ON INTERACTION OF STRONG LHW WITH ELECTRONS

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Abstract. The diffusion of electrons in velocity space induced by a spectrum of electrostatic waves has been numerically investigated in [1]. A strong disagreement between the numerically obtained diffusion coefficient and the quasilinear one has been found at electric field amplitudes exceeding $\approx 10^4$ V/m, narrow spectra and short (spatial) lengths of the interaction region, which are however usual in Lower Hybrid Heating and Current Drive experiments in tokamaks. The diffusion coefficient has been obtained as a statistical average of the velocity spread after a single transit through a spatially limited RF field region. In the present paper, we present a justification of this approach. In particular, stochastization of particle velocities due to spatial discontinuity of RF field is demonstrated by a simple analytical model.

The quasilinear theory (e.g., [2]) is based on two important assumptions, namely, on the Landau absorption of the applied spectrum, and on the approximation of chaotic phases of waves in the spectrum.

The first assumption requires that the wave with the phase velocity $v_{ph} = \omega/k$ (where ω and k is the frequency and wave number of the corresponding spectral line) interacts only with particles with velocity $v_p = v_{ph}$. Consequently, if the spectrum is determined by its broadeness $2\Delta v_{ph}$ and lies in the region $v_{ph,0} \pm \Delta v_{ph}$, the spectrum interacts resonantly with particles in their velocity region $v_{p,0} \pm \Delta v_{ph}$, where $v_{p,0} = v_{ph,0}$.

The second assumption makes the wave-particles interaction diffusive with the well known quasilinear diffusion coefficient D_{ql}. The justification of the assumption itself was many times discussed; contemporary, the Chirikov model of overlaping of resonances, a generic model of nonlinear systems in the regime of the deterministic chaos, seems to be generally accepted by the plasma community. Chirikov model works even in the case of a discrete spectrum.

Lower hybrid waves interaction with a plasma is considered as an almost text-book example of the applicability of the quasilinear approximation (QLA). Especially the electrostatic form of LHW enables simple algorithmization of the interaction. For LHW heating in tokamaks, the assumption of the averaging of the effect of the wave-plasma interaction over magnetic surfaces for the cases, when the LHW conus represents a well-defined geometric object, is generally applied.

The system of the quasilinear equations was developed by means of a perturbative analysis. Consequently, the validity of this approximation may be called into question for high field amplitudes. In [1], we have shown that if realistic values of the electric field within Lower Hybrid cones are considered instead of the values averaged over a magnetic surface, the resulting diffusion coefficient obtained by direct numerical simulation (DNS) becomes smaller and the interaction region broadens already at field amplitudes about 10 kV/m (cf. Fig. 1 as an example). Here, the DNS diffusion coefficient on the RF field area Γ is obtained as

$$D_\Gamma = \left\langle \frac{(v_f - v_{avg})^2}{2\tau} \right\rangle, \qquad (1)$$

where v_f is the final velocity of the electron when leaving the RF area Γ; τ is the time of travel through the region, the average $<>$ is taken over a sufficient number of samples for the same initial velocity v_0 but random choice of the wave phases; $v_{avg} =< v_f >$ is the average final velocity for the set of samples. For details, see

This formulation implicitly contains an assumption that there is no correlation between subsequent passes of a particle through the RF region, so that a statistical average based on a single transit is justified.

To avoid a chance of accumulated numerical errors, we formulate a simple, 1D, analytically tractable model, considering only a single wave which is however localized in space, and a particle orbiting toroidally on a circle with radius $R_0 + r_0$, entering alternatively the region of RF field of the length L and then the region without RF field of the length $2\pi(R_0+r_0)-L$. Both types of the dynamics are therefore integrable. In the RF field free region the motion is simply inertial, in the RF field of the one wave the motion is identical with the mathematical pendulum and can be described by the system of action-angle coordinates J, w, where

