ANALYTIC APPROXIMATED SOLUTION FOR THE REDUCED ENERGY DISTRIBUTION FUNCTION OF ECR HOT ELECTRONS

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Abstract

In this paper we study the electronic function due to the Bremsstrahlung spectrum of hot electrons, produced by microwave heated electrons cyclotron resonance (ECR) discharge. The non-linear differential equation for this distribution is treated here considering the asymptotic and power expansions of that differential equation. Through both these expansions, we apply the two-point quasifractional method in order to find an approximant, which will be a highly accurate approximated solution for that differential equation. Our results compare well to those determined experimentally and the previous approximated solutions, derived for small and large values of the energy.

1. Introduction

Our experimental knowledge of the electronic distribution of the electrons are mainly given by Bremsstrahlung measurements in the X-ray region, performed by Bernhardi and Brinkmann [1], Bernhardi and Wissemann [2], where a strong anisotropic behavior for the distribution function $(\vec{p}_{\|} << \vec{p}_{\perp})$ is pointed out. Using a quasilinear approach they establish (see ref. [1]), through several appropriated approximations, a reduced differential equation in only one variable, $\vec{p}_{\perp}$, where

$$\begin{cases} p_{\perp min} & \leq p_{\perp} \leq p_{\perp max} \\ \mid p_{\|} \mid & \leq \ p_{\| max} \end{cases} \tag{1}$$

Here $p_{\perp min}$ means the lowest impulse value, when the kinetic electron energy reaches the value $K_{min} \cong \ 1keV$, determined through X-ray measurements. $p_{\perp \ max}$ is equivalent to the max energy value $K_{max} \cong 1 \ MeV$. The solutions of this D.E. is, in general, only possible through numerical methods or through plausible approximations at the boundaries.

In this paper we propose a general solution of the problem using the quasi-fractional Padé method [3] in order to obtain an approximant, which represents an analytical approximation solution to the problem, but with a high grade of accuracy (better than 10% max error) in the whole range of energy.

2. Theoretical Treatment and Results

Following the quasi-linear diffusion theory developed by Bernhardi et al. (see ref. [1]), writing down a differential equation for the reduced electronic distribution function F, defined through:

$$F = \int_{-p_{\parallel\ max}}^{+p_{\parallel\ max}} \bar{f}_h d\, p_{\parallel}, \tag{2}$$

in the form:

$$\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(\gamma \bar{K}_{tt}\frac{\partial F}{\partial p_\perp} + \frac{p_\perp}{\gamma m_e^2 c^2}\bar{K}_{\theta\theta}F\right) + \frac{C_I}{p_\perp}\frac{\partial}{\partial p_\perp}\left(\frac{\gamma^2}{p_\perp}F\right) + \frac{C_L \gamma^3}{p_\perp^3}F = 0, \tag{3}$$

where, introducing the dimensionless variable $\chi = p_\perp/m_e c$ and $\gamma(\chi) = (1+\chi^2)^{1/2}$, we get:

$$\frac{1}{\chi}\frac{\partial}{\partial \chi}\left((1+\chi^2)^{3/2}\bar{K}_{tt}\frac{\partial F}{\partial \chi} + \frac{\chi}{(1+\chi^2)^{1/2}}\bar{K}_{\theta\theta}F\right) +$$

$$\frac{C_I}{\chi}\frac{\partial}{\partial \chi}\left(\frac{1+\chi^2}{\chi}F\right) + C_L\frac{(1+\chi^2)^{3/2}}{\chi^3}F = 0, \tag{4}$$

where $\bar{K}_{tt},\ \bar{K}_{\theta\theta},\ C_I$ and C_L are parameters well defined in ref. [1]. Defining now new parameters $\beta_1,\ \beta_2$ and β_3 in order to have a simple form of the D.E. (4):

$$\beta_1 = C_I/\bar{K}_{tt};\ \beta_2 = \frac{C_L - C_I}{\bar{K}_{tt}};\ \beta_3 = \sqrt{\frac{C_I}{\bar{K}_{\theta\theta}}};\ \chi = \frac{p_\perp}{m_e c} \equiv \sqrt{\left(\frac{E}{m_e c^2} - 1\right)^2 - 1}, \tag{5}$$

