DIELECTRIC CHARACTERISTICS OF A DIPOLE MAGNETOSPHERE

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1. Plasma model and Vlasov equation.

To study the wave processes in the Earth's magnetosphere/plasmasphere it is necessary to solve the Maxwell's equations with a "nonlocal" dielectric tensor [1-3]. For high-frequency waves near the ion/electron cyclotron frequencies, this tensor can be obtained by solving the Vlasov equation for the trapped particles taking into account the two-dimensional inhomogeneities of geomagnetic field and plasma parameters. The intrinsic dipole geomagnetic field is given by $B(R,\phi) = B_0\,(R_0/R)^3\sqrt{1+3\sin^2\phi}$. Here R_0 is the radius of Earth, R is the geocentric distance, ϕ is the geomagnetic latitude, B_0 is the magnetic field in an equatorial plane on the Earth's surface ($R=R_0, \phi=0$). To solve the Vlasov equation we use the standard method of switching to new variables associated with the conservation integrals of energy: $v_\perp^2+v_\parallel^2 = const$, magnetic moment: $v_\perp^2/2B = const$, and the equation of the $\mathbf{B}$-field line: $R/\cos^2\phi = const$. Introducing the variables $v = \sqrt{v_\parallel^2+v_\perp^2}$, $\mu = v_\perp^2 B(L,0)/v^2 B(L,\phi)$, and $L = R/R_0\cos^2\phi$ (instead of $v_\parallel, v_\perp, R$) we seek the perturbed distribution function as

$$f(t,R,\phi,\theta,v_\parallel,v_\perp,\alpha) = \sum_s^{\pm 1}\sum_l^{\pm\infty} f_l^s(\phi,L,v,\mu)\exp(-i\omega t + im\theta + il\alpha) \qquad (1)$$

where α is the gyrophase angle in velocity space. So that the linearized Vlasov equation, for harmonics f_0^s and $f_{\pm 1}^s$, can be rewritten by

$$\frac{\sqrt{1-\mu/b(\phi)}}{\cos\phi\sqrt{1+3\sin^2\phi}}\frac{\partial f_l^s}{\partial\phi} - is\frac{LR_0}{v}\left(\omega + \frac{l\,\omega_{co}}{L^3 b(\phi)}\right) f_l^s = Q_l^s, \qquad l = 0, \pm 1 \qquad (2)$$

where $$Q_0^s = \frac{eE_\parallel F_0}{T}\sqrt{1-\mu/b(\phi)}, \qquad Q_{\pm 1}^s = s\frac{eR_0 L\sqrt{\mu}}{2T\sqrt{b(\phi)}}E_{(\pm 1)}(\phi)F_0.$$

Here, $F_0 = N(L)(\pi v_T^2)^{-1.5}\exp(-v^2/v_T^2)$ is the equilibrium distribution function of plasma particles with the density N, temperature T, charge e and mass M; $v_T^2 = 2T/M$; $b(\phi) = \cos^6\phi/\sqrt{1+3\sin^2\phi}$; $E_\parallel = (\mathbf{E}\cdot\mathbf{B})/B$ is the component of a perturbed electric field parallel to $\mathbf{B}$; $E_{(l)} = E_n - lE_b$, where E_n and E_b are the normal and binormal components of $\mathbf{E}$ perpendicular to $\mathbf{B}$. By indexes $s = \pm 1$ we differ the particles with positive and negative values of $v_\parallel = sv\sqrt{1-\mu/b(\phi)}$ relatively to $\mathbf{B}$. In Eq. (2) we neglect the drift corrections assuming the drift frequencies are much smaller than the bounce frequencies, that is valid when $mv_T L^2 R_0^{-1}\omega_{co}^{-1} << 1$ and $mv_T L^{-1} R_0^{-1}|\omega - \omega_{co}/L^3|^{-1} << 1$, where $\omega_{co} = eB_0/Mc$, and m is the azimuthal wave number over θ (east-west) direction. Depending on μ, the domain of perturbed distribution functions is defined by inequalities: $L^{-2.5}(4L-3)^{-0.5} \le \mu \le 1$ and $-\phi_t(\mu) \le \phi \le \phi_t(\mu)$, where $\pm\phi_t(\mu)$ are the local mirror points for the trapped particles at a given (by L) magnetic field line, which are defined by the zeros of parallel velocity: $\cos^6\phi_t - \mu\sqrt{1+3\sin^2\phi_t} = 0$. Due to the Earth's atmosphere, the trapped particles will be thermalized by the collisions with atmospheric molecules and atoms before they reach the Earth's surface. Any particle with $\mu < L^{-2.5}(4L-3)^{-0.5}$ will not survive more than one half of bounce time and will be precipitated into the atmosphere. After solving Eq. (2), the two-dimensional normal, $j_n(\phi, L)$, and binormal, $j_b(\phi, L)$, current density components can be expressed as $j_n = 0.5[j_1 + j_{-1}]$ and $j_b = 0.5i[j_1 - j_{-1}]$, where