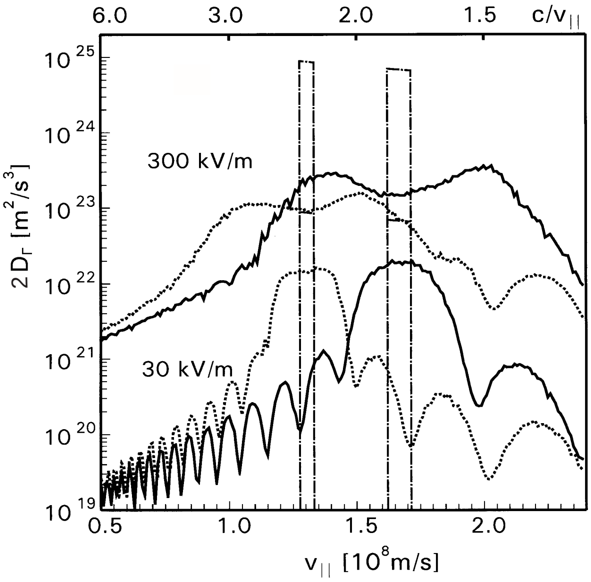

Figure 1. A comparison between the quasilinear (chain lines) and DNS diffusion coefficients of D_Γ vs. $v_\parallel$ for f = 3.7GHz, RF segment length 0.3m, E_0 as indicated, and $\Delta N_\parallel$ = 0.1, $\overline{N_\parallel}$ = 1.8 (solid) and 2.3 (dotted).

$$J = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{2mH + 2me\varphi_0(1-2\sin^2\frac{k_\parallel Q}{2})}dQ; \quad w = \frac{\partial W}{\partial J} \tag{2}$$

and W is the solution of the Hamilton-Jacobi equation

$$H = \frac{1}{2m}(\frac{\partial W}{\partial Q})^2 - e\varphi_0\cos k_\parallel Q. \tag{3}$$

Here H is the Hamiltonian of a particle in the wave-phase velocity frame

$$H = \frac{1}{2m}P^2 - e\varphi_0\cos k_\parallel Q \tag{4}$$

with the transformation to the original coordinate z and momentum p_z

$$P = p_z - m\frac{\omega}{k_\parallel}; \quad k_\parallel z = k_\parallel Q + \omega t \tag{5}$$

and with the corresponing transformation between the new Hamiltonian H and the old Hamiltonian H_0

$$H = H_0 - \frac{P}{k_\parallel}\omega - \frac{m\omega^2}{2k_\parallel^2}. \tag{6}$$

The procedure renders possible to follow particles, trapped or untrapped in the wave, and also the change from trapping to untrapping regime (and vice versa). Inside the RF region, the particle performs the integer number of periods, and the rest of the period. Therefore, for the determination of the dynamics, it is necessary to find only this integer and the effect of the rest period (this would be impossible to perform already for two waves). The numerical part of the procedure is just the evaluation of the complete and incomplete elliptic integrals, which can be done with sufficient accuracy.

Some results are presented in Figs. 2 to 4. For all cases, we take ω = 2.32$\times$10^{10} s^{-1} and k = 1.39$\times$10^{2} m^{-1}.

Figs. 2a,b and Fig. 3a,b briefly present two different regimes. The parallel axis labels time, in which the particle just enters into the RF segment, and the vertical coordinate declares its velocity at this moment (the lines in the figures are only auxiliary). Figs. 2 represent the case when the RF field is distributed homogeneously aroud the whole circumference, Figs. 3 the case, when the RF field is present only in a segment of the circumference.

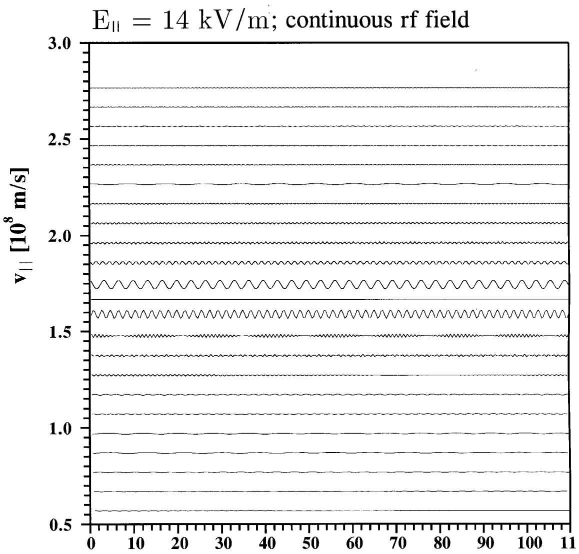

Figure 2 a)

