KINETIC THEORY OF TRANSPORT DRIVEN CURRENT

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Tokamak discharge usually operate with edge or mid radius fuelling. The understanding of energy, mass and charges fluxes in tokamak is the basic issue of the magnetic confinement program. Most of the theoretical efforts are directed toward the identification of the driving mechanisms behind anomalous fluxes. However, besides this challenging issue, the understanding of the coupling between the various fluxes is particularly important to set up a global picture of a working steady state reactor. Up to now tokamak discharges have been operated via edge or mid-radius fuelling. In a reactor there are many advantages to operate via central fuelling in order to peak the pressure profile and to wash out ashes and impurities which have a tendency to accumulate near the magnetic axe. Although, it is not clear now how to perform this central fuelling, it is important to evaluate the physical consequences of central fuelling in a steady state reactor.

Consider a reactor with a fusion power W. In order to operate in steady state a radial flux or matter, from the center (where fuelling and combustion take place) to the edge (where ashes and heat removal is operated), is set up, a lower bound of the electrons flux is given by $\frac{dN}{dt} = \frac{2W}{Q}$, where Q is the energy yield per fusion reaction. In a straight tokamak model this rate of particles injection/extraction corresponds to a radial ambipolar flux :

$$\Gamma_e(r) = 2\Gamma_i(r) = \frac{W}{2\pi^2 QrR},$$

where r is the radius of a drift surface and R the major radius of the tokamak. This flux is a consequence of the steady state and central fuelling requirements and its exact nature, convective, diffusive or non local, remains an open question. A radial velocity is associated with this flux:

$$V_r(r) = \frac{\Gamma_e(r)}{n(r)} = \frac{W}{2\pi^2 QRrn(r)},$$

where $n(r)$ is the electron density. A direct application of Ohm's law predicts a toroidal current due to the poloidal component of the magnetic field: $J_\varphi(r) = \sigma\left[E_\varphi(r) + V_r(r)B_\theta(r)\right]$. This result is questionable as the V cross B term in Ohm's law is associated with a convective flow. Nevertheless let us explore the consequences of this crude fluid model in the collisionless limit $\sigma = \infty$

$$E_\varphi(r) + V_r(r)B_\theta(r) = -\frac{\partial A_\varphi(r,t)}{\partial t} + V_r(r)B_\theta(r) = 0.$$

As a consequence of the frozen-in law a growth of the toroidal vector potential takes place, and the poloidal magnetic field increases with time. Despite the fact that both fluid and collisionless hypothesis are questionable, such a growth of the poloidal field has been observed in PIC simulations and identified as an efficient current generation mechanism [1].

This increase of the poloidal field predicted with the previous fluid model and observed in PIC simulations is due to the lack of dissipative processes. Thus, we have to set up a collisional model, but a collisional fluid model is not sufficient because the velocity involved in the V cross B term of the Ohm's law is a mixing of diffusion and convection,

$$\Gamma_e(r) = -D(r)\frac{\partial n}{\partial r} + V(r)n,$$

where both the diffusion coefficient D and the pinch velocity V are anomalous and unknown. Nevertheless, despite this lack of informations, a collisional kinetic model can be set and solved and the transport driven current can be calculated. Let us reconsider the frozen-in law where electron inertia is taken into account.

$$E_\varphi + V_r B_\theta + \frac{m}{e}\frac{dV_\varphi}{dt} = 0$$

As $E_\varphi = -\frac{\partial A_\varphi}{\partial t}$ and $B_\theta = -\frac{\partial A_\varphi}{\partial r}$ we can rewrite this collisionless condition as the well known invariance of the toroidal momentum,

$$mV_\varphi - eA_\varphi.$$

Now the problem is to identify under which conditions this momentum is preserved by turbulent transport and how this invariance can be accommodated with collisions. The answer to the first question is fairly direct if we consider electric turbulence, described by the electric potential $\Phi(r)e^{il\theta+in\varphi-i\varpi t}$ and magnetic turbulence, described by the magnetic vector potential $\mathbf{A}(r)e^{il\theta+in\varphi-i\varpi t}$, as the sources of the anomalous transport. For each (l, n) mode a new invariant can be constructed:

$$mV_\varphi - eA_\varphi + \frac{nm}{2R\varpi}V^2,$$

where $\frac{mV^2}{2}$ is the kinetic energy of the electron. As the frequency of the electric and magnetic turbulence is far below the cyclotron frequency the adiabatic invariance of the magnetic moment is preserved so that the previous invariant gives to a good accuracy $m\delta V_\varphi\left(1 + \frac{n}{R\varpi}V_\varphi\right) + \frac{e}{m}B_\theta\delta r = 0$. Thus, when radial transport is driven by low n number electric and magnetic fluctuations, increments of the radial displacement and the toroidal velocity are constrained by the relation:

