , \\
Hj &= \mathbf{j} \cdot \operatorname{rot} \mathbf{j} = -\frac{1}{\mu_0} \, \mathbf{j} \cdot \Delta \mathbf{B} ,
\end{aligned}
\qquad
\begin{aligned}
H_E &= \mathbf{E} \cdot \operatorname{rot} \mathbf{E} = -\mathbf{E} \frac{\partial \mathbf{B}}{\partial t} , \\
H_v &= \mathbf{v} \cdot \operatorname{rot} \mathbf{v} \ .
\end{aligned}
\tag{2}
$$

Beltrami property

The vector field fulfilling the Beltrami property

$$\mathbf{K} \times \operatorname{rot} \mathbf{K} = 0 , \quad \text{or} \quad \operatorname{rot} \mathbf{K} = \alpha \, \mathbf{K} \tag{3}$$

seems to be very interesting from the helicity point of view. The coefficient of the proportionality between the field and its rotation may be a function of space and time variables. The Beltrami fields are helical, because

$$H \equiv \mathbf{K} \cdot \operatorname{rot} \mathbf{K} \ = \ \alpha \, \mathbf{K} \cdot \mathbf{K} \ = \ \alpha \, K^2 \, . \tag{4}$$

The α coefficient is the field helicity divided by quadrate of the field absolute value. For $\alpha = \mathrm{const}$ and $\operatorname{div} \mathbf{K} = 0$, the field $\mathbf{K}$ satisfies Helmholtz equation

$$\Delta \mathbf{K} \ + \ \alpha^2 \, \mathbf{K} \ = \ 0 \, . \tag{5}$$

It can be derived from (3) by applying the rot operator. In this case, the field $\mathbf{K}$ is characteristic vector of the Laplace operator in corresponding geometry.

A typical example of the Beltrami fields are ABC flows [2]:

$$\mathbf{K} \ = \ (A \sin z \ + \ C \cos y \ , \ \ B \sin x \ + \ A \cos z \ , \ \ C \sin y \ + \ B \cos x) \, . \tag{6}$$

For this field, $\operatorname{rot} \mathbf{K} = \mathbf{K}$ and $\Delta \mathbf{K} = -\mathbf{K}$. There exist chaotic regions in these flows. In plasma physics, the magnetic field satisfies Beltrami condition in force free configuration with current density aligned along magnetic field, $\mathbf{j} \parallel \mathbf{B}$ (Birkeland current). In this case the Lorenz force density $\mathbf{j} \times \mathbf{B}$ is zero. This configuration has the lowest possible energy and plasma with energy dissipation tends to this configuration. The Beltrami condition can be derived from the Ampere law:
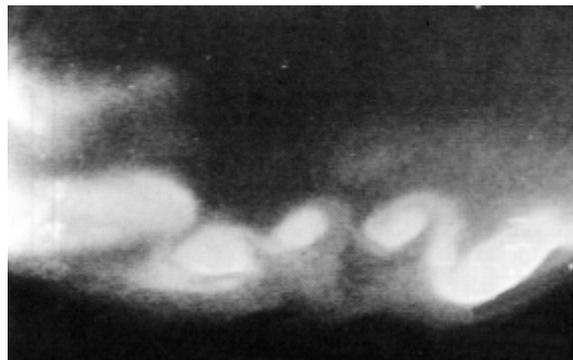
$$\mathbf{j} = \operatorname{rot} \mathbf{B} / \mu_0 \ \ \wedge \ \ \mathbf{j} \parallel \mathbf{B} \ , \ \ \ \ \Rightarrow \ \ \ \ \mathbf{B} \times \operatorname{rot} \mathbf{B} = 0 \, , \ \ \text{resp.} \ \ \operatorname{rot} \mathbf{B} = \alpha \mathbf{B} \, . \tag{7}$$

From the above it is obvious that the force free magnetic field is helical. The force free configuration evolves in final steps of the pinch evolution and matches the onset of helical mode.

Diocotron instability
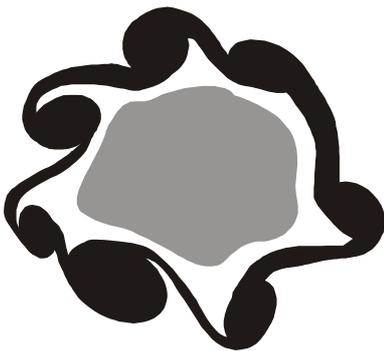
The pinch cross section (diocotron instability)   Aurora. Alaska 1973 [1]

If, in the cylindrical pinch, the charge is separated in radial direction for some reasons, the radial electric field arises. As a consequence, the pinch azimuthally rotates due to particle drift in perpendicular E_r and B_z fields. On the pinch surface Kelvin-Helmholtz like instability called the diocotron instability evolves. The surface is modified into typical vortex

structures [1]. The charge separation can be invoked by particle drifts, pinch radiation accompanying with the temperature gradient and some instabilities. The diocotron instability had been observed in many plasma arrangements and the azimuthal rotation can be the starting mechanism for the onset of the helical mode.

