ECRH AT LARGE POLOIDAL ANGLES

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Detailed ray tracing, of wave propagation in a plasma near electron cyclotron resonances, suggests that refraction can lead to reduced absorption, in some cases. By studying the full wave equation for the ordinary wave near the fundamental, in a slab model, it is shown that such refraction does indeed reduce absorption, for this particular case, but that wave energy tunnelling can significantly modify the result.

1. Wave Equations

Cold plasma ray tracing, of electron plasma waves near the fundamental, predicts that, below critical density, absorption increases with poloidal angle of incidence. However, hot plasma ray tracing, where the ray is calculated from the hot plasma dispersion relation, suggests that above a certain angle refraction causes absorption to fall to zero. [1,2] Here, we examine these results in detail, by taking a simplified model, which enables the solution of the full wave equations, whilst keeping the essential physics which produces the refraction.

The model taken, is of a hot plasma with magnetic field in the z-direction (toroidal) and inhomogeneity in the x-direction (radial) only, on a length scale L. Into this, we launch an electron cyclotron wave in the x-y (poloidal) plane. This wave has its fundamental resonance ($\omega = \omega_c$) on axis (x=0) and enters the resonance region at an angle θ to the x-axis.

The hot plasma ray tracing absorption, for this simplified model, is derived from its dispersion relation,

$$N_\perp^2 = N_x^2 + N_y^2 \;\;=\;\; \frac{1-\alpha}{1+f(x)}, \tag{1}$$

where $\quad f(x) = \dfrac{\alpha}{2} F_{7/2}\left(\mu \dfrac{\omega - \omega_{ce}}{\omega}\right), \quad \mu \;=\; m_{0e}c^2/k_B T_e \;$ and $\; \alpha = \omega_{pe}^2/\omega_{ce}^2.$

The function F is the Dnestrovskii function [3].

As there is homogeneity in the y-direction, N_y is fixed, and so, provided N_y is large enough, N_x falls to zero before the resonance is reached and so the wave is refracted away without being absorbed at all (Figure 1.)

This means that, according to hot plasma ray tracing, there is a critical angle, θ_{crit}, above which waves are refracted away from resonance and experience no absorption. For angles below
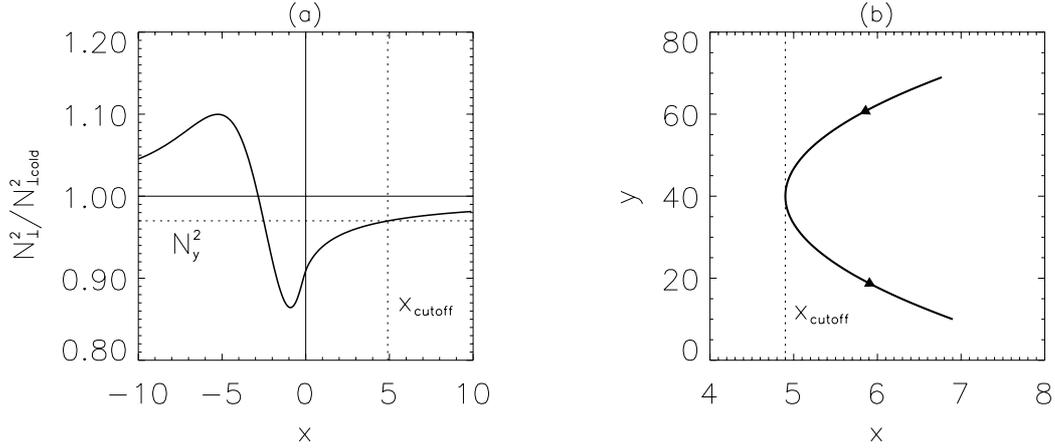
Figure 1. Refraction of wave by x-cutoff for $\alpha = 0.5$. (a) Refractive index, (b) Ray path.

θ_{crit} the wave reaches the resonance and its absorption can be calculated using the anti-hermitian part of the dispersion relation.

The full wave equation is, following [4],

$$\frac{d}{dx}\left[\{1 + f(x)\}\frac{dE_z}{dx}\right] + \left[-\{1 + f(x)\}k_y^2 + \frac{df}{dx}k_y + \frac{\omega^2 - \omega_{pe}^2}{c^2}\right]E_z = 0, \tag{2}$$

where E_z is the toroidal component of the perturbed electric field. This equation is then solved numerically. The key parameter determining the behaviour is

$$\delta' = \frac{\omega L}{\mu c}(1 - \alpha)^{1/2} = 1.15\left(\frac{B}{T}\right)\left(\frac{L}{m}\right)\left(\frac{T_e}{keV}\right)(1 - \alpha)^{1/2}, \tag{3}$$

where $\delta' cos\theta$ can be thought of as the tunnelling parameter. Figure 2 shows the behaviour for three values of δ'.