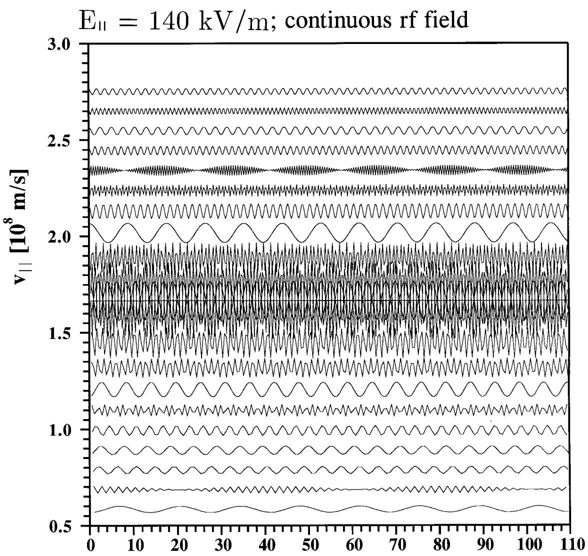

Figure 2 b)

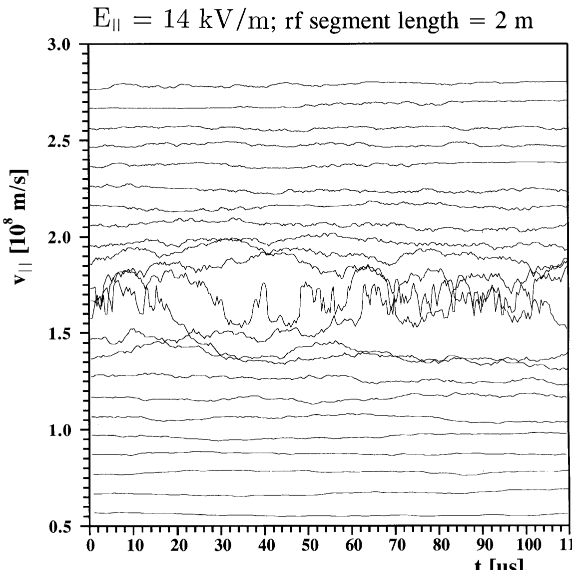

Figure 3 a)

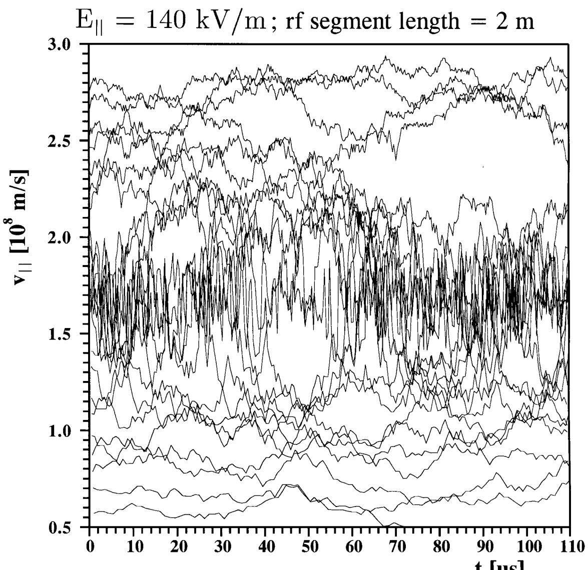

Figure 3 b)

We see that in the first case 2a,b, no chaos appears, regardless of the wave amplitude (this is obvious because such system is integrable). The comparison between Figs. 2 and 3 shows that the mechanism of the generation of chaos consists in the combination of segments with and without RF field. Of course, in the latter case, chaotic changes of velocities are more significant for larger RF fields.

To demonstrate the influence of the length of the RF segment, we present a set of figures, Figs. 4a-d for lengths L =0.005 m, 0.015 m, 0.03 m, and 20 m, and for the potential Φ = 10 V. Since the wave length λ is λ = 4.5$\times$10^{-2} m, we see that threshold of the stochasticity appears for $L \approx \lambda$. There is no dramatic change for longer L (cf. Fig. 4d).