Radiation

In pinch, the Joule heating is compensated by the radiation processes. There are three basic mechanisms of the radiation: recombination, bremsstrahlung and synchrotron radiation. Corresponding power densities for cylindrical pinch can be written as [4]

$$
\begin{aligned}
P_R &= P_{R0}n^2T^{-1/2} &&\text{recombination,} \\
P_B &= P_{B0}n^2T^{1/2} &&\text{bremsstrahlung,} \\
P_S &= P_{S0}n^2T^2 &&\text{synchrotron radiation.}
\end{aligned}
\tag{8}
$$

The proportionality constants P_{R0}, P_{B0}, P_{S0} depend on the plasma type and some estimates of the values are given in [4]. The recombination is dominant for the temperatures bellow 10^5 K, the bremsstrahlung in the temperature interval $(10^5 \div 10^7)$ K and synchrotron radiation above 10^7 K [5].

The radiation dominantly influences the behavior of the pinch. The pinch does not have the Bennett profile [4,5], its boundary is not sharp, the pressure tends to zero very slowly. In some cases the depletion of the concentration in the pinch center occur (hollow pinch). Due to the radiation, the temperature gradient cannot be neglected. The temperature gradient leads to the chemical and charge separation in the radial direction and can result in diocotron instability and the onset of the helical mode.

α effect

The velocity field fluctuations cause nonzero current component in magnetic field mean value direction. This current component behaves like Birkeland current and generates azimuthal magnetic field. The statistical plasma fluctuations together with magnetic reconnection processes enable transformation between axial and azimuthal magnetic field components and so the nonzero pinch helicity [3].

Equilibrium model

In [4] a simple model of the pinch equilibrium with radiation processes was suggested and some important results were obtained (soft edge, central depletion in concentration, polytropic behavior). The natural extension to the helical pinch in the steady state flow equilibrium is:

$$\begin{aligned}
\operatorname{rot} \mathbf{B} &= \mu_0\, \mathbf{j}, \\
\operatorname{rot} \mathbf{E} &= 0, \\
mn\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} &= -\nabla p + \mathbf{j}\times\mathbf{B}, \\
\operatorname{div} n\mathbf{v} &= 0,
\end{aligned}
\qquad
\begin{aligned}
\mathbf{j}\cdot\mathbf{E} &= n^2\, c_\beta(z) T^\beta, \\
\mathbf{j} &= \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \\
p &= nkT\ .
\end{aligned}
\qquad (9)$$

The fundamental equations are: the Ampere's law, the Faraday's induction law, the equation of the motion, the continuity equation, the energy balance with the radiation, the Ohm's law and the equation of the state. The number of the unknown variables (p, n, T, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{j}$, $\mathbf{v}$) can be adequately reduced in the cylindrical geometry. The electric and magnetic fields have to meet the conditions div $\mathbf{B}$ = div $\mathbf{E}$ = 0.
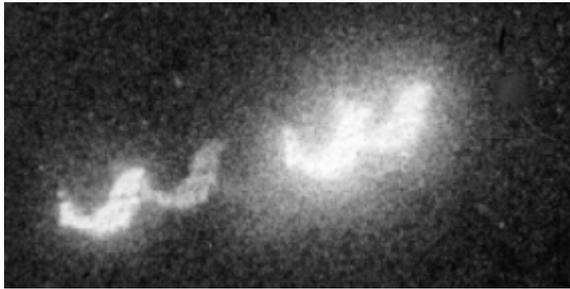
Cu fiber discharge spiral, $\varnothing$ 110×10^{-6} m, 1.3 MA, RTG, S-300 device, RRC Kurchatov Institute, 1998. Shot No. 19059801. Camera obscura $\varnothing$ 100×10^{-6} m, foil 24×10^{-6} m Mylar + 120 nm Al.

Conclusion

The onset of the helical mode can be caused by a variety of phenomena. In the final pinch stages the helical structure seems to be predetermined, relatively stable pinch configuration.

Acknowledgements

References

[1] Perratt A.L.: Physics of the Plasma Universe. Springer-Verlag, 1991

[2] Feudel F., Seehafer N., and Schmidtmann O.: "Fluid helicity and dynamo bifurcations", Phys. Lett. A **202**, 73-78, 1995

[3] Moffatt H.K.: Magnetic field generation in electrically conducting fluids. Cambridge University Press, 1978

[4] Koller J., Kulhánek P., Žáček M.: "Z pinch equilibrium with radiation processes", *12. konference českých a slovenských fyziků*, VŠB Ostrava, 1996 (*in Czech*)

[5] Žáček M., Koller J., Kulhánek P.: "Present state of the z pinch equilibrium calculations", *Workshop 97*, CTU Prague, 1997

[6] Kulhánek P., Břeň D., Kubeš P., Koller J.: "Pinch - The Universe in the Lab", (*in Czech*) http://aldebaran.feld.cvut.cz/, 1997

[7] Kubeš P., Kravárik J. et al.: "Generation of VUV Radiation of a Rod Corona from a Small Magnetic Pinch", *this ICPP*, Prague, 1998.