THE EXPANSION OF THE UNDERDENSE PLASMA INTO A VACUUM UNDER THE ACTION OF 'EM' WAVE FIELD

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1. Introduction

The interaction of high-intense EM radiation with a plasma has received significant attention over the last decades. This process has been investigated in a great number of publications. Most of the latest papers devoted to theoretical consideration of numerous phenomena accompaning the interaction, present the results of the computer simulation. Interaction of short powerful laser pulses with the plasma surface causes a lot of interesting nonlinear effects (rise of different modes of surface waves and instabilities, appearance of quasi-stationary vortex structures, generation of high harmonics in a boundary layer of an overdense plasma etc.). All these effects strongly depend on a distribution of the plasma density near the surface. Therefore, for a clear understanding of cited phenomena it is necessary to have simple analytical models of the edge plasma dynamics. At the same time, this models should take into account the main physical processes accompaning the interaction of radiation with plasmas.

Here, we present an exact analytical solution of 1D problem of underdense collisionless plasma expansion into a vacuum under the action of EM radiation. The proposed solution is obtained in the fluid theory approximation (ion temperature equals to zero) assuming quasi-neutrality. The flow of the cold ions is described by the continuity and momentum equations. We consider two limiting cases of expansion for hot electrons: isothermal and adiabatic. The spatial distribution of the EM field is described by the EM wave equation without absorbtion. We neglect the influence of decay instabilities.

2. Plasma expansion

Consider an initially homogeneous plasma with the density ρ_0 that occupies the half-space $z < 0$. At the moment $t = 0$ the radiation is switched on and plasma starts expanding. The momentum and continuity equations are

$$\begin{aligned} &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\left(c_s^2\frac{\partial \rho}{\partial z} + \frac{1}{4}\varepsilon_0\frac{\omega_p^2}{\omega^2}\frac{\partial |E|^2}{\partial z}\right), \\ &\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0 \end{aligned} \tag{1}$$

where ω and $\omega_p = \sqrt{e^2\rho/mM\varepsilon_0}$ are the radiation and plasma frequency, respectively (we consider the underdense plasma, so that $\omega > \omega_p(\rho_0)$); $c_s = \sqrt{\gamma p/\rho}$ is the sound speed, γ is the adiabatic index (for isothermal electrons c_s equals ion acoustic speed and $\gamma = 1$); m and M are the electron and ion mass, respectively. The last term in the right-hand side of the momentum equation corresponds to a high-frequent pressure of the radiation. Electric field of the EM wave E obeys the stationary wave equation (as the ratio of the electromagnetic time-scale to the gasdynamic one is of order of c_s/c)

$$\frac{\partial^2 E}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon(z,t)E = 0 \tag{2}$$

where c is the velocity of light, $\varepsilon = 1 - \omega_p^2/\omega^2$ is the plasma permittivity.

Let us assume *a priori* that the density is a monotonical function of z. Consequently, ρ can not exceed ρ_0, so that the expanding plasma remains underdense. Therefore, since the spatial scale of the plasma inhomogeneity $L_\rho = \rho/|d\rho/dz|$ is much greater than the wavelength c/ω, we may use WKB-method to solve the wave equation (2) in the expanding plasma [1]

$$E = \frac{c_1}{\varepsilon^{1/4}} \exp\left\{ik_0 \int \sqrt{\varepsilon}\, dz\right\} + \frac{c_2}{\varepsilon^{1/4}} \exp\left\{-ik_0 \int \sqrt{\varepsilon}\, dz\right\} \tag{3}$$

where $k_0 = \omega/c$. In a vacuum (at $z \to \infty$) a spatial distribution of the electric field is determined by combination of the incident and the reflected waves

$$E(z) = E_0 \exp\{i(\omega t + k_0 z)\} + R_0 \exp\{i(\omega t - k_0 z)\}$$

where E_0 and R_0 are amplitudes of the incedent and reflected waves, respectively. Using the continuity conditions for E and $\partial E/\partial z$ on the plasma-vacuum boundary we obtain, that $R_0 \sim E_0/k_0 L_\rho$ (since $\mathrm{Im}\sqrt{\varepsilon} = 0$). Neglecting the terms $O(1/k_0 L_\rho)$ we have finally from the formula (3)

$$|E|^2 \simeq \frac{E_0^2}{\sqrt{\varepsilon}} \equiv \frac{E_0^2}{\sqrt{1 - \omega_p^2/\omega^2}} \tag{4}$$

We see that $|E|$ is a single-valued function of density ρ. As the expansion is either isothermal or adiabatic we may also assume that the velocity v is a single-valued function of density. So that the solution of the presented problem should be similiar to the general Riemann solution of a 1D travelling waves problem [2]. The equations (1) and formula (4) yield

$$\begin{aligned} &\frac{dv}{d\rho}\frac{\partial\rho}{\partial t} + c_{s0}\left(v\frac{dv}{d\rho} + \frac{c_s^2}{\rho} + \mathcal{A}(1 - \Omega^2\rho)^{-3/2}\right)\frac{\partial\rho}{\partial z} = 0, \\ &\frac{\partial\rho}{\partial t} + c_{s0}\left(\rho\frac{dv}{d\rho} + v\right)\frac{\partial\rho}{\partial z} = 0 \end{aligned} \tag{5}$$

where dimensionless parameters are introduced

$$\mathcal{A} = \frac{1}{8}\frac{\varepsilon_0 E_0^2}{\rho_0 c_{s0}^2}\Omega^4, \qquad \Omega = \frac{\omega_p(\rho_0)}{\omega}$$

In equations (5) the density ρ is normalized to ρ_0, the velocity v and sound speed c_s are normalized to $c_{s0} = c_s(\rho_0)$. Taking into account the relation $c_s^2 = \rho^{\gamma-1}$ we obtain from the equations (5)

$$\rho\left(\frac{dv}{d\rho}\right)^2 = \rho^{-(2-\gamma)} + \mathcal{A}(1-\Omega^2\rho)^{-3/2},$$
$$z = c_{s0}\left(v + \rho\frac{dv}{d\rho}\right)t + f(\rho) \tag{6}$$

where $f(\rho)$ is an arbitrary function of ρ. It is worth to note that the equations (6) are the general solution of 1D problem of the underdense plasma flow in a presence of EM radiation. This solution is valid when v is a single-valued function of ρ and when the geometric optics approximation (WKB-method) may be used. In the limit $\mathcal{A} \to 0$, equations (6) correspond to the general Riemann solution.

