ENHANCED PARTICLE ACCELERATION VIA CASCADE OF AUTORESONANCE DETRAPPINGS

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Abstract

We propose to enhance the acceleration of electrons by repeating consecutively a basic accelerating mechanism. This mechanism consists of trapping the particles in a traveling ponderomotive well, then autoresonance detrapping them allowing for considerable acceleration. The traveling well is generated by two counter-propagating electromagnetic waves along a uniform magnetic field.

The possibility of accelerating charged particles is presently attracting a great amount of research interest. A very promising approach for the realization of such types of accelerations has been forwarded, based essentially on the mechanism of an autoresonance process (ARP). As is well known, a major obstacle in arriving at feasible accelerating devices is the intrinsic problem of dephasing in ARP. This dephasing problem is also hunting the scheme proposed by us in our recent publications [1,2]. In this scheme we proposed to utilize two electromagnetic circularly polarized waves counter-propagating along a magnetic field. Such a configuration was shown to enhance the effectiveness of the ARP allowing for multiple AR accelerating processes. However, the dephasing problem leading to the deceleration of the particle appearing in every step of this multiple process, could not be avoided.

In this letter, we uncover a subtle mechanism which, in fact, is conditioning the system so as to bring it in a position which enables it to participate in the AR process. We show that this very mechanism itself could be used to challenge the dephasing problem in our scheme. To show this explicitly, we consider the relativistic motion of an electron due to its interaction with a constant uniform magnetic field taken in the z direction and two electromagnetic circularly polarized waves having different frequencies and different wavenumbers counter-propagating along the magnetic field in the vacuum. Using a Hamiltonian formalism, the equations of motion could be written as follows:

$$\dot{\psi} = \frac{\partial \overline{H}}{\partial P_\psi} = \frac{1}{\gamma}\left[-\bar{k}_2\left(\bar{k}_1 P_\phi - \bar{k}_2 P_\psi\right) + 1 + \frac{\overline{A}_1 \sin\phi + \overline{A}_2 \sin\psi}{\sqrt{2\left(P_\phi + P_\psi\right)}} \right] - \overline{\omega}_2 \ , \tag{1}$$

$$\dot{\phi} = \frac{\partial \overline{H}}{\partial P_\phi} = \frac{1}{\gamma}\left[+\bar{k}_1\left(\bar{k}_1 P_\phi - \bar{k}_2 P_\psi\right) + 1 + \frac{\overline{A}_1 \sin\phi + \overline{A}_2 \sin\psi}{\sqrt{2\left(P_\phi + P_\psi\right)}} \right] - \overline{\omega}_1 \ , \tag{2}$$

$$\dot{P}_\psi = -\frac{\partial \overline{H}}{\partial \psi} = -\frac{\overline{A}_2}{\gamma}\left[\sqrt{2\left(P_\phi + P_\psi\right)} \cos\psi + \overline{A}_1 \sin\left(\phi - \psi\right) \right] , \tag{3}$$

$$\dot{P}_\phi = -\frac{\partial \overline{H}}{\partial \phi} = -\frac{\overline{A}_1}{\gamma}\left[\sqrt{2\left(P_\phi + P_\psi\right)}\cos\phi - \overline{A}_2 \sin\left(\phi-\psi\right)\right] , \tag{4}$$

where
$$\gamma = \Big[1+\left(\bar{k}_1 P_\phi - \bar{k}_2 P_\psi\right)^2 + 2\left(P_\phi + P_\psi\right) + \overline{A}_1^2 + \overline{A}_2^2$$
$$+2\sqrt{2\left(P_\phi + P_\psi\right)}\left(\overline{A}_1 \sin\phi + \overline{A}_2 \sin\psi\right) + 2\overline{A}_1\overline{A}_2 \cos\left(\phi-\psi\right)\Big]^{1/2} = \overline{H} + \overline{\omega}_1 P_\phi + \overline{\omega}_2 P_\psi . \tag{5}$$

These equations have been derived from a conservative Hamiltonian:

$$\overline{H}\left(\psi, \phi, P_\psi, P_\phi\right) = \Big[1+\overline{A}_1^2 + \overline{A}_2^2 + \left(\bar{k}_1 P_\phi - \bar{k}_2 P_\psi\right)^2 + 2\left(P_\phi + P_\psi\right)$$
$$+2\sqrt{2\left(P_\phi + P_\psi\right)}\left(\overline{A}_1 \sin\phi + \overline{A}_2 \sin\psi\right) + 2\overline{A}_1\overline{A}_2 \cos\left(\phi-\psi\right)\Big]^{\frac{1}{2}} - \overline{\omega}_1 P_\phi - \overline{\omega}_2 P_\psi . \tag{6}$$

For details, notations and normalization, we refer the reader to references [1] or [2].

