STOCHASTIC ION HEATING IN STOCHASTIC TOKAMAK MAGNETIC FIELD

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1. Introduction

The accumulated experimental results from tokamak experiments during the last two decades support the fact that RF heating at ion cyclotron frequency and lower hybrid heating of ions provide an efficient way to heat tokamak plasmas. Previous work has shown that ions are accelerated by the lower hybrid wave through a collisionless stochastic process and this is the origin of hot ion tail formation in the ion distribution function during LH heating experiments [1]. Similarly other authors [2], [3] have proved that ion cyclotron heating can become stochastic. We present here a way to generalize these works to include the drift orbit motion and the stochastic magnetic field lines in tokamaks.

2. Basic equations

The guiding center ion drift orbit in tokamak magnetic configuration is given by the following hamiltonian equations using magnetic coordinates [4] $(\psi_p, \rho_{//}, \vartheta, \phi)$:

$$\dot{\psi}_p = \frac{g}{D}\left(\rho_{//}^2 B + \mu\right)\frac{\partial B}{\partial \vartheta} - g\,\frac{\rho_{//} B^2}{D}\frac{\partial \alpha}{\partial \vartheta} + I\,\frac{\rho_{//} B^2}{D}\frac{\partial \alpha}{\partial \phi} + \frac{g}{D}\frac{\partial \Phi_{MHD}}{\partial \vartheta} - \frac{I}{D}\frac{\partial \Phi_{MHD}}{\partial \phi} \tag{1a}$$

$$\dot{\vartheta} = \frac{\rho_{//} B^2\left(1+\rho_c g'\right)}{D} - \left(\mu + \rho_{//}^2 B\right)\frac{\partial B}{\partial \psi_P}\frac{g}{D} - \frac{g}{D}\frac{\partial \Phi_{MHD}}{\partial \psi_p} + \rho_{//} B^2\,\frac{\partial \alpha}{\partial \psi_p}\frac{g}{D} \tag{1b}$$

$$\dot{\rho}_{//} = \frac{\left(\rho_{//}^2 B + \mu\right)}{D}\left[\left(-1 - \rho_c g' - g\,\frac{\partial \alpha}{\partial \psi_p}\right)\frac{\partial B}{\partial \vartheta} + \left(g\,\frac{\partial \alpha}{\partial \vartheta} - I\,\frac{\partial \alpha}{\partial \phi}\right)\frac{\partial B}{\partial \psi_p}\right] + \frac{\left(\rho_c I' + q\right)}{D}\frac{\partial \Phi_{MHD}}{\partial \phi}$$
$$- \frac{\left(1+\rho_c g'\right)}{D}\frac{\partial \Phi_{MHD}}{\partial \vartheta} + \left(I\,\frac{\partial \Phi_{MHD}}{\partial \phi} - g\,\frac{\partial \Phi_{MHD}}{\partial \vartheta}\right)\frac{\alpha'}{D} + \left(g\,\frac{\partial \alpha}{\partial \vartheta} - I\,\frac{\partial \alpha}{\partial \phi}\right)\frac{\Phi'_{MHD}}{D} \tag{1c}$$

$$\dot{\phi} = -\frac{\rho_{//} B^2\left(q + \rho_c I'\right)}{D} + \left(\mu + \rho_{//}^2 B\right)\frac{\partial B}{\partial \psi_p}\frac{I}{D} + \frac{I}{D}\frac{\partial \Phi_{MHD}}{\partial \psi_p} - \rho_{//} B^2\,\frac{\partial \alpha}{\partial \psi_p}\frac{I}{D} \tag{1d}$$

where μ is the magnetic moment, $\rho_{//}$ is the parallel gyro radius, ψ_p is the poloidal flux function, ϕ the toroidal angle and ϑ is a magnetic poloidal angle different from the physical poloidal angle θ ; $\rho_c = \rho_{//} + \alpha$; $D = \rho_{//}\left(Ig' - gI'\right) - gq + I$, g and I are respectively the poloidal and toroidal plasma current. Primes denote derivations with respect to ψ_p . We choose a parabolic profile for the safety factor q. The function

$$\alpha = \sum_{n,m} \alpha_{nm}(\psi)\cos(n\phi + m\vartheta) \tag{2}$$

represents the magnetic perturbation leading on resonance to magnetic field line stochasticity; with ψ the toroidal flux function and Φ_{MHD} the static electric potential of MHD activities in the plasma. The expression of the magnetic field in magnetic coordinates for a large aspect ratio tokamak is given by

$$B = 1 - \sqrt{2\psi}\cos\left(\vartheta + \sqrt{2\psi}\sin\vartheta\right) \tag{3}$$

