A NEW SELF-CONSISTENT, INTEGRAL-EQUATION TECHNIQUE FOR THE EVALUATION OF INPUT IMPEDANCE OF ICRH ANTENNAS

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Abstract

The self-consistent method used for the evaluation of the current distribution, and the input impedance of the straps assumes a slab geometry and the presence of a back-wall and a Faraday shield. The calculation is based on the solution of an Electric Field Integral Equation (EFIE) in the spectral domain, applying the Galerkin, weighted residual method. Adopting first-order interpolating vector basis and test functions ("rooftops"), no assumptions have been made on the current distribution. The fully three-dimensional Green's function is derived from the cold plasma surface impedance by means of an equivalent transmission lines model. A key feature is an efficient hybrid spectral-spatial extraction technique of the asymptotic Green's function behavior, leading to a drastic reduction of the spectral integration support and global computational effort.

The implemented code is modular both in its theory background and implementation, and therefore open to significant improvements as the introduction of the antenna recess and to the adoption of any plasma model.

An optimization of the IGNITOR machine antenna has been developed using this new code at this stage. The optimization has been carried on maximizing the power transmitted to the antenna, i. e. the standing wave ratio in the coaxial cables feeding the straps. The optimized parameters are the strap dimensions, the relative position of the straps, and the distance between the antenna and the Faraday shield. A comparison of the antenna performances with a not self-consistently calculated current distribution is presented.

Introduction

The strap antennas in the ion cyclotron range of frequency (ICRF) are radiators rather than transmission lines: in the presence of a strongly magnetized plasma (the field on the antenna straps couples to the plasma waves) and of a complicated geometry, it is no longer permissible to speak of transmission lines [1]. For this reason the current on the antenna strap should be calculated in a self-consistent way. This is particularly important in the calculation

of the input impedance at each feeding point of the antenna. Stated in general, the problem we are aiming at is the solution of the Maxwell equations in a structure consisting of planar (perfectly conducting) metalizations (straps and Faraday shield) occasionally linked by means of radial (perfectly conducting) metalizations embedded in a vacuum region infinitely extended in the (y, z) plane and extended from a perfectly conducting plane (back-wall) to a plasma region along the x direction. The problem is formulated as an Electric Field Integral Equation (EFIE), in which the metalizations are replaced by equivalent electric surface currents $\underline{J}_t$ that produce a scattered field expressed via the three-dimensional Green's function $\underline{\underline{G}}$ of the structure itself. The enforcement of the proper boundary conditions on the total electric field, on the areas occupied by the metalizations, ensures the self-consistency of the equivalent current distribution, and give rise to an integral equation:

$$\iiint \underline{\underline{G}}(x, x', y - y', z - z') \cdot \underline{J}_t(x', y', z') dx' dy' dz' = -\underline{E}_t^i(x, y, z), \tag{1}$$

where $\underline{E}_t^i$ is the tangential field generated by the sources in absence of the structure.

The Weighted Residual Method

Equation (1) can be solved via approximation of the unknown current $\underline{J}_t$ by a set of basis functions $\underline{f}_n$ with unknown coefficients I_n. The EFIE (1) is tested over the same set of basis functions (Galerkin method [2]) by an inner product, obtaining a linear system $[Z][I] = [V]$ for the unknown current coefficients I_n, where

$$Z_{mn} = \left\langle \underline{w}_m, \underline{\underline{G}} * \underline{f}_n \right\rangle, \quad V_m = \left\langle \underline{w}_m, -\underline{E}_t^i \right\rangle. \tag{2}$$

Since the Green's function is naturally expressed in the k_y and k_z spectral domain, we employ this spectral domain approach throughout. We employ a discretization of the metalizations geometry using subdomain functions defined on the cells of the obtained rectangular mesh. These functions are typically vector-valued, linear interpolating functions, called "rooftop" [3]. The number of subdomain basis function (and thus of unknown coefficients) is, in general, equal to the square of the number of cells in the mesh, and the cell side is a fraction of the wavelength ($\approx \lambda/10$). Many unknowns (≥ 500) and a wide spectrum makes numerical efficiency necessary.

The Spectral Green's Function

For the magnetized plasma, we adopt the field representation of a cold plasma with two normal modes (FW, SW). Continuity conditions at plasma-vacuum interface are enforced, to obtain a relationship between tangential fields called plasma surface impedance [4], that completely characterizes the plasma half-space. The fully three-dimensional Green's function

of the system is obtained as the solution of the Maxwell's equations in vacuum; the fields are decomposed into their transverse electric (TE) and transverse magnetic (TM) parts with respect to one given direction z (direction of the static magnetic field); the equations are subjected to metallic boundary conditions at the back-wall and to a general boundary condition at the plasma-vacuum interface. The interest of the present formulation is that for any current distribution the electric field can be expressed in terms of the elements of only 4 piecewise analytic matrices [5].

