NUMERICAL INVESTIGATION OF MINORITY RF HEATED TOKAMAK PLASMAS

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Abstract

The use of the compressional hydromagnetic mode (also called the fast wave mode) is examined in some detail with respect to the heating of minority Helium-3 species in a fully catalyzed deuterium Tokamak. Our study develops a rapid approximate finite element solution to analyse the two dimensional, steady state Fokker- Planck equation, with quasilinear ICRH heating terms. The resulting distribution function develops a high, anisotropic, in the perpendicular velocity direction, energy tail as a result of ICRH.

1. Introduction

The use of waves in the ion cyclotron range of frequencies (ICRF) is a leading method of heating plasmas to thermonuclear temperatures in toroidal devices. The D-T reaction appears to be the fusion reaction, which leads most directly to a power producing reactor. This is exemplified by the constant progress of Tokamak investigators in achieving high ion temperatures of a minority Helium-3 component in a deuterium plasma. In this paper, we develop a rapid solution to analyse the anisotropic effects of ICRH on a minority Helium-3 species in a fully catalyzed deuterium plasma. The two component plasma fast-wave heating has already been considered by Stix [1]. However, Stix's analysis does not give a full determination of the velocity distribution for thermal energies and above. Only, a simple solution is obtained. Furthermore, Stix's analysis does not include a source of energetic minority ions, which is present in a fully catalyzed deuterium system. To formulate a steady-state model, we assume an ion sink at some much lower energy, Ws, which absorbs test ions at the same rate that the source emits them.

2. Basic Equations

The standard kinetic equation for the ion velocity distribution incorporating the combined effects of quasilinear heating and Coulomb thermalization is:

$$\frac{\partial f}{\partial t} = C(f) + Q(f) \qquad (1)$$

where

$$C(f)=-\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\left(\left\langle\Delta v_{//}\right\rangle+\frac{1}{2v}\left\langle(\Delta v_\perp)^2\right\rangle\right)f\right]+\frac{1}{2v^2}\frac{\partial^2}{\partial v^2}\left(v^2\left\langle(\Delta v_{//})^2\right\rangle f\right)$$
$$+\frac{1}{4v^2}\frac{\partial}{\partial \mu}\left(1-\mu^2\right)\frac{\partial}{\partial \mu}\left(\left\langle(\Delta v_\perp)^2\right\rangle f\right) \qquad (2)$$

is the Fokker- Planck collision operator, f is the velocity distribution of the minority component of Tritons in a thermal Deuterium plasma, μ is the particle cosine angle and v is the particle velocity. $\left\langle\Delta v_{//}\right\rangle$ and $\left\langle\Delta v_\perp\right\rangle$ are the Coulomb diffusion coefficients [2].

$$Q(f)=\sum_{mn}\frac{R}{\pi\omega_{ci}\, r\left|Sin\theta_0\right|}Q_{mn}\quad\left(x_0,\left|y_0\right|\right)R_m(f) \qquad (3)$$

is the quasilinear diffusion operator, calculated by Kennel and Engelman for cyclotron absorption and averaged over a toroïdal magnetic surface of minor radius *r* and a major radius *R*, where $\theta = Tg^{-1}\left(y_0 / x_0\right)$ and x_0 , y_0 are the coordinates of plasma cross-section where the selected magnetic surface intersects the central resonant surface. The expressions of Q_{mn} and R_m are given in Ref. [1].

To the right hand side of Eq. (1), we add a net source term:

$$S(v)=\frac{s}{4\pi v^2}\delta(v-v_0)-\frac{s}{4\pi v^2}\delta(v-v_s) \qquad (4)$$

$S(v)$ is the net number of minority ions born in the $d^3\vec{V}$ about $\vec{V}$ per unit volume per unit time at the source speed v_0 . The source is monoenergetic and isotropic in velocity space. The sink is isotropic in velocity space and of magnetitude equal to the source.

3. Numerical Resolution

Following the conventional manner of obtaining solutions to equation (1), we first expand the distribution function for the minority ions in a series of Legendre polynomials in μ :

$$f(\vec{v},t)=\sum_{l=0}^{\infty} g_{el}(v,t)\;\; P_{2l}(\mu)$$

Restricting ourselves to the fundamental cyclotron frequency and keeping only the first two terms in μ expansion, we find:

$$f(v,\mu,t)=A(v,t)+\frac{1}{2}\left(3\mu^2-1\right)B(v,t) \qquad (5)$$

Substituting Equation (5) into (1), then averaging over a circular magnetic surface and separating variables, we obtain from the P_0 (μ) and P_2 (μ) moments of Eq. (1), two coupled linear, differential equations in *v*.