For the considered problem of plasma expansion into a vacuum $f(\rho) = 0$, so that we have the case of a similarity flow depending on the variable z/t [2]. According to the first equation (5), the effective sound speed is represented by

$$c_s^* = \sqrt{c_s^2 + \mathcal{A}\rho(1-\Omega^2\rho)^{-3/2}} > c_s$$

From (6) we obtain the co-ordinate of the rarefaction wave front ($\rho = 1$) moving into the plasma

$$-\left.\frac{z}{c_{s0}t}\right|_{\rho=1} = c_s^*|_{\rho=1} = \sqrt{1 + \mathcal{A}(1-\Omega^2)^{-3/2}} \tag{7}$$

The co-ordinate of the plasma boundary ($\rho = 0$) is

$$\left.\frac{z}{c_{s0}t}\right|_{\rho=0} = \int_0^1 x^{-(3-\gamma)/2}\sqrt{1 + \mathcal{A}x^{2-\gamma}(1-\Omega^2 x)^{-3/2}}dx$$

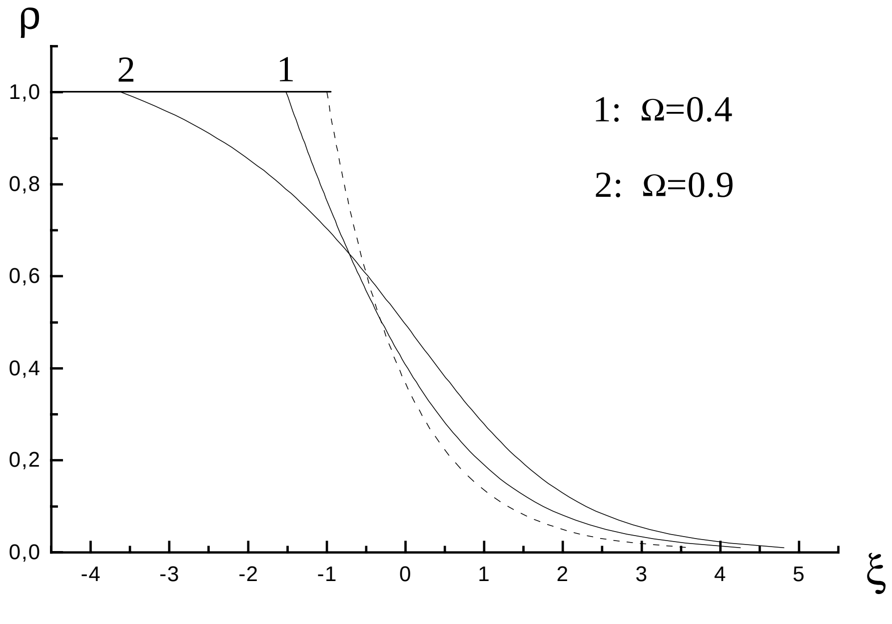

Figure 1. Density ρ vs $\xi = x/c_{s0}t$ in the isothermal rarefaction wave for the underdense plasma. The dashed line represents the self-similar solution in absence of radiation.

Both values are greater than those for the case of free plasma expansion [3,4]. An inhomogeneous plasma layer between the plasma boundary and the wave front is the region of a rarefaction wave. The figure 1 shows the dependence of the plasma density in the rarefaction wave on the variable $\xi = z/c_{s0}t$. This distribution corresponds to the isothermal case $\gamma = 1$ for $\Omega = 0.4$ (curve 1) and $\Omega = 0.9$ (curve 2); for both curves $\mathcal{A} = 1$. The dashed line represents the self-similar solution of the plasma expansion in absence of radiation [5].

Finally, we should make a short remark regarding the obtained solution. The above results can be immediately generalized for the case of any gas where we may neglect the absorbtion of the EM wave and assume $\varepsilon > 0$ (for a given ω). Indeed, the polarisability of the gas $\varepsilon - 1$ may be regarded as proportional to its density [6]

$$\varepsilon - 1 = \chi\rho$$

where χ is a function of ω depending on an optical properties of the gas. This function may be both positive and negative (for a plasma $\chi = -\Omega^2$). Then the force of the high-frequency pressure is given by [6]

$$F_{hf} = \frac{1}{4}\varepsilon_0(\varepsilon - 1)\frac{\partial|E|^2}{\partial z}$$

Therefore, replacing Ω^2 by $-\chi$ in equations (6) we obtain the general solution of the presented problem for the gas.

3. Conclusions

For the underdense plasma, the high-frequency pressure of EM wave causes strong acceleration of the plasma expansion making the profile of plasma density more smooth in comparison to the case of expansion without radiation. In a plasma $d\varepsilon/d\rho < 0$, so that the high-frequency pressure increases towards the unperturbed region. This results in rapid acceleration of the rarefaction wave front with Ω, $z/t|_{\rho=1} \propto \Omega^2/(1-\Omega^2)^{3/4}$ for $\mathcal{A} \geq 1$ (see formula (7) and curve 2 in Figure 1).

References

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