EFFECT OF PLASMA FLUCTUATIONS ON SOLAR NEUTRINO SUPPRESSION BY MSW MECHANISM

Amita Das[1], **Predhiman Kaw**[1] and Jitendra Parikh[2]

[1] *Institute for Plasma Research, Bhat, Gandhinagar 382 428, India.*
[2] *Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India*

1. Introduction

One of the outstanding puzzles in solar astrophysics is the observed deficit in solar neutrino flux in experiments which have been set up to detect them. Neutrino physics predicts two dominant flavours for neutrinos, namely the electron neutrino ν_e and the muon neutrino ν_μ which are produced in reactions involving the respective leptons. The muon neutrino, being inert, can not be detected. Experiments have been designed to detect ν_e and show that their numbers are far suppressed than what is predicted by Standard Solar Models (SSM).

A prominent candidate for explaining the solar neutrino puzzle is the Mikheyev - Smirnov - Wolfenstein (MSW) resonance mechanism involving neutrino flavour oscillations [1]. Recent experimental results [2] from the SuperKamiokande have given very strong evidence in favour of a finite neutrino mass and associated flavour oscillations. This strengthens the case of MSW mechanism as an explanation for the Solar neutrino puzzle. In this mechanism , the electron neutrinos ν_e produced in the core of the sun can get converted into muon neutrinos through flavour oscillations modified by the weak interaction with matter along its path, especially around certain resonant densities. Since the muon neutrino can not be detected, we get a suppression in the observed solar neutrino flux. In recent years it has been pointed out that the MSW mechanism can be significantly modified by the presence of density fluctuations in the sun. It has been shown that coherent density perturbations may induce additional flavour transitions through parameteric coupling effects [3] and that random perturbations may lead to damping of pure flavour states and new effects associated with broadening of resonances [4]. Most of this work has been done for low frequency acoustic disturbances in the sun. In this paper we give a general treatment which unifies a lot of the earlier work on the MSW mechanism and also extends it to allow one to consider the effect of a broader variety of solar plasma disturbances (viz the entire range of interesting frequencies, wavelengths, coherence etc)

2. MSW Suppression Model

We now briefly describe the MSW suppression model. First of all we recognize that ν_e and

ν_μ, representations of neutrinos in the flavour basis are not eigenstates propagating unaltered in free space and/or the solar interior. The freely propagating states are ν_1 and ν_2 which are linear combinations of the above two flavours : $\nu_e = \nu_1 \cos(\tilde{\theta}) + \nu_2 \sin(\tilde{\theta}), \nu_\mu = \nu_1 \sin(\tilde{\theta}) + \nu_2 \cos(\tilde{\theta})$ where the mixing angle $\tilde{\theta}$ is given by $\tan(2\tilde{\theta}) = \Delta \sin(2\theta)/(\Delta \cos 2\theta - A); \Delta = m_2^2 - m_1^2$ is the mass difference of the two flavour states, $A = 2\sqrt{(2)} G_F n_e E$ describes effects due to 'weak' interaction of a neutrino of energy E with solar matter of density n_e particles/cc and $G_F = 8.86 \times 10^{-47} Gev/cm^3$ is the Fermi-coupling parameter . Note that for propagation in vacuum, $n_e = 0, A = 0$ and we have $\tilde{\theta} = \theta$, the vacuum mixing angle. The propagation of flavour neutrino is thus described by the evolution equation (in the frame moving with neutrinos)

$$\frac{d\nu_f}{dx} = H\nu_f$$

where, $H \equiv E - p \equiv M/(E+p) \simeq M/2E$ and the elements of the 2×2 mass matrix in the flavour basis are given by $M_{11} = 2E(a+b), M_{22} = 2E(a-b), M_{12} = M_{21} = 2Ec, a = (m_1^2 + m_2^2 + A)/4E, b = (A - \Delta \cos 2\theta)$ and $c = (\Delta \sin 2\theta)/4E$. The conventional method to proceed is through an appropriate diagonaliztion of H by a unitary transformation and subsequent integration. The magnitude of $| \nu_e |^2$ is then directly related to the survival probability of electron neutrinos, as modified by passage through matter. When $A = 0$, these equations describe the standard flavour oscillation of neutrinos in vacuum. When A is a function of x and t because of inhomogeneities in density and propagating solar disturbances, the above treatment gets progressively more cumbersome.

We now present a novel unified approach in which we rewrite the two coupled first order evolution equations as a single second order differential equation in $\Psi = \nu_e \exp(-i \int a dt)$, *viz*,

$$\frac{d^2\Psi}{dt^2} + \left[b^2 + c^2 + \frac{i}{4E}\frac{dA}{dt} \right] \Psi = 0 \tag{1}$$

Note that t and x are interchangeable because velocity of light is 1. The non-uniformities of solar matter in the moving neutrino's frame now enter through the t dependence of A (through parameter b) and $\dot{A} \equiv dA/dt$.

When $b^2 + c^2 = f^2(t)$ is a slowly varying function and $\dot{A}$ is negligible, we may use the WKB ansatz to write the solutions $\Psi_{1,2} \sim \exp[\pm i \int f(t) dt]$. The true state is given by a linear combination of these functions, the coefficients being determined by the initial state function. The survival probability for electron neutrinos is thus given by

$$P^{ad}_{\nu_e \nu_e} = | \cos(\theta_0)\cos(\theta) + \sin(\theta_0)\sin(\theta)\exp(2i \int f(t) dt) |^2 \tag{2}$$

where θ_0 is the mixing angle at the source given by $\tan 2\theta_0 = \Delta \sin(2\theta)/(\Delta \cos(2\theta) - A_0)$, A_0 corresponding to the matter density in sun's core and θ is the mixing angle at detector (taken to be vacuum mixing angle as $A \approx 0$). Eqn(2) gives the correct description for vacuum flavour oscillations (if θ_0 is chosen to be equal to θ) and the well known adiabatic result for solar neutrino suppression, when $\theta_0 \neq \theta$.

