MHD EQUILIBRIUM AND STABILITY OF FORCE-FREE ATMOSPHERES OF INTERSTELLAR MAGNETIC CLUMPS

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Abstract

Giant molecular clouds (GMCs) contain weakly ionized (10^{-4} – 10^{-6}) mass condensations of scale length R_0 ~ 0.1pc = $3.086{\cdot}10^{17}$cm, called clumps. They are sites of massive star formation. The Jeans mechanism of purely gravitational collapse predicts a too large star formation rate in the Galaxy, exceeding by orders of magnitude the observed rate. As a matter of fact the gravitational collapse of clumps is opposed by strong magnetic fields (β< 1) and by Alfven waves turbulent energy (Alfven Mach number m_A ~ 1). Magnetized clumps can still condense via ambipolar diffusion of the magnetic field, which decouples the ionized component of the cloud from the self gravitating neutrals; but the ambipolar diffusive timescale for a clump is $\geq 2{\cdot}10^7$ yr, longer than the lifetime of a GMC, yielding a too small star formation rate. However the field often appears to be in filaments: the folding of the filaments by MHD instabilities and their break-off in fast reconnection processes, with timescales ~ (1-3) $\cdot 10^6$ yr, can be a faster trigger of massive star formation. A model for longitudinal current instabilities of magnetic clumps is proposed.

1. Evidence for magnetic support (static and turbulent) of the clumps

Large GMCs, with mass M=l0^6 $M_\odot$ (Solar mass = $M_\odot$=$1.989{\cdot}10^{33}$g), are concentrated on a narrow disk (250 pc) in plan of the Galaxy. They contain smaller scale condensations : dark clouds (M=l0^3 $M_\odot$) and clumps (M=7 $M_\odot$), which are site of active star formation. Clumps are up to l0^6 time denser than the average interstellar matter (which has n ~ 1 cm^{-3} as number density). The primary composition of GMCs is 60% H and 30% He, but also dust grains and molecules are present. Nonthermal line broadening σ is observed in all the molecular line emissions (CO, NH_3, CS, OH): σ^2_{NT} = σ^2 - $kT/\mu_i m_H$> $kT/\mu_i m_H$ where $\mu_i m_H=2.3m_H$ is the mean molecular mass and T is the spin temperature measured at λ=21.11 cm (hyperfine H transition).

	R_0 (pc)	M($M_\odot$)	n(cm^{-3})	σ_{NT} (cm/s)	T (^{o}K)
GMC	60	10^6	10-100	$4 \cdot 10^5$	100
Dark cloud	1.7	10^3	10^3	10^5	10
Clump	0.1	7	$3 \cdot 10^4$	$1.4 \cdot 10^4$	10

Table 1. Observed trends of M, n, σ_{NT} and T for interstellar clouds of different scale length R_0

The virial theorem (in axial symmetry) predicts that, in absence of rotation and of B field, the gravitational collapse occurs if a cool and dense mass M, embedded in a more hot and dilute cloud with pressure p_{ext}, exceeds the Jeans mass M_J: $M>M_J= 1.18\ \sigma^4/\sqrt{G^3 p_{ext}}$ (gravitational constant G=6.672 $\cdot 10^{-8}$cgs) [1]. The Jeans collapse occurs in a free-fall time: τ_{ff} = 1.38 $\sqrt{10^3/n \cdot 10^6}$ yr, which for a clump with $n=3 \cdot 10^4$ cm^{-3} is ~$2.5 \cdot 10^5$ yr. In the Galaxy most clouds exceed the Jeans critical mass, predicting a star formation rate >130-400 $M_\odot$/yr. The observed galactic star formation rate is only l-4 $M_\odot$/yr, indicating that the Jeans collapse is contrasted by magnetic field: starlight polarization shows that the galactic field (~2μG) is highly perturbed and amplified in GMCs. Static magnetic field B provides support: the matter of a clump is weakly ionized and the magnetic flux freezing opposes the gravitational collapse (Fig. la). Massive star formation occurs if the mass to poloidal magnetic flux ratio M/Ψ of a clump exceeds a critical ratio $(M/\Psi)_{crit}$ ~0.12/$G^{1/2}$ [1]. This criterion translates into a critical ratio between B and the columnar density N=4nv3: $(B/N)_{obs} \approx 2.4\cdot10^{-21}$ μGcm^2. Observations of the cloud W3 of Perseus GMC [2] yield $(B/N)_{obs} \approx 3\cdot10^{-21}$ μGcm^2, near the critical ratio. Furthermore the prolate form of the clumps [3] suggests that also a fluctuating magnetic field δB provides support: the nonthermal line broadening $\sigma_{NT}^2 \approx v_A^2/3$ is in agreement with Alfven wave isotropic turbulent pressure $p=(\delta B)^2/8\pi$; (v_A=Alfven speed) [4]. A model with turbulent, gravitational and magnetic energy in equipartition [5] fits with B~30μG the observed trends of σ_{NT} and n (Table 1).