We obtain:

$$\frac{\partial^2 F}{\partial \chi^2} + A(\chi)\frac{\partial F}{\partial \chi} + B(\chi)F = 0, \tag{6}$$

meaning here:

$$A(\chi) = \beta_1\frac{(1+\chi^2)^{1/2}}{\chi} + \left(1 + \frac{\beta_1}{\beta_3^2}\right)\frac{\chi}{1+\chi^2}$$

$$B(\chi) = \frac{\beta_1+\beta_2}{\chi^2} - \frac{\beta_1}{\chi^2(1+\chi^2)^{1/2}} + \frac{\beta_1}{(1+\chi^2)^{1/2}} + \frac{\beta_1}{\beta_3^2}\frac{1}{(1+\chi^2)^2} + (\beta_1+\beta_2). \tag{7}$$

Now in order to get the approximant we apply, for the potential series solution of the D.E. (6), the mathematical expression:

$$F_{pot}(\chi) = \chi^{s_-}\sum_{\nu=0}^{\infty} a_\nu \chi^{2\nu} + \chi^{s_+}\sum_{\nu=0}^{\infty} c_\nu \chi^{2\nu}, \tag{8}$$

where, $s_{\pm} = \frac{(1-\beta_1)}{2} \pm \sqrt{\frac{(1-\beta_1)^2}{4} - \beta_2}$ are obtained through the indicial equation. For the asymptotical series solutions we use:

$$F_{asym}(\chi) = \frac{e^{\sigma\chi}}{\chi^t} \sum_{\nu=0}^{\infty} \frac{b_\nu}{\chi^\nu}, \tag{9}$$

therefore $\sigma = -\left(\frac{\beta_1}{2} + \sqrt{\left(\frac{\beta_1}{2}\right)^2 - (\beta_1 + \beta_2)} \right) < 0$, and $t = \frac{(1+\beta_1/\beta_3^2)\sigma + \beta_1}{2\sigma + \beta_1}$ are determined from the leading term of the asymptotical expansion (9). Using the quasi-fractional technique discussed elsewhere [3], we propose in this paper a second order approximant in the form:

$$\hat{F}(\chi) = \left(\frac{\chi^{s-}}{(1+\chi)^{s-+t}} \frac{a_0 + p_1\chi + p_2\chi^2}{1+q\chi^2} + \frac{\chi^{s+}}{(1+\chi)^{s++t}} \frac{c_0 + P_1\chi + P_2\chi^2}{1+q\chi^2} \right) e^{\sigma\chi} \tag{10}$$

In order to determine the coefficient of the approximant we take four terms of the potential expansions and two terms of the asymptotical series to get a linear system of equations very easy to solve. The parameter β_3 is obtained through a minimum square method in order to fit $\hat{F}(\chi)$ to the experimental data. In box ahead, there are shown several relevant experimental and numerical data for the argon (see ref. [1]).

$\beta_1 = 1.81;\ \beta_2 = -8.79;\ \beta_3 = 1.73344;\ q = 0.98;\ s_+ = -3.39733;\ s_- = 2.58733$
$\sigma = 2.58733;\ t = 0.337769 + 1.19828r;\ \alpha_1 = -(1.34728 + 0.54r);\ \alpha_1 = a_1/a_0;$
$\gamma_1 = -(0.04157 + 0.406596r);\ \gamma_1 = c_1/c_0;\ \delta_1 = b_1/b_0;\ r = 1/\beta_3^2$
$\delta_1 = (10.2122 - 0.1982r + 0.7330r^2)/(9.3607 + 13.3856r)$

Fig 1 shows our results compared to the experimental data given by ref. [1]. We obtain very good agreement to these results. In fact this method is general and could be applied to each distribution defined before, where two regions of energy are clearly identified and typified through different models: a power law for the low energy part, and a maxwellian profile for the high energy part. We reproduce the whole range with our approximant, which allows a better analysis of the physical situations derived from these type of distributions.