MSW pointed out the importance of resonant transitions around the densities $\Delta \cos(2\theta) = A$ where $b \approx \dot{A}t/4E$. In this case, eqn(2) becomes a parabolic cylinder equation

$$\frac{d^2\Psi}{d\tau^2} + \left[\frac{i}{2} + \frac{\tau^2}{4} + \frac{\Delta^2}{8AE}\sin^2 2\theta\right]\Psi = 0 \tag{3}$$

with $\tau = (\dot{A}t/4E)^{1/2}$. Relating $\Psi(+\infty)$ to $\Psi(-\infty)$ by using asymptotic form of parabolic cylinder functions we find a transition probability

$$X = \exp(-\gamma_r) \equiv \exp\left(-\frac{\pi}{4}\frac{\Delta \sin^2 2\theta}{E\cos 2\theta}\frac{1}{\mid \frac{d}{dt}\ln n \mid}\right) \tag{4}$$

This leads to the survival probability for electron neutrinos,

$$P^{NA}_{\nu_e\nu_e} = (1 - X)P^{ad}_{\nu_e\nu_e} + X(1 - P^{ad}_{\nu_e\nu_e}) \tag{5}$$

Eqs(4) and (5) give the basic MSW result for solar neutrino suppression when the neutrinos encounter a single MSW resonance. It may be suitably modified for multiple resonances.

3. Effect of Coherent Waves on MSW Suppression

We first consider the situation when a coherent density disturbance is propagating in the path of the neutrino. The density disturbance could be associated with any one of a variety of plasma disturbances such as electron plasma waves, ion-acoustic waves, Alfven or magnetosonic waves, upper or lower hybrid waves etc. We may thus write $A = \bar{A} + A_1 \cos\Omega t$ where $\Omega = (\omega - k)$, is the Doppler shifted frequency, ω being the natural frequency of the disturbance and k its wave vector along the direction of neutrino propagation (with $c = 1$). We have used $\xi = x - t$ and replaced the phase factor $kx - \omega t$ by $k\xi - (\omega - k)t$ and ignored the $k\xi$ piece as being a relatively unimportant initial phase. Note that all the earlier work has been done in the limit of low frequencies $\omega << k$, when the wave period is much longer than the time of transit of neutrinos across a wavelength. The Ψ equation may now be put in the form

$$\frac{d^2\Psi}{d\tau^2} + [a - h\cos 2\tau + ig\sin 2\tau]\Psi = 0 \tag{6}$$

where

$$a = \frac{4}{\Omega^2}\left[\frac{(\bar{A} - \Delta\cos 2\theta)^2}{16E^2} + \frac{\Delta^2\sin^2 2\theta}{16E^2}\right]$$

$$h = -\frac{4}{\Omega^2}\left(\frac{\bar{A} - \Delta\cos 2\theta}{8E^2}\right)A_1, g = -\frac{A_1}{\Omega E}.$$

and all the small terms proportional to A_1^2 have been ignored.

Eqn.(6) is of the form of a Hill's equation, which can be solved by the method of multiple time scales. Resonanaces arise when $a = k^2$ where k is an integer. At the resonances, the survival probability suffers a significant decay in a manner similar to Bragg scattering. The resonant condition may be written as $\lambda_f = \frac{2\pi}{\Omega} = kl_m$ where $l_m = (4\pi E/\Delta)[(\cos 2\theta - \pi/2)^2 + \sin^2 2\theta]^{1/2}$ is the oscillation length in matter and λ_f is the fluctuation wavelength in the limit when $\omega << k$. The above resonant condition is consistent with the results obtained earlier by Krastev and Smirnov[3]. Note however that a new feature arises when ω is retained, as we have done. If the plasma disturbance is propagating with a phase velocity close to the propagation speed of the neutrino (i.e $\sim c$), then even very short waves can satisfy the above resonance condition. Thus even plasma disturbances with wavelength much shorter than the oscillation length of neutrinos can significantly alter the transition probability of electron neutrinos, if they have phase speeds comparable to the velocity of light. This is a new result. The survival probability for $k = 1$ is readily obtained by a multiple time scale analysis and shows $X = \mid \Psi \mid^2 \simeq \exp(-\frac{h}{2}\tau_1)$. This gives a parametric decay length l_p comparable to that calculated by earlier workers [3]. As one scans neutrinos of different energy, the parametric resonance condition can be satisfied for various discrete values of k. Thus the energy dependence of survival probability of neutrinos shows characteristic peaks and valleys.

4. Conclusion

We have given a unified treatment for analyzing the influence of solar plasma disturbances on the MSW mechanism of solar neutrino suppression. This treatment is now being extended to inhomogeneous solar plasma and to the case when the plasma disturbances are turbulent in nature.

References

[1] T. K. Kuo and J. Pantaleone, Revs. of Modern Physics **61**, 937 (1989).

[2] J. G. Learned et al (SuperKamiokande Collbn.) Neutrino' 98 Conference (4-9 th June 1998), Tokayama, Japan.

[3] P. I. Krastev and A. Yu Smirnov, Phys. Letters **B 226**, 341 (1989).

[4] C. P. Burgess and D. Michand, hep-ph 9606295 (1996);
H. Nunokawa et al, Nucl. Phys. **B 472**, 495 (1996).