STOPPING OF GYRATORY FAST PARTICLE IN MAGNETIZED COLD PLASMA

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Abstract. The energy loss by a test gyratory particle in a cold plasma in the presence of a homogeneous magnetic field is considered, and analytical and numerical results for the rate of energy loss are presented. This is done for strong fields and for weak fields (i. e., for the electron cyclotron frequency greater than, and less than, the plasma frequency), and in case, when the test particle velocity is greater than the electron thermal velocity. It is shown that the rate of energy loss may be much higher than in the case of motion through plasma in the absence of magnetic field.

The energy loss of fast charged particles in a plasma has been a topic of great interest since the 1950s because of its considerable importance for the study of the basic interactions of charged particles in real media; moreover, recently, it has also become a great concern in connection with heavy-ion driven inertial fusion research [1, 2].

The nature of experimental plasma physics is such that experiments are usually performed in the presence of magnetic fields, and consequently, it is of interest to investigate the effects of a magnetic field on the rate of energy loss.

The stopping of charged particles in a magnetized plasma has been the subject of several papers [3-6]. The stopping of a fast test particle moving with a velocity v much higher than the electron thermal velocity v_T was studied in Refs. [3, 5]. The energy loss of a charged particle moving with arbitrary velocity was studied in [4]. The expression derived there for the Coulomb logarithm corresponds to the classical description of collisions.

In Ref. [6] expressions were derived describing the stopping power of a slow charged particle in Maxwellian plasma with a classically strong (but not quantizing) magnetic field, under conditions such that the scattering processes must be described quantum-mechanically.

In the present report we consider the rate of energy loss of a nonrelativistic gyratory charged particle in a magnetized cold plasma. Also, this problem is important for the construction of models of *X*-ray pulsars [7] and the study of processes in the atmospheres of

magnetic white dwarfs the magnetic fields on the surfaces of which can attain strengths of 10^5-$10^{10}kG$.

A uniform plasma is considered in the presence of a homogeneous magnetic field $\mathbf{B}_0$ (directed in the positive z-direction) which is assumed sufficiently small so that $\lambda_B \ll a_c$ (where λ_B and a_c are respectively the electron de Broglie wavelength and Larmor radius). From these conditions we can obtain $B_0 < 10^5 T$ (T is the plasma temperature), where T is measured in eV and B_0 in kG. Also, due to the high frequencies involved, the very weak response of the plasma ions is neglected and the Vlasov-Poisson equations to be solved for the perturbation to the electron distribution function and the scalar potential ϕ. The solution is (see, for example, [8])

$$\phi(\mathbf{r},t) = \frac{4\pi Ze}{(2\pi)^4} \int d\mathbf{k} \int_{-\infty}^{+\infty} d\omega \frac{\exp[i(\mathbf{kr} - \omega t)]}{k^2 \varepsilon(\mathbf{k},\omega)} \int_{-\infty}^{+\infty} d\tau \exp[i\omega\tau - i\mathbf{kr}_0(\tau)], \tag{1}$$

where $\mathbf{r}_0(t)$ is the radius-vector of the test particle having the components $x_0(t) = a\sin(\Omega_c t)$, $y_0(t) = a\cos(\Omega_c t)$, $z_0(t) = 0$ ($\Omega_c = ZeB_0/Mc$, $a = \mathrm{v}/\Omega_c$, Ze and v are the Larmor frequency, the Larmor radius, the charge and the velocity of the test particle respectively), $\varepsilon(\mathbf{k},\omega)$ is the longitudinal dielectric function of magnetized cold plasma

$$\varepsilon(\mathbf{k},\omega) = \varepsilon(\omega)\cos^2\alpha + h(\omega)\sin^2\alpha \tag{2}$$

with

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\upsilon)}, \tag{3}$$

$$h(\omega) = 1 + \frac{\omega_p^2(\omega + i\upsilon)}{\omega[\omega_c^2 - (\omega + i\upsilon)^2]}. \tag{4}$$

Here, α is the angle between the wave vector $\mathbf{k}$ and the magnetic field, $\omega_p = \sqrt{4\pi n_0 e^2/m}$, ω_c and υ are the plasma frequency, Larmor frequency and the effective collision frequency of the plasma electrons respectively.

The rate of energy loss S of a plasma against a fast charge is defined by the energy loss of the charge in a unit time due to interactions with the plasma electrons. From Eq. (1) it is straightforward to calculate the electric field $\mathbf{E} = -\nabla\phi$, and the stopping force acting on the particle. Then, the rate of energy loss of the test particle becomes

$$S = \frac{2Z^2 e^2 \Omega_c^2}{\pi \mathrm{v}} \sum_{n=1}^{\infty} n\, Q_n(s) \operatorname{Im}\left[\frac{-1}{\varepsilon(n\Omega_c)T(n\Omega_c)}\right], \tag{5}$$

where

$$T(\omega) = \sqrt{\frac{|P(\omega)| + \operatorname{Re} P(\omega)}{2}} + i \operatorname{sgn}\left[\operatorname{Im} P(\omega)\right]\sqrt{\frac{|P(\omega)| - \operatorname{Re} P(\omega)}{2}}, \tag{6}$$

$$Q_n(s) = \pi \int_0^s dx\, J_n^2(x), \tag{7}$$

$P(\omega) = h(\omega)/\varepsilon(\omega)$, $J_n(x)$ is the nth order Bessel function and $s = k_{\max} a$ with $k_{\max} = 1/r_{\min} = 2mv/\hbar$, where $r_{\min}$ is the effective minimum impact parameter. Here $k_{\max}$ has been introduced to avoid the divergence of the integral caused by the incorrect treatment of the short-range interactions between the test particle and the plasma electrons within the linearized Vlasov theory.