An important point to notice, when considering this configuration is that in addition to the interaction of the particle with any of the waves, the particle is also affected by the ponderomotive potential generated from the nonlinear interaction of the two waves. The interaction of the particle with this ponderomotive potential is at the heart of the mechanism for conditioning the system for an AR interaction and for future acceleration of the particle as we will explain later on.

In order to reveal these features of the system, we proceed to evaluate the changes in time of the energy of the system γ and the parallel component of the momentum $\overline{P}_z$.
One deduces from the equations of motion that:

$$\frac{d\beta_z}{d\bar{t}} = \frac{\sqrt{2\left(P_\phi + P_\psi\right)}}{\gamma^2}\left[-\overline{\omega}_1\overline{A}_1\left(1-\beta_z\right)\cos(\phi) + \overline{\omega}_2\overline{A}_2\left(1+\beta_z\right)\cos(\psi)\right] + \frac{\overline{A}_1\overline{A}_2}{\gamma^2}\left[\overline{\omega}_1\left(1-\beta_z\right) + \overline{\omega}_2\left(1+\beta_z\right)\right]\sin\left(\phi-\psi\right) . \tag{7}$$

Inspecting Eq.(7), one observes three different oscillating terms depending on the phases ϕ , ψ and their difference: $\xi = \phi - \psi$ respectively. The last term depending on ξ and involving the product $\overline{A}_1\overline{A}_2$ is describing the effect of the ponderomotive potential generated from the interaction of the two waves, independently of the external magnetic field. The two other terms depending linearly on the amplitudes of the waves might, under appropriate conditions, be responsible for AR accelerations of the particle.

Surprisingly, we find that the system has intrinsically built in a mechanism, which conditions it to arrive to the appropriate conditions for AR interaction. This mechanism has its origin in the trapping of the particle in the ponderomotive well when its parallel velocity lies in the vicinity of the phase velocity of the ponderomotive propagating potential $v_\xi = c(\overline{\omega}_1 - \overline{\omega}_2)/(k_1 + k_2)$. When the particle is trapped, its sweeps a continuous range of $\overline{P}_z$ values between a minimum and a maximum values. In the course of its sweep, the particle might of course arrive at values for which the AR condition is satisfied, for example: $\gamma\left(1-\beta_z\right) \cong 1/\overline{\omega}_1$. Once, the particle arrives at such a value it can get into AR and one of the two terms of the Eq.(7), depending linearly on the amplitude of the waves, starts to become dominating the forcing terms. This is associated with a slow variation of its phase. The parallel momentum starts to be larger and larger till the potential can no more maintain it, and the particle gets detrapped. We call this phenomenon: AR detrapping.

The " search " of the particle for an AR condition ($\gamma\left(1-\beta_z\right)\cong 1/\bar{\omega}_1$) during its trapped motion is shown in Fig.1. In Fig. 1(a), the motion of the particle in phase space $\left(\bar{P}_z,\xi\right)$ is shown during a time interval when the particle undergoes a trapped orbit centered around the phase velocity of the ponderomotive well v_ξ . In Fig. 1(b), we plot for the same time interval the value of $\gamma\left(1-\beta_z\right)$ as a function of time. As is clear from this figure, during the trapping time the values of $\gamma\left(1-\beta_z\right)$ change smoothly. Once, the particle arrives at values for which $\gamma\left(1-\beta_z\right)\cong 1/\bar{\omega}_1$, it enters into an AR and gets detrapped (see Fig. 1 (a)). It starts then to be accelerated and naturally the AR acceleration is limited due to the dephasing of the interaction.