$$\delta V_{\varphi} = -\frac{e}{m} B_{\theta} \delta r.$$

This velocity-position coupling is similar to the one which is put at work in alpha particles free energy channeling $[2]$. On the basis of this relation it is straightforward to set up a collisional kinetic model. We introduce the distribution function $F(p, \mu, r)$ where μ is the electron pitch angle and p the electron momentum ($p\mu = V_{\varphi}$). This distribution function is the solution of the following kinetic equation:

$$\frac{Z+1}{2p^3} \frac{\partial}{\partial \mu} \left(1 - \mu^2\right) \frac{\partial}{\partial \mu} F + \frac{1}{r} \frac{\partial}{\partial r} \left[-rD \frac{\partial n}{\partial r} + Vn\right] F +$$

$$+ De^2 B_{\theta}^2 \frac{\partial}{\partial V_{\varphi}} \frac{\partial}{\partial V_{\varphi}} F - VeB_{\theta} \frac{\partial}{\partial V_{\varphi}} F = \frac{2W}{Q} \frac{\left[\delta\left(r\right) - \delta\left(r - a\right)\right]}{4\pi^2 rR}.$$

The first term is the pitch angle scattering operator describing electrons-ions collisions, the two Dirac functions describe steady state central injection and edge extraction and the unknown anomalous radial transport coefficients D and V give rise to toroidal velocity diffusion and drift in order to fulfill the invariance of the toroidal momentum (the unit of time is the slowing down time, $\tau_s = (4\pi Ln(\Lambda) n r_e^2 c)^{-1}$, the unit of velocity is c, the velocity of light, the unit of mass is $m,$ the electron mass. Despite its apparent complexity this type of equations describing both collisional velocity relaxation and turbulent radial transport can be solved analytically $[3, 4]$.

Here, in order to simplify, we will assume that collisional velocity relaxation occurs on a time scale shorter than anomalous radial transport. Such an ordering is fulfilled in tokamak and will provide a lower bound of the current.

The turbulent electric and magnetic fluctuations induce a combination of radial diffusion and drift which can be described as a correlated random walk occurring on a typical time scale much longer than the collisional velocity relaxation time. This random walk can be decomposed as a set of elementary radial steps δr. Consider an electron which is at time t on the drift surface r with a pitch angle μ_0, as a result of the anomalous transport this electron jump from r to $r + \delta r$ and this creates: (i) a hole on the drift surface r at pitch angle μ_0, (ii) an additional electron on the drift surface $r + \delta r$ at pitch angle $\mu_0 + \delta\mu$ with

$$\delta\mu = -\frac{eB_{\theta}}{mp} \delta r.$$

This electron-hole current carrying structure will rapidly decay on a collisional time scale and we have to calculate the elementary current which has been created. To do so we consider the following equation describing the electron-hole history $f(\mu, t)$:

$$\frac{\partial}{\partial t}f(\mu,t)-\frac{Z+1}{2p^3}\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu}f(\mu,t)=\left[\delta\left(\mu-\mu_0-\delta\mu\right)-\delta\left(\mu-\mu_0\right)\right].$$

The solution of this equation can be easily expressed as a series of Legendre polynomials P_l :

$$f(\mu,t)=\delta\mu\sum_{l=0}^{l=\infty}\frac{2l+1}{2}\exp\left[-\frac{l(l+1)(Z+1)}{2p^3}t\right]P_l(\mu)\frac{dP_l(\mu_0)}{d\mu_0}.$$

The current density resulting from the continuous anomalous flux of dN electrons, per unit time, from r to $r+\delta r$ is given by the moment $\int p\mu f d\mu$,

$$J_\varphi(\delta r)=\frac{e^2B_\theta}{m}\delta r\int_0^\infty\exp\left[-\frac{(Z+1)}{p^3}t\right]\frac{dN}{dt}dt.$$

This current density J_φ is associated with a toroidal current I_φ. In order to evaluate the scaling of this transport driven current we assume that the poloidal field is constant so that we finally obtain:

$$I_\varphi=\sum_{RW}\frac{e^2B_\theta}{m}\frac{dN}{dt}\frac{p^3}{Z+1}\frac{\delta r}{2\pi R}=\frac{e^2B_\theta}{m}\frac{dN}{dt}\frac{p^3}{Z+1}\frac{a}{2\pi R},$$

where *RW* means a sum over all the steps of the electrons random walk from the center to the edge. As the electron are injected at the center and removed at the edge we have $\sum\delta r=a$ and this sum is independent of the detail of the correlated random walk (independent of D and V) describing turbulent transport. To display the scaling of this transport driven current the previous result can be rewritten in term of $I_0=\frac{e2W}{Q}$, the equivalent current injected at the center.

$$\left[\frac{I_\varphi}{I_0}\right]=\frac{\nu_\theta\tau_s}{Z+1}\frac{a}{R}\left[\frac{3T}{511KeV}\right]^{1.5}\frac{a}{R},$$

Where *T* is the temperature and ν_θ the cyclotron frequency associated with the poloidal field.

If we consider typical reactor parameters $\tau\approx 28ms, T\approx 10KeV, I_0\approx 300A$, we end up with a transport driven current of the order of several *Mega-Ampere*.

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