RELATIVISTIC ELECTRON-POSITRON PLASMA DYNAMICS IN THE PULSAR MAGNETOSPHERE

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Abstract

The investigation of a relativistic electron-positron plasma dynamics in the pulsar magnetosphere is given. The equation of the motion of the magnetospheric plasma particles is discussed. As it turned out, if the particle radial velocity $V_r > c/\sqrt{2}$ (where c is the speed of light), their centrifugal acceleration changes its sign and the particle braking begins. Also the stability of the magnetospheric plasma with respect to the radially oriented potential perturbations is discussed and the possibility of the radial electric field generation in the pulsar magnetosphere plasma along the magnetic field lines is shown.

1. Introduction

As it is well known pulsar is a rotating neutron star with extremely high magnetic field, about 10^{12} - 10^{13} G. Pulsar is surrounded by the magnetosphere, which is filled with the relativistic electron-positron plasma. This is the superconductive plasma, therefore the pulsar magnetic field lines are frozen in the magnetospheric plasma. The matter inside the pulsar is also in the superconductive state and the magnetic field is frozen in it too. So the solid body type rotation, corotation of pulsar, its magnetic field and the magnetospheric plasma takes place.

We think that in such case the centrifugal force, which was disregarded in the previous papers, will play the important role in the pulsar magnetosphere relativistic plasma dynamics near the star surface. Really, as we will show below, taking into account the centrifugal force causes the appearance of the effect, unknown before for the rotating relativistic plasma. Namely, this is the effect of the magnetospheric relativistic plasma particle radial braking under some condition.

As a result of this effect, the magnetospheric plasma becomes unstable. We have considered the stability of the pulsar magnetosphere relativistic plasma with respect to the radial potential perturbations. As it turned out, under some condition, which we have derived, the aperiodic instability development, i.e. the generation of the exponentially increasing radial electric field is possible in the pulsar magnetosphere.

The effect of the magnetospheric plasma particle radial braking and the generation of the increasing radial electric field together cause the appearance of the increasing radial electric current and corresponding toroidal magnetic field in the pulsar magnetosphere.

2. Pulsar magnetosphere relativistic plasma dynamics near the star surface

We discuss the case, when the pulsar rotation axis and the magnetic axis are perpendicular (for example Crab Nebula pulsar PSR 1921+31) and treat only the polar cap region. Besides we assume that the pulsar magnetic field lines are the radial straight lines located in the plane which is perpendicular to the pulsar rotation axis. This assumption is justified because we discuss the processes in the thin layer of the magnetosphere, the thickness of which is much less than the curvature radius of the magnetic field lines. The magnetospheric plasma particles move along the magnetic field lines and also rotate together with them. The electric field, generated by the pulsar rotation together with its magnetic field, is screened by the magnetospheric plasma.

It is convenient to discuss the pulsar magnetosphere relativistic electron-positron plasma dynamics in two frames - in the noninertial rotating frame and in the rest inertial frame.

Let us discuss the problem for the first time in the noninertial rotating frame. Namely this is the reference frame of a rotating magnetic field line and its metric has the following form:

$$dS^2 = -(1-\Omega^2 r^2)dt^2 + dr^2, \tag{1}$$

where Ω is the pulsar rotating frequency. Here and below we use so called geometric units $c = G$ = 1.

The equation of the motion of the pulsar magnetosphere relativistic electron-positron plasma in the metric (1) has the following form [1]:

$$\frac{\partial \vec{p}}{\partial t} + (\vec{V}\vec{\nabla})\vec{p} = -\gamma m \frac{\vec{\nabla}\alpha}{\alpha} + e(\vec{E} + [\vec{V}\vec{B}]). \tag{2}$$

Here $\vec{V}$ and $\vec{p}$ are the plasma particle three-velocity and momentum, γ, m and e are the plasma particle Lorentz-factor, mass and electric charge respectively, $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, and $\alpha = \sqrt{1-\Omega^2 r^2}$ is the so called "lapse function".

As we have already mentioned above the pulsar magnetic field lines are frozen in the magnetospheric plasma, because this is the superconductive plasma. Thus the freezing-in condition

$$\vec{E} + [\vec{V}\vec{B}] = 0 \tag{3}$$

is fulfilled in the pulsar magnetosphere.

Now let us expand all physical quantities in such manner: $\vec{p} = \vec{p}_0 + \vec{p}_1$, where $\vec{p}_0$ is the main (unperturbed) term and $\vec{p}_1$ is the small perturbation in the first approximation of the weak turbulence.In this approximation the small parameter is ${E_1}^2/mn\gamma$.