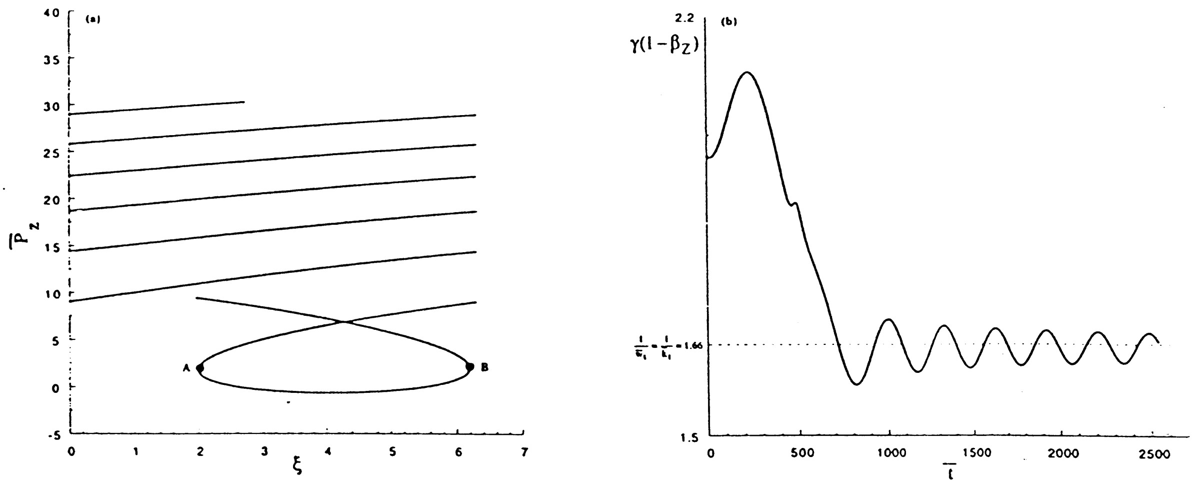

Fig. 1. (a) The trajectory of a particle in phase space ($\bar{P}_z,\xi$). The normalized parameters of the system and the initial conditions are: $\bar{A}_1$=0.9; $\bar{A}_2$=0.07 $\bar{k}_1$=0.06; $\bar{k}_2$=0.2; $\bar{\omega}_1$=0.6; $\bar{\omega}_2$=0.2; $P_{\phi 0}$=3.7; $P_{\psi 0}$=1.5; ϕ_0=3.1; ψ_0=0.1. The normalized velocities of the particle at points A and B on the trajectory are equal to the normalized phase velocity of the ponderomotive well β_ξ, showing that the trajectory of the particle, when trapped, is centered around $\beta_\xi = (\bar{\omega}_1-\bar{\omega}_2)/(\bar{k}_1+\bar{k}_2) = 0.5$. (b) The autoresonance factor versus time for the parameters and initial values of (a). The time interval is the same as in (a). The dashed line shows the value of $\gamma(1-\beta_z)$ for exact AR condition: $\gamma(1-\beta_z) = 1/\bar{\omega}_1 = 1/\bar{k}_1 = 1.66$.

This dephasing problem is a severe one, since it causes the particle to loose the energy and momentum it had gained. A way to overcome this limitation could be to catch the particle in a " protective " environment so that it cannot loose the energy it has just gained. Such an environment could be of course the ponderomotive well. If we could generate in the system an additional ponderomotive well propagating with a phase velocity comparable now to the parallel velocity of the particle which was gained via AR acceleration, the particle could be readily trapped in this second well. Consequently, by introducing in the system additional waves having appropriate frequencies, we can generate ponderomotive wells with different phase velocities. To this end, we have considered a system with three electromagnetic waves having parameters appropriate for the generation of ponderomotive wells which will allow for an acceleration of the particle in cascade.

In order to test the validity of our scheme, we have to check two main points: (a) whether a particle gets accelerated a second time before it falls back to its initial stochastic base from which the acceleration started in the first place, and (b) in case that (a) is fulfilled, whether the value of the parallel velocity of the particle before getting accelerated a second time, is in the vicinity of the phase velocity of the relevant ponderomotive well. In order to check these two points, we have solved numerically the system of equations for an appropriate set of parameters and initial conditions of the three wave configuration. The results are presented in Fig. 2. As is clear from this figure the particle gets AR accelerated, exhibiting the beginning of a typical bell structure characterizing an AR acceleration.