where use has been made for the transformation equations between the physical toroidal coordinates (x,ϕ,θ) and the magnetic coordinates (ψ,ϑ,ϕ):

$$\begin{aligned} x &= \sqrt{2\psi} \\ \vartheta &\approx \theta - \sqrt{2\psi}\sin\theta \end{aligned} \tag{4}$$

with $x = r/R_0$; r and R_0 are the minor and major radius respectively. All quantities are in normalized units; where ω_0^{-1} is the time unit with ω_0 the on axis Larmor frequency and R_0 is the unit of length. It is important to distinguish the topology of the magnetic field and that of the drift orbits. Indeed one can find after integrating the drift equations (1a)-(1d) and the corresponding magnetic field line equations (obtained by taking $\mu = 0$ and the limit $\rho \to 0$ in Eqs. 1a-1d) that there exist drift islands and stochastic drift orbits where the magnetic lines are regular tori. The effect of the LH wave is considered through the electrostatic potential in almost perpendicular propagation

$$\Phi = \tilde{\Phi}(\phi)\cos\left[k_{//}\phi + k_\perp\sqrt{2\psi}\cos\left(\vartheta + \sqrt{2\psi}\sin\vartheta\right) + k_\perp\rho\cos\xi - \tfrac{\omega}{\omega_0}t\right] \tag{5}$$

where $k_{//}$ and $k_\perp$ are the parallel and perpendicular wave number respectively; ρ is the Larmor gyro radius and ω is the wave frequency. Thus to Eqs. (1) we add the magnetic moment and gyro phase evolution equations

$$\dot{\mu} = k_\perp\sqrt{2\mu/B}\,\tilde{\Phi}(\phi)\sin\xi\cos\left[k_{//}\phi + k_\perp\sqrt{2\psi}\cos\left(\vartheta + \sqrt{2\psi}\sin\vartheta\right) + k_\perp\rho\cos\xi - \tfrac{\omega}{\omega_0}t\right] \tag{6}$$

$$\dot{\xi} = B - k_\perp\left(\cos\xi / B\sqrt{2\mu/B}\right)\tilde{\Phi}(\phi)\sin\left[k_{//}\phi + k_\perp\sqrt{2\psi}\cos\left(\vartheta + \sqrt{2\psi}\sin\vartheta\right) + k_\perp\rho\cos\xi - \tfrac{\omega}{\omega_0}t\right] \tag{7}$$

For the ion cyclotron heating Whang and Morales considered the interaction of a test ion with the electric field of a left circular polarized perpendicular propagating wave in the local orthogonal coordinate system with $\mathbf{e}_1$ along the magnetic field

$$\mathbf{E} = E_+\cos(k_\perp x_\perp)\left[\cos(k_{//}x_{//} - \omega t)\mathbf{e}_2 + \sin(k_{//}x_{//} - \omega t)\mathbf{e}_3\right] \tag{8}$$

Thus one can obtain the evolution equations for μ and ξ given here in magnetic coordinates as

$$\dot{\mu} = E_+(\phi)\sqrt{2\mu/B}\cos k_\perp\left[1+\sqrt{2\psi}\cos\left(\vartheta+\sqrt{2\psi}\sin\vartheta\right)-\sqrt{2\mu/B}\sin\xi\right]$$
$$\times\cos\left[\xi - k_{//}\phi - k_{//}\sqrt{2\psi}\cos\left(\vartheta+\sqrt{2\psi}\sin\vartheta\right)\phi+\tfrac{\omega}{\omega_0}t\right] \tag{9}$$

$$\dot{\xi} = -B + \rho_{//}B/\left(q + q\sqrt{2\psi}\cos\left(\vartheta+\sqrt{2\psi}\sin\vartheta\right)\right) - E_+(\phi)\sqrt{2\mu/B}$$
$$\times\cos k_\perp\left[1+\sqrt{2\psi}\cos\left(\vartheta+\sqrt{2\psi}\sin\vartheta\right)-\sqrt{2\mu/B}\sin\xi\right]$$
$$\times\sin\left[\xi - k_{//}\phi - k_{//}\sqrt{2\psi}\cos\left(\vartheta+\sqrt{2\psi}\sin\vartheta\right)\phi+\tfrac{\omega}{\omega_0}t\right] \tag{10}$$

In both cases the wave electric field induced change in the coordinates (ψ_p , $\rho_{//}$) of the guiding center is neglected in comparison with the change in μ since the main change in particle velocity is perpendicular.