Taking into account the freezing-in condition (3) in the equation of the motion, one can easily get for the plasma particle radial acceleration in the zeroth approximation of the weak turbulence the following expression:

$$\frac{d^2r}{dt^2} = \frac{\Omega^2 r}{1-\Omega^2 r^2}\left[1-\Omega^2 r^2 - 2\left(\frac{dr}{dt}\right)^2\right]. \tag{4}$$

As it is evident from the equation (4), when the plasma particle radial velocity

$$V_r = \frac{dr}{dt} > \frac{1}{\sqrt{2}}, \tag{5}$$

the plasma particle radial acceleration changes sign and becomes negative, i.e. when the condition (5) is fulfilled the plasma particle radial braking begins in the pulsar magnetosphere. This is the purely relativistic effect and its reason is the following: when the pulsar magnetosphere plasma particle velocity increases, their mass also increases because of the relativistic effect and when the condition (5) is fulfilled, the particle mass is such a large that their radial braking begins.

While discussing the pulsar magnetosphere relativistic plasma behaviour in the rest inertial frame, which is described by the Minkowskian metric, one finds that in the right hand side of the equation of the motion there is only the Lorentz force. There is not in the evident form the force, which plays the same role in the plasma dynamics in this frame, as the centrifugal force in the noninertial rotating frame. But, if we take into account the inhomogeneity of the magnetic field in the rest inertial frame, in the limits of the drift approximation it is possible to mark out this force from the Lorentz force in the evident form. After this, for the plasma particle radial acceleration in the rest inertial frame one gets the same expression (4), as it was in the noninertial rotating frame (see in detail [2]). Therefore, when the condition (5) is fulfilled, the magnetospheric relativistic plasma radial braking begins.

3. Generation of the electric field in the pulsar magnetosphere

As we have seen above, when the pulsar magnetosphere relativistic electron-positron plasma particle radial velocity $V_r > 1/\sqrt{2}$, their radial braking begins. It is clear that such kind of dynamics cause the plasma perturbation, so the development of instabilities is possible in this plasma. It is most interesting to discuss the stability of the magnetospheric relativistic plasma with respect to the radial potential perturbations. For this purpose one needs three equations, the equation of the motion, the continuity equation and the Poisson equation:

$$\frac{\partial \vec{p}}{\partial t} + (\vec{V}\vec{\nabla})\vec{p} = -\gamma m \Omega^2 \vec{r} + e(\vec{E} + [\vec{V}\vec{B}]), \tag{6}$$

$$\frac{\partial n}{\partial t} + div n\vec{V} = 0, \tag{7}$$

$$div\vec{E} = 4\pi en. \tag{8}$$

From this set of equations one can obtain in the first approximation of the weak turbulence the following (see in detail [3]):

$$\left[\frac{\partial}{\partial t} + ik_r V_{0r}\right]^2 p_{1r} = \frac{\Omega {V_{0i}}^2}{2}\left[\frac{\partial}{\partial t} + ik_r V_{0r}\right] p_{1r} sin 2\Omega t - \frac{{\omega_p}^2}{\gamma_0} p_{1r} sin^2\Omega t, \tag{9}$$

where ω_p is the plasma frequency and V_{0i} is the plasma particle initial velocity.

After this, using the Bessel function techniques and the Fourier transformation, at the end one can obtain the dispersion relation in the following form [3]:

$$\omega^2 = \frac{\omega_p^2}{2\gamma_0} - \frac{k_r^2 {V_{0i}}^2}{2}. \tag{10}$$

As we know $E_1 \sim exp(-i\omega t)$, so when it is fulfilled the following condition:

$$k_r^2 V_{0i}^2 > \frac{\omega_p^2}{\gamma_0}, \tag{11}$$

the aperiodic instability is developing in the pulsar magnetosphere relativistic plasma, i.e. the exponentially increasing radial electric field has been generated.

4. Discussion

So the dynamics of the pulsar magnetosphere relativistic electron-positron plasma near the star surface has the following peculiarity: when the plasma particle radial velocity $V_r > 1/\sqrt{2}$, their radial braking begins. This effect makes the magnetospheric relativistic plasma unstable with respect to the radial potential perturbations and the aperiodic instability is developing in the pulsar magnetosphere, i.e. the exponentially increasing radial electric field generates in it.

These two processes together cause the toroidal magnetic field generation in the pulsar magnetosphere. Really, during the plasma particle radial braking both - electrons and positrons are braked in the same manner. The appearance of the increasing radial electric field (as a result of the aperiodic instability development in the pulsar magnetosphere) causes the additional braking of the charged particles of one sign and the decreasing of braking of the charged particles of another sign. As a result the increasing radial electric current and corresponding toroidal magnetic field appear in the pulsar magnetosphere.

The appearance of the toroidal magnetic field in the pulsar magnetosphere will cause the change of the pulsar magnetic field configuration from the radial to the spiral. After this change of the pulsar magnetic field configuration the corotation will be disturbed in the pulsar magnetosphere. The radial electric field, radial electric current and corresponding toroidal magnetic field will increase until the corotation disturbance in the pulsar magnetosphere.

References

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