2. Magnetic flux shedding

Massive star formation therefore requires that magnetized clumps collapse by shedding their poloidal magnetic flux. In dark clouds the ionization by cosmic rays balances the dissociative recombination, producing an ionized component whose mass density ρ_{ion} is related to the neutral mass density ρ by: $\rho_{ion}=\mathcal{C}\rho^{1/2}$($\mathcal{C}=3\cdot l0^{-16}$cgs). The delay between signals at different frequencies emitted by pulsars in the background of dark clouds confirms that the matter is a weakly ionized plasma ($\rho_{ion}/\rho=l0^{-5}$-$l0^{-6}$), with Coulombian logarithm lnΛ~15. The most obvious shedding mechanism is the ambipolar diffusion [6], which decouples the ionized component from the neutrals, balancing the $(\vec{\mathbf{u}}_{ion}-\vec{\mathbf{u}})$ velocity drag between ions and neutrals with the Lorentz force $\gamma\rho_{ion}(\vec{\mathbf{u}}_{ion}-\vec{\mathbf{u}})=(\vec{\nabla}\wedge\vec{\mathbf{B}})\wedge\vec{\mathbf{B}}/4\pi$ ($\gamma=3.5\cdot10^{13}$cgs); at T= l0ºK ambipolar diffusion dominates ohmic resistive diffusion for $n/B^{4/3}<1.7\cdot10^{16}$cgs. The ambipolar diffusion can be written [7]:

$$\partial\Psi/\partial t+\vec{\mathbf{u}}\cdot\vec{\nabla}\Psi=\left|\vec{\nabla}\Psi/r\sin\theta\right|^2\left[\partial^2\Psi/\partial r^2+(1/r^2)\partial^2\Psi/\partial\theta^2-(\mathrm{ctg}\theta/r^2)\partial\Psi/\partial\theta\right]/16\pi^3\,\mathcal{C}\,\gamma\rho^{3/2},$$

when axially symmetric and expressed in spherical coordinates (r,θ,ϕ). The effective diffusivity $\chi_{AD}=\mathbf{B}^2/(4\pi\,\mathcal{C}\gamma\rho^{3/2})$ implies an ambipolar timescale $\tau_{AD}=4\pi\,\mathcal{C}\gamma\rho^{3/2}R_0^2/B^2$, which for clump is never less than $2\cdot l0^7$ yr [5]. This timescale is too long, as it is comparable to the lifetime of GMCs: galactic shocks form and redisperse the clouds every few $l0^7$ yr; furthermore the most luminous OB stars formed in a GMC radiate so much energy as to unbind the cloud in less than $2\cdot l0^7$ yr [8]. However the interstellar clouds are often observed as magnetized filaments [9]. The folding of the filaments by MHD instabilities [l0] and their break-off in reconnection processes could shed the magnetic flux. In force-free laboratory plasmas, flux loops can be twisted off magnetic filaments by driving large longitudinal currents [11], corresponding to MHD winding numbers q_{MHD} <l-2. As the fast reconnection time [121 τ_{rec}~ln(R_m)(2R_0/2πv_A) depends weakly on the magnetic Reynolds number Rm=4πv_A $R_0/\eta_\perp c^2$ ($\eta_\perp \sim 2\cdot10^{-7}\,T^{-3/2}$ =resistivity), MHD unstable clumps (R_m~10^{11}) could shed their flux and collapse in τ_{rec}~(1-3)·$l0^6$yr ($3\cdot l0^4<v_A<1\cdot l0^5$cm/s).

3. Model for longitudinal current instabilities

The analytical Mestel model of magnetized gravitational contraction [13] has been used (Fig..1). A uniform mass density ρ_i, embedded in a uniform field B_i, undergoes a spherical contraction ($r_i \rightarrow r_0$, θ=constant, $x_i=r_i/R_0$, $x_0=r_0/R_0$. In the contraction it produces a gaussian mass density distribution $\rho(x_0) = \rho_i\{1+ (\rho_c/\rho_i)\exp(-x_0^2)\}$, with a central mass density ρ_C, and a poloidal flux $\Psi^{MESTEL}(x_0)= \pi B_i R_0^2 x_0^2 [1+\rho_C G(x0)/\rho_i x_0^2]^{2/3}$, where $G(x_0) = 3\int_0^{x_0} x^2 \exp(-x^2)dx$. The radial range $(r_0^{ATM}/R_0) = x_0^{ATM} \sim 2 < x_0 < 1.3(\rho_c/\rho_i)^{1/3}$, is nearly a force-free magnetic atmosphere [14], in which turbulent magnetic pressure compensates gravity, with $\vec{\nabla}\wedge\vec{\mathbf{B}} \sim \mu(\Psi^{MESTEL})\vec{\mathbf{B}}$ and $\rho/\rho_c \sim 10^{-2}$.