3. Results

Fig. 1 shows the topology of the drift orbits in the presence of magnetic perturbation. These orbits are invariant tori in the absence of magnetic perturbations. In this later case and upon considering the LH wave for parameters in the stochastic heating regime we obtain the magnetic moment variation depicted on Fig. 2. One can see the poloidal positions of the wave-particle resonances where μ evolves stochasticaly. These resonance regions are parallel planes in the poloidal section of the tokamak corresponding to the different resonance order. The wave potential and electric field amplitude in Eqs. (5),(9),(10) is taken in a Gaussian form

$$\tilde{\Phi}(\phi) = \Phi_0 \exp\left[-(\phi-\pi)^2/2\sigma^2\right] \tag{11}$$

with $\sigma=\pi/20$ to model the finite toroidal extension of the wave field. The interval of variation of the magnetic moment [$1.5\cdot10^{-7}\rightarrow6\cdot10^{-7}$] when the drift orbit is on the regular surface ψ_0=0.01 modifies to [$4\cdot10^{-7}\rightarrow8\cdot10^{-7}$] (not depicted here) in the presence of the magnetic perturbation given in Fig. 1 but with $\alpha_0=1.25\cdot10^{-4}$. In this later case the ion explore the stochastic region beyond the original drift surface ψ_0=0.01 where it encounters more wave resonance regions incresing therefore the wave energy. We obtain qualitatively the same results in the case of ion cyclotron heating above the stochasticity threshold [2] for E_+=0.108 $V\cdot cm^{-1}$. The stochastic augmentation of the ion magnetic moment results in the creation of a larger than normal population of trapped (or detrapped) ions leading even to the inversion of parallel velocity.

4. Conclusion

The stochasticity of magnetic field lines and drift orbits in tokamaks provide a kind of pumping for the evolution of the magnetic moment. This is in some way similar to the Arnold diffusion generic to multidimensional hamiltonian systems [5].

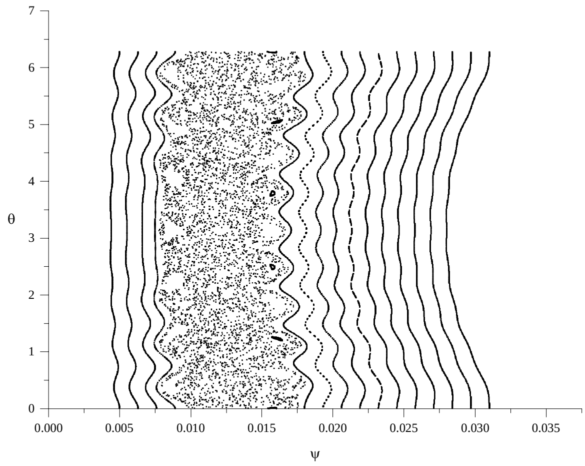

Fig 1: *Surface of section (ψ,ϑ) at $\phi=\pi$ of the drift orbits of a co-moving ion with $\mu=5.04\cdot10^{-7}$. The perturbation modes (8,4) and (9,4) have the amplitude* $\alpha_{nm}(\psi)=\alpha_0\exp\left[-(\psi-\psi_r)^2/2\cdot10^{-4}\right]$ *with $\alpha_0=3.25\cdot10^{-5}$ and ψ_r the resonant magnetic surface defined by* $q(\psi_r)=m/n$.

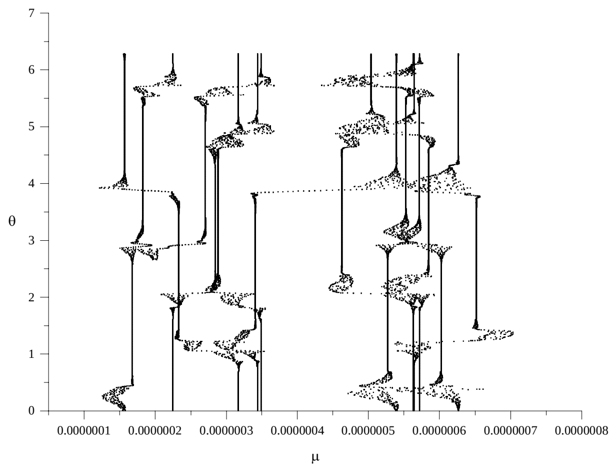

Fig 2: *Surface of section (μ,ϑ) at $\xi=\pi/2$ for 22000 gyro periods with $\psi_0=0.15$, $\rho_{//0}=1.46\cdot10^{-3}$, $\mu_0=5.04\cdot10^{-7}$, $\alpha_0=0$, $\Phi_{MHD}=0$ and $\Phi_0=3.25\cdot10^{-8}$.*

References

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