$$j_l(\phi, L) = \frac{\pi e}{2b^{1.5}(\phi)} \sum_s^{\pm 1} \int_0^\infty v^3 \int_{\mu_0}^{b(\phi)} \frac{\sqrt{\mu} f_l^s(\phi, L, v, \mu)}{\sqrt{1 - \mu/b(\phi)}} \, d\mu \, dv, \qquad l = \pm 1. \tag{3}$$

For the parallel current density component, we have

$$j_\parallel(\phi, L) = \frac{\pi e}{b(\phi)} \sum_s^{\pm 1} s \int_0^\infty v^3 \int_{\mu_0}^{b(\phi)} f_0^s(\phi, L, v, \mu) \, d\mu \, dv, \tag{4}$$

where $\mu_0 = L^{-2.5}(4L - 3)^{-0.5}$. Taking into account that the trapped particles, with a given parameter μ, execute the bounce periodic motion with the bounce period proportional to

$$\tau_b = \tau_b(\mu) = 4 \int_0^{\phi_t} \cos\phi \frac{\sqrt{1 + 3\sin^2\phi}}{\sqrt{1 - \mu/b(\phi)}} d\phi, \tag{5}$$

the solution of Eq. (2), for example, for $f_{\pm 1}^s$, is

$$f_l^s(\phi) = \sum_{p=-\infty}^{+\infty} f_{p,s}^l \exp\left(ip \frac{2\pi}{\tau_b} \tau(\phi) + isl \frac{\omega_{co} R_0}{L^2 v} C(\phi) \right), \tag{6}$$

$$f_{p,s}^l = \frac{-iseR_0 L v \sqrt{\mu} F_0 G_{p,s}^l(\mu)}{2v_T T\pi \, (pv/v_T - sZ_l)}, \qquad Z_l = \frac{\omega}{\omega_b} + l \frac{2R_0 \omega_{co}}{\pi v_T L^2} \int_0^{\phi_t} \frac{\cos\psi}{b(\psi)} \sqrt{\frac{1 + 3\sin^2\psi}{1 - \mu/b(\psi)}} d\psi,$$

$$\tau = \int_0^\phi \cos\eta \sqrt{\frac{1 + 3\sin^2\eta}{1 - \mu/b(\eta)}} d\eta, \; G_{p,s}^l = \int_{-\tau_b/2}^{\tau_b/2} \frac{E_{(l)}(\tau)}{\sqrt{b(\tau)}} \exp\left[-ip \frac{2\pi}{\tau_b} \tau - isl \frac{\omega_{co} R_0}{L^2 v} C(\tau) \right] d\tau,$$

$$\omega_b = \frac{2\pi v_T}{R_0 L \tau_b}, \quad C(\phi) = \int_0^\phi \frac{\cos\psi}{b(\psi)} \sqrt{\frac{1 + 3\sin^2\psi}{1 - \mu/b(\psi)}} d\psi - 4 \frac{\tau(\phi)}{\tau_b} \int_0^{\phi_t} \frac{\cos\psi}{b(\psi)} \sqrt{\frac{1 + 3\sin^2\psi}{1 - \mu/b(\psi)}} d\psi.$$

As a result, the contribution of unspecified kind of plasma particles to the transverse current density component is given by

$$\frac{4\pi i}{\omega} b(\phi) j_l(L, \phi) = \frac{\omega_{po}^2 R_0 L}{4\omega \pi^{1.5} v_T} \sum_s^{\pm 1} \sum_{p=-\infty}^{+\infty} \int_{\mu_0}^{b(\phi)} \frac{\mu d\mu}{\sqrt{b(\phi) - \mu}} \times$$
$$\times \int_0^\infty \frac{u^4 \exp(-u^2) G_{p,s}^l}{pu - sZ_l} \exp\left[ip \frac{2\pi}{\tau_b} \tau(\phi) + isl \frac{\omega_{co} R_0}{L^2 u v_T} C(\phi) \right] du, \tag{7}$$

where $\omega_{po}^2 = 4\pi N e^2/M$. Note, Eq. (7) is written in the general form and can be applied to define the contribution of trapped particles to the perpendicular current density components in an axisymmetric magnetosphere with an arbitrary configuration of the closed magnetic field lines. The perturbed longitudinal current can be derived by analogy [4].