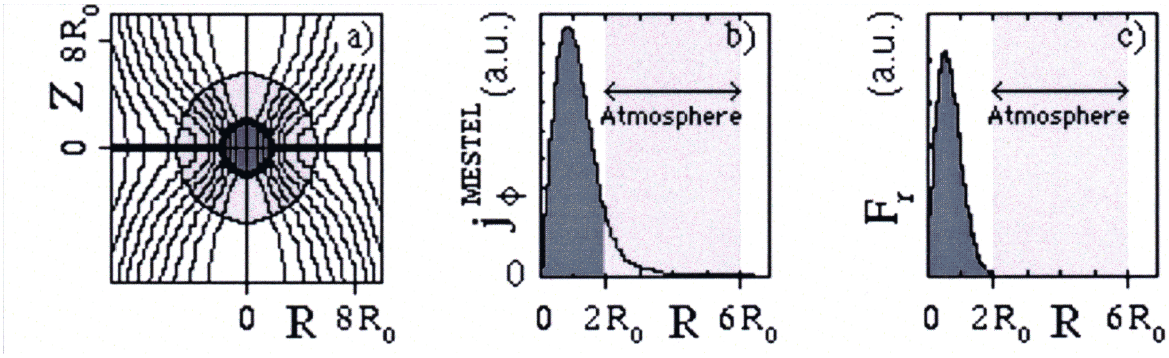

Fig.1: Magnetic clump with central condensation ρ/ρ_C =l00 and atmospheric radial range $[2R_0<r_0<6R_0]$. (a) Poloidal flux Ψ^{MESTEL}. Thick curves: base of atmosphere and accretion disk. (b) Toroidal current density j_ϕ^{MESTEL} and (c) radial force $F_i = \vec{\mathbf{j}}\wedge\vec{\mathbf{B}}/c$ versus the radius R on the equator.

Torsional Alfven waves can drive longitudinal currents by adding an azimuthal field B_ϕ to the poloidal field of a clump [15]. A torsional Alfven wave is modeled to propagate through the densest part of the clump, maintaining the magnetic atmosphere force-free. The nonlinear force-free MHD equilibrium

$$\left[\partial^2\Psi/\partial r^2 + (1/r^2)\partial^2\Psi/\partial\theta^2 - (\mathrm{ctg}\theta/r^2)\partial\Psi/\partial\theta\right] = -\mu(\Psi)\int_0^{\Psi}\mu(\Psi')d\Psi'$$

is solved outside the boundary composed by the spherical base of the atmosphere $r_0= r_0^{ATM}$ and the outer equatorial plane (accretion disc). The poloidal flux Ψ and the local relaxation parameter $\mu(\Psi)=(\vec{\mathbf{B}}\cdot\vec{\nabla}\wedge\vec{\mathbf{B}})/B^2)$ are fixed on the boundary [16] as $\Psi\big|_b = \Psi\big|_b^{MESTEL}$ and $\mu(\Psi)=\mu(\Psi\big|_b^{MESTEL})$.

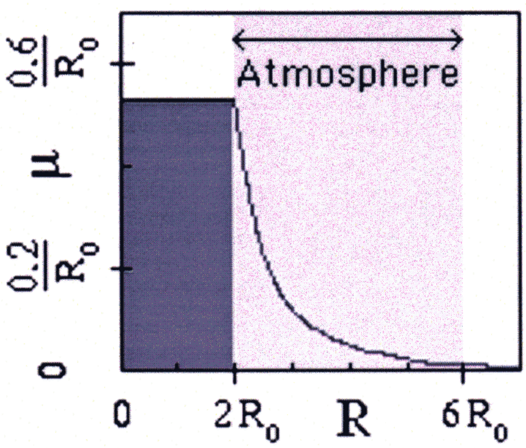

Fig. 2: $\mu(\Psi)$ versus radius R on ihe equator is related to: i) poloidal current $f(\Psi)=(c/2)r \sin\theta B_\phi$ as $\mu(\Psi)=(c/4\pi)df/d\Psi$

ii) j_ϕ as $j_\phi(\Psi, r\sin\theta)=(c/8\pi^2 r\sin\theta)\,\mu(\Psi)\int_0^\theta \mu(\Psi')d\Psi'$.