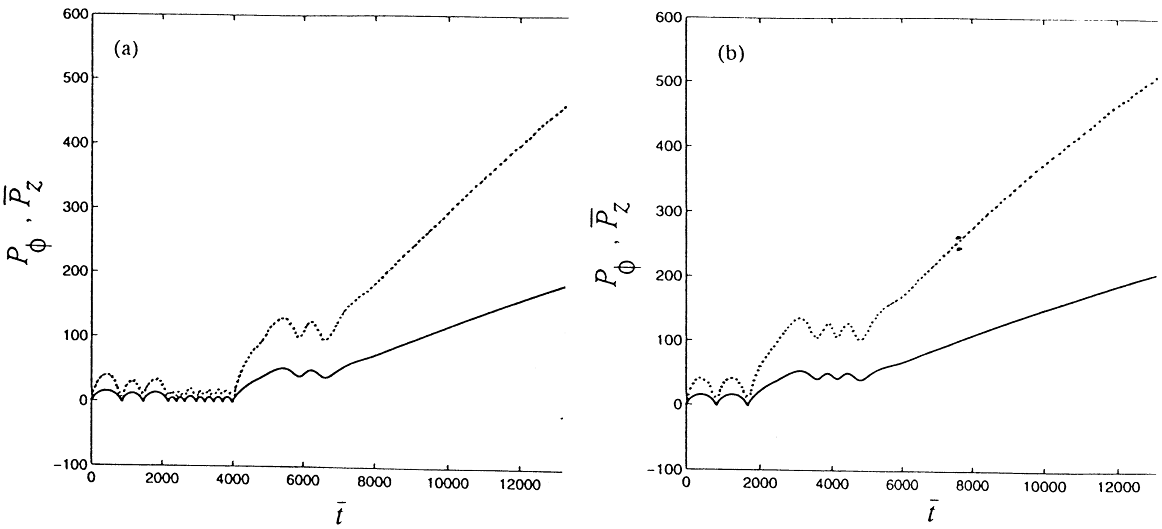

Fig. 2. The change with normalized time $\bar{t}$ of the momenta P_ϕ (the dashed line) and the normalized $\bar{P}_z$ (the full line) for a three wave configuration having the following set of parameters and initial conditions: $\bar{A}_1$ =0.3; $\bar{A}_2$ =0.02; $\bar{A}_3$ =0.18; $\bar{k}_1$ =0.4; $\bar{k}_2$ =0.2; $\bar{k}_3$ =0.0095; $\bar{\varpi}_1$ =0.4; $\bar{\varpi}_2$ =0.2; $\bar{\varpi}_3$ =0.0095; $P_{\phi 0}$ =1.7; $P_{\psi 0}$ =1.6; $P_{\eta 0}$ =0.15; ϕ_0 =3.1; ψ_0 =3.1; η_0 =3.1. Notice the two acceleration steps that the particle undergoes. Discussion in the text (b) The same as (a) with: $P_{\phi 0}$ =0.8; $P_{\psi 0}$ =1.8; $P_{\eta 0}$ =0.1; ϕ_0 =3.5; ψ_0 =1.6; η_0 =1.1. The parameters of the waves are unchanged.

However, after arriving at the peak of the bell and starting to decrease as in the standard acceleration process, the momentum stops to decrease and starts to oscillate for sometime. It then undertakes a second strong AR acceleration course. Checking now, the validity of point (b), we have calculated the value of the normalized parallel velocity $\beta_z = P_z/\gamma$ during the oscillating time interval before the particle gets into the second acceleration stage. We found β_z to be in the range 0.94 < β_z < 0.96. Since the phase velocity of the well is $\beta_\xi = v_\xi / c = (\bar{\omega}_1 - \bar{\omega}_2)/(\bar{k}_1 + \bar{k}_2) = (0.4 - 0.095)/(0.4 + 0.095) = 0.9536$, one realizes indeed the validity, in principle, of the scheme. We present in Fig. 2 the change in time of the action P_ϕ and the momentum $\bar{P}_z$ for two different sets of initial values of the particle. One clearly sees the operation of the cascading acceleration mechanism.

References

[1] Y. Gell and R. Nakach: Physics Letters A **207**, 342 (1995).
[2] Y. Gell and R. Nakach: Physical Review E **55**, 5915 (1997).