For small δ', the plasma is optically thin and hence there is strong transmission below θ_{crit}, in good agreement with ray tracing. However, due to tunnelling, strong transmission continues even above θ_{crit}, and so we see a marked difference from hot plasma ray tracing. For intermediate δ', the picture is more complicated. Full wave effects mean that we now see reflection for $\theta < \theta_{crit}$ and, once more, there is significant transmission, due to tunneling, for $\theta > \theta_{crit}$. Most significantly, there is appreciable absorption (over 30%) at θ_{crit}, where hot plasma ray tracing predicts complete reflection. For large δ', the large tunneling region means that, above θ_{crit}, there is almost complete refraction and so negligible absorption. However, refraction effects actually in the resonance region mean that we do not see full absorption until significantly below θ_{crit}.

2. Discussion

Full wave effects can give rise to significant departures from hot plasma ray tracing, dependent on the value of the tunnelling parameter $\delta' cos\theta$; low values giving large transmission above the critical angle, high values to major refraction even before the critical angle.
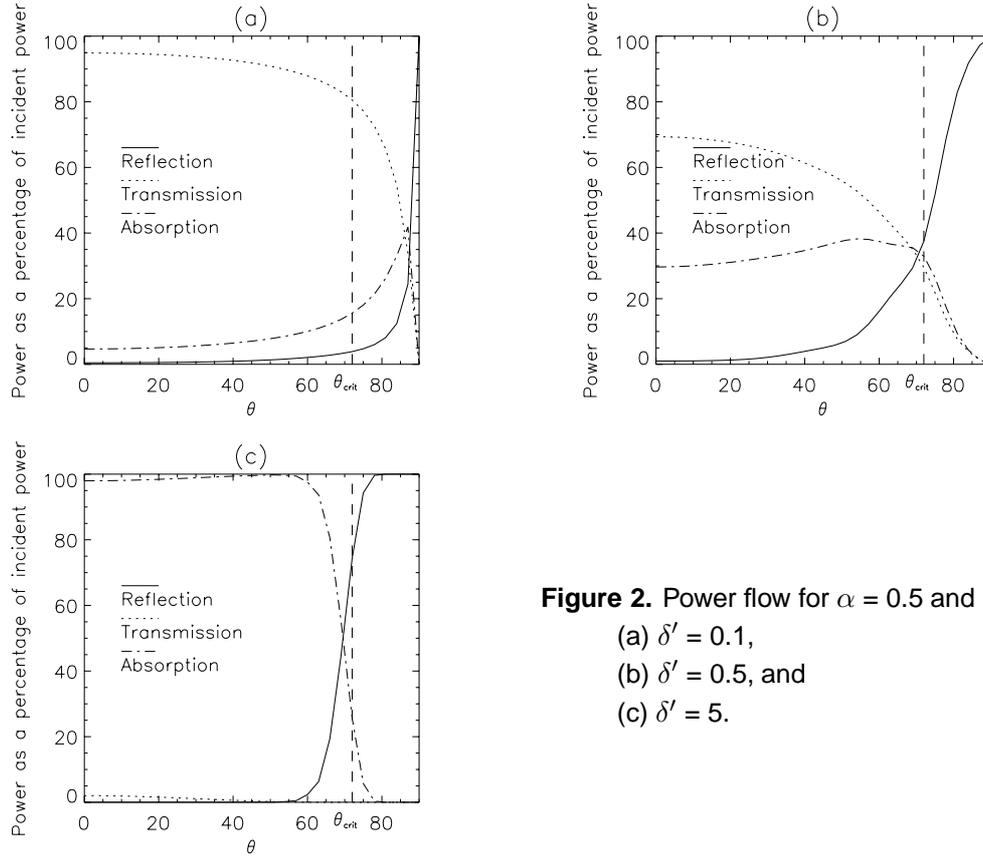
Figure 2. Power flow for $\alpha = 0.5$ and
(a) $\delta' = 0.1$,
(b) $\delta' = 0.5$, and
(c) $\delta' = 5$.

COMPASS-D like parameters ($R_0 = 55.7cm$, $B_0 = 2.4T$, $T_e = 0.5keV$ to $T_e = 5keV$, and $n_{e0} = 3.1 \times 10^{19} m^{-3}$) give $\delta' = 0.13$ rising to $\delta' = 1.3$ towards the end of the heating phase. Initially, we would see a small tunnelling region [Figure 2(a)], hence sizeable transmission, and so we would need high angles of launch to see significant absorption. At higher temperatures, δ' is around one [Figure 2(b)] and we see wave splitting.

JET like parameters ($R_0 = 3\ m$, $B_0 = 2.7\ T$, $T_e = 0.5\ keV$ to $5\ keV$, and $n_{e0} = 5.0 \times 10^{19}\ m^{-3}$) give $\delta' = 0.9$ rising to $\delta' = 9$ towards the end of the heating phase. Here we have a large tunnelling region [Figure 2(c)] and so good agreement with hot plasma ray tracing, only with major refraction expected even for angles significantly below θ_{crit}.

Acknowledgements

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