Spectral-Spatial Asymptotic Extraction

The asymptotic behavior of the Green's function is divergent in k_y and k_z; this yields to a very broad spectrum, poor convergence of the integrals (2), and poor numerical efficiency. The large number of unknowns (see previous sections) forces us to adopt a technique for reducing the time necessary to fill the matrix. An extraction of the Green's function asymptotic behavior in vacuum $\underline{\underline{\widetilde{G}}}^a$, and its successive evaluation in the spatial domain can help to get this goal:

$$\underline{\underline{\widetilde{G}}} = \underline{\underline{\widetilde{G}}}^a + \underline{\underline{\widetilde{G}}}^r, \quad \underline{\underline{\widetilde{G}}}^r = \underline{\underline{\widetilde{G}}} - \underline{\underline{\widetilde{G}}}^a, \quad Z_{mn} = Z_{mn}^a + Z_{mn}^r. \tag{3}$$

Considering R the distance between two points in space, the last terms in (3) becomes

$$Z_{mn}^a = -\frac{Z_0}{2\pi k_0} \iiint \nabla_t \underline{f}_m (x, y, z) dx dy dz \iiint \nabla_t \underline{f}_n (x', y', z') \frac{1}{R} dx' dy' dz' \tag{4}$$

that can be evaluated in closed form, and

$$Z_{mn}^r = \frac{1}{(2\pi)^2} \iiiint \underline{\widetilde{w}}_m^* (x, k_y, k_z) \cdot \underline{\underline{\widetilde{G}}}^r (x, x', k_y, k_z) \cdot \underline{\widetilde{f}}_n (x', k_y, k_z) dx dx' dk_y dk_z . \tag{5}$$

that is rapidly convergent in k_y and k_z.

The radiation and input impedance are calculated via the usual variational expressions, that, in this case, translate into

$$Z_{in} = \frac{[I_n]^{T^*}[Z_{mn}][I_n]}{\left|I_n^{in}\right|^2}, \quad Z_{rad} = 2\frac{[I_n]^{T^*}[Z_{mn}][I_n]}{\left|I_n^{max}\right|^2}. \tag{6}$$

Source Modeling

The source terms are represented by current driving terms, i.e. the current distribution produced by the coaxial feeding lines. This is the so-called "incident" current distribution that would be present in absence of the structure. In our analyses this current is modeled as an half-rooftop function at the end of each "feeding" feeder (different from "shorting" feeder) (see Fig. 1). The electric field produced by the driving terms is

$$E_t^i(x,y,z) = \iiint \underline{\underline{G}}(x,x',y-y',z-z') \cdot \underline{J}_t^i(x',y',z')dx'dy'dz', \quad [V] = [Z][I^i]. \qquad (7)$$

Ignitor Results and Conclusions

We have analyzed the IGNITOR machine ICRF antenna in a plane stratified geometry with a 50% T, 47% D, 1% H, 1% ^{3}He plasma at $\omega = 2\Omega_{CT} = \Omega_{C^3He} = 131$ MHz. This antenna consists of four strap to form a poloidal and toroidal phased array; each strap is directed along the y direction and is current fed at one end, and short-circuited to the back-wall at the other end. In this simulation we have used ten basis functions for each strap and one basis function on each feeder. Fig. 2 shows the real part of the radiation impedance vs. distance between straps and plasma compared with previous semi-consistent calculation [6]. This code is modular and applicable to all the ICRF antenna structure, both in the analysis phase and in the design phase. Improvements are under way, as the retaining of antenna recesses and the adoption of more accurate plasma models.

References

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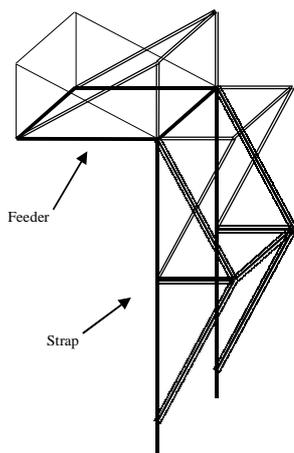[6] R. Maggiora, et al.: Fusion Engineering and Design **38** (1998) 353.

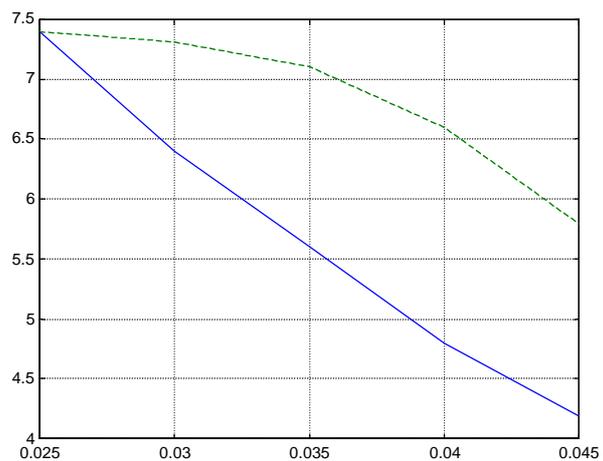**Figure 1.** Basis function distribution the "feeding" feeder and the strap.

Figure 2. Radiation resistance in ohm vs. distance on strap-plasma in meter using the self-consistent technique (—) and the semi-consistent technique (--).