$$L_1(A) = \frac{\partial A}{\partial t} = \frac{\partial}{\partial v}\left[-\alpha v^2 A + \frac{1}{2}\frac{\partial}{\partial v}\left(\beta v^2 A\right) + Kv\frac{\partial}{\partial v}v\left(A - \frac{B}{5}\right) - K\left(A + \frac{2}{5}B\right)v\right]$$

$$+ \frac{s}{4\pi v^2}\delta(v - v_0) - \frac{s}{4\pi v^2}\delta(v - v_s) \qquad (6)$$

$$L_2(B) = \frac{\partial B}{\partial t} = -\frac{1}{v^2}\frac{\partial}{\partial v}\left(\alpha v^2 B\right) + \frac{1}{2v^2}\frac{\partial^2}{\partial v^2}\left(\beta v^2 B\right) - \frac{3}{2}\frac{\gamma}{v^2}B + \frac{K}{v^2}\frac{\partial}{\partial v}v\frac{\partial}{\partial v}v\left(-A + \frac{5}{7}B\right)$$

$$-\frac{K}{v^2}\left(3A + \frac{30}{7}B\right) + \frac{K}{v^2}\frac{\partial}{\partial v}v\left(4A - \frac{5}{7}B\right) \qquad (7)$$

where
$$K = \frac{2R}{3\pi\,\omega_{ci}\,r\left|Sin\theta_0\right|}Q_{01}\left(x_0, \left|y_0\right|\right) = \frac{\langle P\rangle}{3mn} \qquad (8)$$

$$\alpha = \langle \Delta v_{//}\rangle + \frac{1}{2}\left\langle (\Delta v_{\perp})^2\right\rangle; \quad \beta = \left\langle (\Delta v_{//})^2\right\rangle; \quad \gamma = \left\langle (\Delta v_{\perp})^2\right\rangle \qquad (9)$$

and P is the absorbed RF power density averaged over the magnetic surface.

3.1. Method of solution

We assume a steady state and solve for the equilibrium of the two coupled linear equations by an iterative process. The solution of this system will be $A(v) = A(v, B(v))$. So, we first chose an arbitrary solution for $B(v)$, insert it into the first two equations and solve for $A(v)$. We then take $A(v)$, normalize it to aid in convergence, insert it into the second equation, solve for $B(v)$ and normalize it. This solution is inserted into the first equation to find a new $A(v)$. This process is repeated until convergence occurs. A finite element technique is used to find $A(v)$ and $B(v)$ at each step. As its elements, we consider “Hat functions” which satisfy the boundary conditions $A(0)= B(0)=A(v_{\mathrm{Max}})= B(v_{\mathrm{Max}})=0$. We seek an approximate solution of the form:

$$A(v) = \sum_i C_i \psi_i(v) \qquad (10)$$

To find the coefficients C_i of the assumed finite element expansion, we substitute the above trial solution for $A(v)$ in its governing differential equation. This will result in a residual, which we write:

$$R = L_1\left(\sum_i C_i \psi_i(v)\right)$$

To obtain the best solution, we then attempt to distribute the residuals through the domain of solution (from zero to v_{Max}) of the function $A(v)$ by trying to minimize the integral of the residual. We employ Galerkin’s method of using a weighted value of the residuals technique with the hat functions themselves as weight functions. For a mesh points (m), corresponding to the hat function ψ_m , we have to take the sum over all the k’s for which the product of the two hat functions $\psi_m\psi_k$ is not zero. So, we have at each time a sum of three terms, each involving integrals over v in the range where the hat function products are non zero. The coefficients involved for the mesh point (m) are: C_{m-1} , C_m and C_{m+1}.

This approach results, at each iteration, in a linear system of equations that can be represented by tridiagonal matrix where all matrix elements have been calculated analytically. The system to solve is:

$$\sum_i C_i \left(L_1 \psi_i , \psi_j\right) = 0 \qquad (i = 1,...M) \qquad (11)$$

where

$$\left(L_1 \psi_i , \psi_j\right) = \int_0^{v_{Max}} \left(L_i \psi_i(v)\right) \psi_j(v) dv$$

To find the distribution $A(v)$ and $B(v)$, Eq.(6) and Eq.(7) are first multiplied by $4\pi v^2$; then normalized to the new variables: $x = \dfrac{v}{v_{th}}$; $A(x) = \dfrac{A(v)}{v_{th}^3}$; $B(x) = \dfrac{B(v)}{v_{th}^3}$.

v_{th} is the background deuterium thermal velocity.

To invert the matrix (Eq. (10)) at each iteration, we apply the Gaussian elimination method.

3.2. Results and conclusions

We present in this section results obtained solving the Fokker- Planck equation for ICRH effects on a minority Helium-3 species in a deuterium plasma. The isotropic part of the distribution, $A(x)$, is peaked at the source (x_0) and drops to zero at the sink (x_s). When there is no RF heating, this solution corresponds to a Maxwellian distribution. When the Helium-3 is Fast-wave heated, an anistropic term, $B(x)$, appears. The resulting distribution function develops a high, anistropic, in the perpendicular velocity direct ion, energy tail as a result of ICRH.

To conclude, we have presented a fast 2-D Fokker- Planck solver, which can have import potential applications for parametric studies, various possible diagnostics and physical quantities calculation.

References

[1] T.H. Stix: Nuclear Fusion **15**, 748 (1975).

[2] L. Spitzer: The Physics of Fully Ionized Gases. 2nd Edition, Interscience, New-York, 1962.

[3] M. Tribeche and T.H. Zerguini: *Proceedings of the 21st EPS Conference*, Montpellier, 1994; *IEEE ICOP'S 95*, Madison, USA, 1995; *22nd EPS Conference*, Bournemouth, 1995.