2. Transverse permitivity

In order to solve two-dimensional wave equations, we should expand preliminary the perturbed values in a Fourier series over ϕ. So, for the transverse components of the current density $j_l b(\phi)$ and electric field E_l, we have:

$$b(\phi) j_l(L, \phi) = \sum_n^{\pm\infty} j_l^{(n)}(L) \; \exp\left[\frac{i\pi n\phi}{\phi_0(L)} \right], \quad E_l(L, \phi) = \sum_{n'}^{\pm\infty} E_l^{(n')}(L) \; \exp\left[\frac{i\pi n'\phi}{\phi_0(L)} \right], \tag{8}$$

where the points $\pm\phi_0(L) = \pm\arccos(1/\sqrt{L})$ are the beginning and the end of a given (by L) magnetic field line on the Earth's surface. This procedure converts the operator, representing the dielectric tensor, into a matrix whose elements are calculated independently on the solutions of Maxwell's equations. As a result, $(4\pi i/\omega)j_l^{(n)} = \sum_{n'}^{\pm\infty}\epsilon_l^{n,n'}E_l^{(n')}$, and, after the s-summation, the contribution of a given kind of plasma particles to the transverse permittivity elements, $\epsilon_l^{n,n'}(L)$, is

$$\epsilon_l^{n,n'}(L) = \frac{\omega_{po}^2 R_0 L}{2\omega\pi^{1.5}v_T\phi_0}\sum_{p=1}^{\infty}\int_{\mu_0}^{1}\mu d\mu\int_{-\infty}^{\infty}D_p^{n,l}\hat{D}_p^{n',l}\frac{u^4\exp(-u^2)}{pu - Z_l}du - \\ -\frac{\omega_{po}^2 R_0 L}{2\omega\pi^{1.5}v_T\phi_0}\int_{\mu_0}^{1}\frac{\mu d\mu}{Z_l}\int_{0}^{\infty}D_0^{n,l}\hat{D}_0^{n',l}u^4\exp(-u^2)du, \quad (9)$$

where $$D_p^{n,l} = \int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} - 2\pi p\frac{\tau(\phi)}{\tau_b} - il\frac{\omega_{co}R_0}{L^2uv_T}C(\phi)\right)\frac{d\phi}{\sqrt{b(\phi)-\mu}} + \\ +(-1)^p\int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} + 2\pi p\frac{\tau(\phi)}{\tau_b} + il\frac{\omega_{co}R_0}{L^2uv_T}C(\phi)\right)\frac{d\phi}{\sqrt{b(\phi)-\mu}}, \quad (10)$$

$$\hat{D}_p^{n,l} = \int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} - 2\pi p\frac{\tau(\phi)}{\tau_b} - il\frac{\omega_{co}R_0}{L^2uv_T}C(\phi)\right)\frac{\cos\phi\sqrt{1+3\sin^2\phi}}{\sqrt{b(\phi)-\mu}}d\phi + \\ +(-1)^p\int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} + 2\pi p\frac{\tau(\phi)}{\tau_b} + il\frac{\omega_{co}R_0}{L^2uv_T}C(\phi)\right)\frac{\cos\phi\sqrt{1+3\sin^2\phi}}{\sqrt{b(\phi)-\mu}}d\phi. \quad (11)$$

Thus we see that, due to geomagnetic field inhomogeneity, the full spectrum of an electric field (by $\sum_{n'}^{\pm\infty}$) is present in a given (by n) current density harmonic.

3. Longitudinal permitivity

To evaluate the longitudinal permittivity elements, we should expand the perturbed longitudinal (parallel to $\mathbf{B}$) components of the current density $j_\| b(\phi)$ and electric field $E_\|$ in the following Fourier series over ϕ:

$$b(\phi)j_\|(L,\phi) = \sum_{n}^{\pm\infty} j_\|^{(n)}(L)\ \exp\left[\frac{i\pi n\phi}{\phi_0(L)}\right], \qquad E_\|(L,\phi) = \sum_{n'}^{\pm\infty} E_\|^{(n')}(L)\ \exp\left[\frac{i\pi n'\phi}{\phi_0(L)}\right], \quad (12)$$