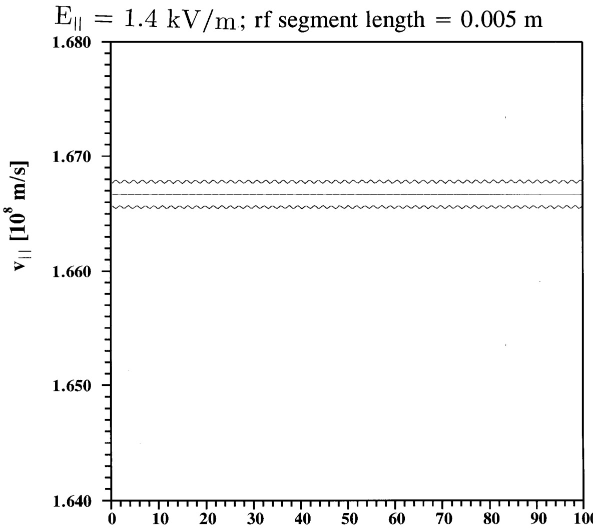

Figure 4 a)

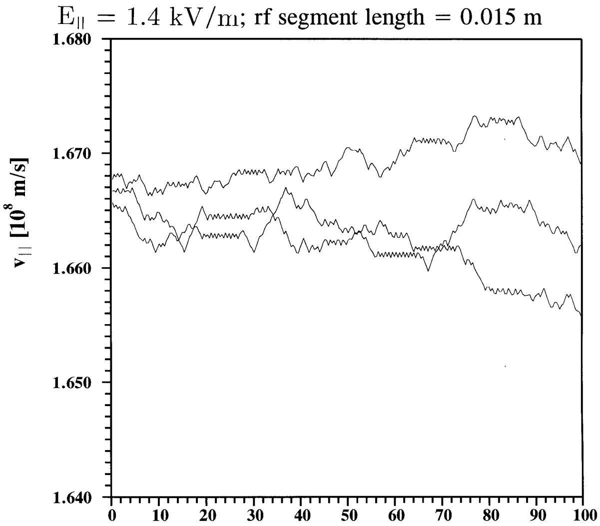

Figure 4 b)

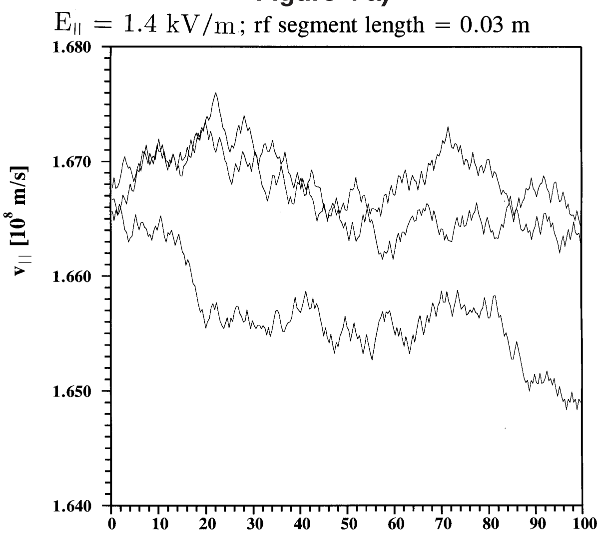

Figure 4 c)

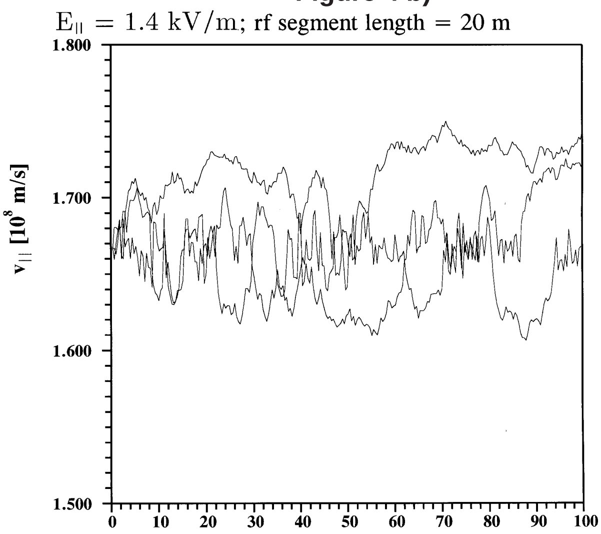

Figure 4 d)

To conclude, we have shown that the alternation of segments with and without RF field can be a source of the chaotic particle motion. Consequently, the original Chirikov mechanism, requiring a set of waves, cannot be directly applied. There is probably an interesting connection with recent paper of Fuchs et al. [3]; see also our paper [4]. Here, the effect of a limited grill length results in appearing of higher harmonics in $k_{||}$. Among these harmonics, Chirikov stochasticity can appear and, consequently, the acceleration of particles also. Nevertheless, for larger amplitudes, strong changes of velocities appear already after rather few periods, and it is questionable, whether the simple Fourier expansion is applicable.

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