At atmospheric base $[x_0= x_0^{ATM}, O<\theta<\pi/2]$:

$f^2(\Psi)=cR_0\int_0^\theta j_\phi^{MESTEL}(x_0^{ATM})\sin\theta\,(d\Psi/d\theta)d\theta$.

On the accretion disk $[x_0> x_0^{ATM}, \theta=\pi/2]$:

$$f^2(\Psi)=cR_0\int_0^{\pi/2} j_\phi^{MESTEL}(x_0^{ATM},\theta)x_0^{ATM}\sin\theta(d\Psi/d\theta)d\theta$$

. $$+ cR_0\int_{x_0^{ATM}}^{x_0(\Psi} j_\phi^{MESTEL}(x,\pi/2)\ (d\Psi/dx)dx$$

The effect of the longitudinal return currents can be approximated: i) either by solving inside the atmosphere $x_0^{max}<1.3(\rho_c/\rho_i)$, allowing $j_\phi>0$ only for $\Psi>0$ and matching Ψ on the boundary by a series of external spherical multipoles; ii) or by solving inside a sphere $x_0^{max} > 1.3(\rho_c/\rho_i)^{1/3}$ (Fig.3), allowing $j_\phi<0$ for $\Psi<0$, as $j_\phi(\Psi,r\sin\theta)= -c\mu(\alpha\Psi/\Psi_{min}) \int_0^{-\Psi} \mu(\alpha\Psi'/\Psi_{min})d\Psi'/8\pi^2 r\sin\theta$ (α such that $j\to 0$ at Ψ_{min}), and matching Ψ on the boundary by a single external spherical dipole.

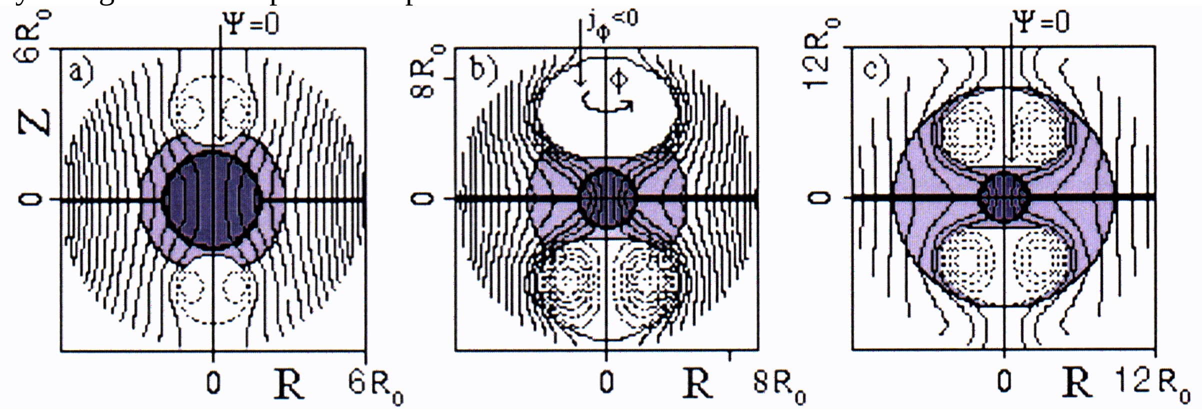

Fig. 3: Force free atmospheres of clumps increasing condensation: (a) ρ_c/ρ_i = 10; (b) ρ_c/ρ_i = 100, (c) ρ_c/ρ_i = 300.

In the latter approximation the force-free equilibria form two rarefied regions with reversed azimuthal current density j_ϕ on top and bottom of the clump; in both approximations two singular curves with Ψ=0 limit the atmosphere. The MHD stability is evaluated by calculating the winding number q_{MHD} along the field lines, from the lower to the upper edge of the atmosphere: q_{MHD} is only weakly influenced by the arbitrary elements of the model (x_0^{ATM}, x_0^{max}, return currents). For clumps with central condensations ρ_c/ρ_i ~10-100, $q_{MHD}\geq 4$ and no global scale MHD instabilities should be excited; when ρ_c/ρ_i ~100-1000, $q_{MHD}<2$ for R≥0.75 R_0 and $q_{MHD}< 1$ for R>1.5 R_0 on the equatorial plane: global MHD instabilities should appear.

4. Conclusion

Magnetic clumps can become unstable to global MHD instabilities due to torsional Alfvén waves if their central mass density ρ_c exceeds by 100-1000 the cloud mass density ρ_i.

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