As a result, $(4\pi i/\omega)j_\|^{(n)} = \sum_{n'}^{\pm\infty}\epsilon_\|^{n,n'}E_\|^{(n')}$, and the contribution of a given kind of plasma particles to the longitudinal permittivity elements, $\epsilon_\|^{n,n'}(L)$, is

$$\epsilon_\|^{n,n'}(L) = \frac{\omega_{po}^2 L^2 R_0^2}{2\pi^2 v_T^2\phi_0}\sum_{p=1}^{\infty}\frac{1}{p^2}\int_{\mu_0}^{1}\tau_b A_p^n\hat{A}_p^{n'}\left\{1 + \frac{2\omega^2}{p^2\omega_b^2}\left[1 + i\sqrt{\pi}\frac{\omega}{p\omega_b}W\left(\frac{\omega}{p\omega_b}\right)\right]\right\}d\mu, \quad (13)$$

where $W(z) = \exp(-z^2)\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z\exp(t^2)dt\right)$ is the plasma dispersion function,

$$A_p^n = \int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} - 2\pi p\frac{\tau(\phi)}{\tau_b}\right)d\phi + (-1)^{p-1}\int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} + 2\pi p\frac{\tau(\phi)}{\tau_b}\right)d\phi, \quad (14)$$

$$\hat{A}_p^n = \int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} - 2\pi p\frac{\tau(\phi)}{\tau_b}\right)\cos\phi\sqrt{1+3\sin^2\phi}\,d\phi + \\ +(-1)^{p-1}\int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} + 2\pi p\frac{\tau(\phi)}{\tau_b}\right)\cos\phi\sqrt{1+3\sin^2\phi}\,d\phi. \quad (15)$$

As was noted above, the expressions (9) and (13) describe the contribution of any one kind of trapped particles to the dielectric tensor elements. The corresponding expressions for plasma electrons and ions can be obtained from Eqs.(9)-(15) by replacing T (temperature), N (density), M (mass) by the electron T_e, N_e, m_e and ion T_i, N_i, M_i parameters. To obtain the total expressions of transverse and longitudinal dielectric tensor elements, as usual, it is necessary to carry out the summation over all kinds of plasma particles.

4. Conclusions

The dielectric characteristics of a dipole magnetosphere (as is for magnetospheric plasmas with circular magnetic field lines [5,6]) depend substantially on the geomagnetic field nonuniformity. If $\omega << \omega_b = 2\pi v_T/R_0 L\tau_b$, the imaginary part of the longitudinal permittivity decreases as $\sim v_T^{-5}$. This decrease is stronger than $\sim v_T^{-3}$ for plasmas in a straight magnetic field. It should be noted, since the bounce resonances are not effective in this frequency range, the drift corrections become substantial for the ultralow-frequency waves with the large transverse wave numbers m. As was shown in Refs. [2] and [3], the excitation of the low-frequency geomagnetic pulsations in the range of Pc-3 and Pc-5 oscillations is associated with an effective bounce-drift interaction between the wave and energetic protons. If $\omega \sim \omega_b$, the numbers of the basic bounce resonances are defined by $p \sim \omega/\omega_b$. In this case, Im $\epsilon_{\parallel}^{n,n}(L)$ has a maximum for the waves with longitudinal wave numbers $n \sim p$. It means that the resonant condition for the effective wave-particle interaction in magnetospheric plasmas should understand as the condition when the wave performs the integer number (p) of oscillations during one bounce period $2\pi/\omega_b$ of the trapped particles.

Information related to the basic cyclotron resonance effects is included in the transverse dielectric tensor components, which are derived by solving the Vlasov equation taking into account the cyclotron and bounce oscillations of the trapped particles. However, if $\omega << \omega_{co}/L^3$, it is possible to use the "local" approximation for the transverse permittivity elements. In this case, the bounce resonant wave-particle interaction is associated only with the Cherenkov bounce resonances between the particles (mainly the electrons) and longitudinal electric field. The effective cyclotron-bounce interaction between the wave and the trapped particles becomes possible in the frequency range $\omega \sim \omega_{co}/L^3$. In this case, the numbers of the basic bounce resonances are defined by $p \sim |\omega - \omega_{co}/L^3|/\omega_b$. It means that the cyclotron resonant condition should understand as the condition when the transverse electric field performs the integer number (p) of oscillations during one bounce period of particles. In particular, if $\omega > \omega_{co,i}/L^3$, there are two symmetric ICR-ICR points (at the considered magnetic field line) where the ion-cyclotron resonance (ICR) is realized exactly. The damping rate of such waves will be defined by the level of ion energy (temperature); i.e., the interaction will be effective if the bounce-period of the trapped ions and their transit time between the ICR-ICR points are compared one another.

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