The function $Q_n(s)$ is exponentially small at $n > s$. Therefore the series in the Eq. (5) is cut at $n_{\max} \cong s$ and the rate of energy loss is determined by harmonics with $n < n_{\max}$.

Consider now the Eq. (5) for strong and weak magnetic fields. In the case of weak magnetic field ($\Omega_c < v$) one may substitute the summation in expression (5) by integration in $\omega = n\Omega_c$. Since also $Q_n(s) \cong \ln(s/n)$ when $s > n$, in the limit $\Omega_c \to 0$ the Eq. (5) is transformed into a known Bohr's expression.

Consider the case of strong magnetic field and let $c = \omega_c/\Omega_c$ be non-integer. In this case, from Eq. (5) we find:

$$S \cong \frac{Z^2 e^2 \omega_p^2}{\pi v} \frac{v}{\Omega_c} \sum_{n=1}^{\infty} \frac{1}{n^2} Q_n(s)\left[1 + \frac{n^4}{(n^2 - c^2)^2}\right]. \tag{8}$$

From Eq. (8) it follows, that energy loss decreases inversely proportional to the magnetic field. In the case when $c = 1$ (electron test particle), from Eq. (5) we find:

$$S \cong \frac{Z^2 e^2 \omega_p^2}{\pi v} \frac{\Omega_c}{v} Q_1(s). \tag{9}$$

Note that the rate of energy loss increases proportionally to the magnetic field.

These examples of asymptotic dependence of energy loss rate on the value of magnetic field show strong dependence of energy loss on mass of the test particle.

From the Eq. (5) it is straightforward to trace qualitatively the behavior of energy loss rate as a function of magnetic field in the general case. Note, as it follows from Eq. (5) that the rate of energy loss is maximal for those values of magnetic field for which $\varepsilon(n\Omega_c)$ has small values. The small $\varepsilon(n\Omega_c)$ means, that in the dependence of energy loss from magnetic fields, maximums at integer values of parameter $b = \omega_p/\Omega_c$ can be observed. It corresponds

to the case, when on test particle's Larmor orbit includes integer number of plasma oscillation wavelengths $(\lambda_p = 2\pi v / \omega_p)$.
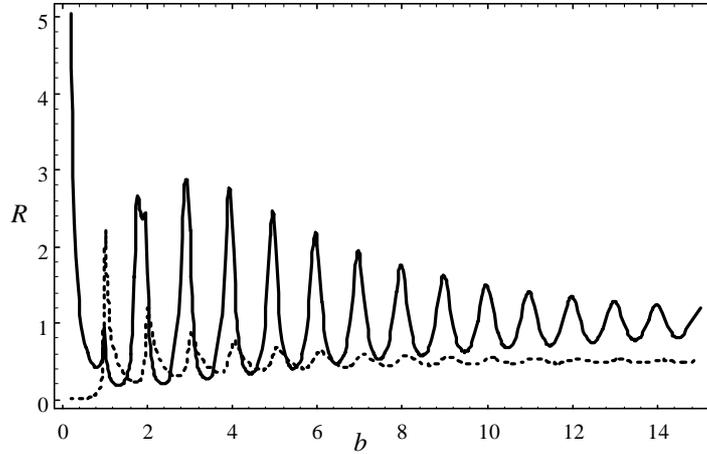

Fig.1. The ratio of $R = S/S_B$ as a function of parameter b in two cases; for electron test particle (solid line), and for proton test particle (dotted line). The parameters are: $\upsilon/\omega_p = 0.01$, $n_0 = 10^{18} cm^{-3}$, $T = 100 eV$ and $v/v_T = 10$.

Fig. 1 shows the ratio $R = S/S_B$ (where S_B is the well-known Bohr result [8]) as a function of parameter b in two cases; for electron test particle (solid line), and for proton test particle (dotted line). The plasma/particle parameters are: $\upsilon/\omega_p = 0.01$, $n_0 = 10^{18} cm^{-3}$, $T = 100 eV$ and $v/v_T = 10$. As it follows from Fig. 1, the rate of energy loss oscillates as a function of magnetic field and many times exceeds the usual Bohr losses of energy.

References

[1] J. D'Avanzo, M. Lontano and P.F. Bortignon: Phys. Rev. E **47,** 3574 (1993).

[2] C. Deutsch and P. Fromy: Phys. Rev. E **51,** 632 (1995).

[3] N. Honda, O.Aona, and T. Kihara: J. Phys. Soc. Jpn. **18,** 256 (1963).

[4] R.M. May and N.F. Cramer: Phys. Fluids **13,** 1766 (1970).

[5] G.G. Pavlov and D.G. Yakovlev: Sov. Phys. JETP **43,** 389 (1976).

[6] S.V. Bozhokin and E.A. Choban: Sov. J. Plasma Phys. **10,** 452 (1984).

[7] M.M. Basko and R.A. Syunyaev: Sov. Phys. JETP **41,** 52 (1975).

[8] N. Rostoker and M.N. Rosenbluth: Phys. Fluids **3,** 1 (1960).