3. Conclusions

In this paper we find an approximant $\hat{F}(\chi)$ solution of the D.E. (6), that matches very well the experimental data from Bernhardi et al. in energy's whole range. Such a solution is general due to the analytical nature of the approximant, which anables us to apply it in a variety of problems, where this distribution is involved. On the other hand, since it is expressed in terms of elementary functions, it can be calculated by using simple pocket calculators and is easy to extend to other models related to similar problems [4], [5]. To our knowledge, it is the first time a new form of attacking the problem of finding a general solution for the electronic distribution function in ECR heating models.

References

[1] K. Bernhardi and R.P. Brinkmann: SFB Plasma Phys. Report "Stochastiche Beschleunigung von Elektron-Zyklotron-Resonanz im Inhomogenen Magnetfeld." Bochum/Jülich (1988).

[2] K. Bernhardi and K. Wiesemann: Plasma Phys. **24**, 867 (1976).

[3] P. Martin and G. Baker: J. Math. Phys. **32**, 1470 (1992).

[4] W.M. Manheimer: "A Simple Scheme for Implementing Wave Absorption in Quasi-Neutral PIC Simulations of ECR Plasma." NRL/MR/6707–98-8122, (1998).

[5] A. Girard: "Plasma Diagnosis Related To Ion Sources". Rev. Sci. Instrum. **63**, 2886 (1992).

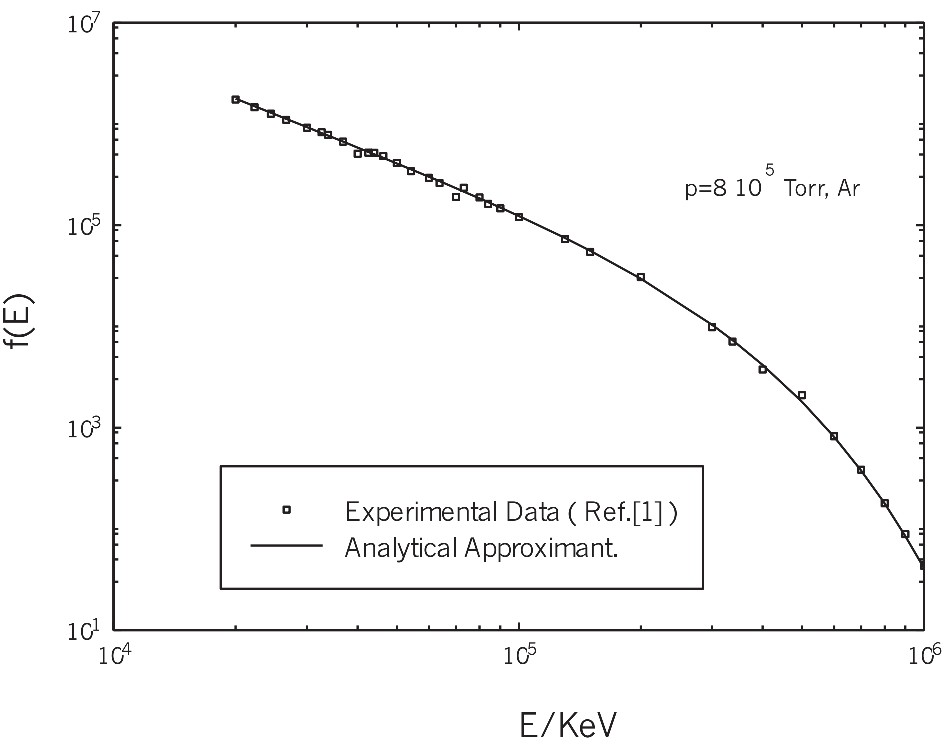

Figure 1. Energy distribution function $f(E)$.