A NEW INSIGHT FOR THE QUASILINEAR DIFFUSION COEFFICIENT FOR THE LOWER HYBRID CURRENT DRIVE

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Abstract

The spectral gap problem arises when one tries to explain the experimental problem using the standard self-consistent and non self-consistent quasilinear theory for the driven noninductive radio frequency current. We have revised the well known lower hybrid current drive (LHCD) spectral gap problem, calculating the quasilinear diffusion coefficient using a resonance function (smooth resonance border) rather than the Dirac's Delta Function (sharp resonance border). A proper calculation of the quasilinear diffusion coefficient, taking the complete form as given by Swanson (1989), has produced a good size wing for the diffusion coefficient in the lower side of the wave spectrum. This wing produces a considerable difference on the value of the driven current approaching the experimentally measured values. Our calculation may be one of possible ways of solving the spectral gap problem for lower hybrid current drive which has been under consideration since last decade. This problem is not yet solved completely (Ide et al. 1992, Watari 1993, Ide et al. 1994).

1. Quasilinear Fokker-Planck Equation

We consider the deposition of energy and momentum of a spectrum of slow electromagnetic waves propagating in a non-equilibrium hot magneto-active plasma, with a uniform magnetic field $\vec{B}$, in a weak turbulence and weak collisional regime. If we also assume that the electrostatic aproximation is valid, that is, $\vec{E} = -i\vec{k}\Phi$, then it can be described by the quasilinear Fokker-Planck equation such as given by Akhiezer (1975), Swanson (1989):

$$\frac{\partial f_{\alpha 0}}{\partial t} = \frac{\partial}{\partial \vec{v}} \cdot \boldsymbol{D} \cdot \frac{\partial f_{\alpha 0}}{\partial \vec{v}} + \left.\frac{\delta f_{\alpha 0}}{\delta t}\right|_{coll} \tag{1}$$

with

$$\boldsymbol{D} = \frac{q_\alpha^2}{m_\alpha^2} \sum_{n=-\infty}^{+\infty} \frac{1}{V} \int \frac{d^3k}{(2\pi)^3} \frac{\gamma_k J_n^2(\lambda) \|\Phi_k\|^2}{(\omega_{rk} - n\epsilon\omega_{c\alpha} - k_\parallel v_\parallel)^2 + (\gamma_k)^2} \vec{b}_{n,k}\vec{b}_{n,k} \tag{2}$$

where, $\boldsymbol{D}$ is the quasilinear coefficient for electrostatic slow waves, α indicating the particle species, $f_{\alpha 0}$ is the slow time dependent equilibrium distribution function for species α, and the last term of (1) is the small particle-particle collision term; $\vec{k}$ is the wave vector assumed that it lies on the x-z-plane, $\vec{B} = B_0\vec{e}_z$, $k_\perp = k_x$ is chosen, $k_\parallel \equiv k_z$, (x, y, z) referring to the Cartesian coordinates, q is the particle charge and m particle mass, V is the volume, $\vec{e}_u$ is the unit vector in the direction of a vector $\vec{u}$, $J_n(x)$ is the Bessel's function of order n, $\|\Phi_k\|^2$ is the wave power spectrum; $\epsilon = q/|\,q\,|$ is the charge sign, $\vec{b}_{nk} = (n\epsilon\omega_{c\alpha}/v_\perp)\vec{e}_{v_\perp} + k_\|\vec{e}_z$, $\vec{E}_k = -i\vec{k}\Phi_k$, $\omega_{c\alpha} = e_\alpha B_0/m_\alpha c$ is the cyclotron frequency, $\lambda_\alpha = k_\perp v_\perp/\epsilon\omega_{c\alpha}$, $\omega_k = \omega_{rk} + i\gamma_k$ is the complex wave frequency with γ_k indicating the damping or growth rate, $v_\| = \omega_{rk}/k_z$.

$$k_\perp^2 \sim k_\|^2(-K_3)/K_1 \tag{3}$$

is the relation between the $k_\|$ and $k_\perp$ where $K_3 = K_{33}$ and $K_1 = K_{11}$ and the components of the cold plasma dielectric tensor $\boldsymbol{K}$, with $K_3 \sim 1 - \omega_{Pe}^2/\omega_{rk}$ and $K_1 \sim 1 + \omega_{Pi}^2/(\omega_{ci}^2 - \omega_{rk}^2) + \omega_{Pe}^2/(\omega_{ce}^2 - \omega_{rk}^2)$.

2. Hot Plasma Dispersion Relation for Electrostatic Waves

The quasi-electrostatic waves dispersion relation in a hot magnetoactive plasma can be written, as in cold plasma

$$\vec{k} \cdot \boldsymbol{K} \cdot \vec{k} = 0 \tag{4}$$

where $\boldsymbol{K}$ is the dielectric tensor. This relation is valid when $|\,\vec{E}_\|\,| >> |\,\vec{E}_\perp\,|$. This relationship is valid for the hybrid (upper or lower) resonances.

For the warm plasma and the magnetic field in the z-direction, we can write

$$k_\perp^2 K_{11} + 2k_\perp k_\| K_{13} + k_\|^2 K_{33} = 0 \tag{5}$$

For the case of the lower-hybrid waves, and considering zero fluid velocity, $V_{\|\alpha} = 0$, we can simplify the dispersion relation to:

$$k_\perp^2 + k_\|^2\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right) + i\frac{\omega_{pe}^2}{v_{\theta\|e}^2}\sqrt{\pi}\eta_{0e}e^-\eta_{0e}^2 + \frac{\omega_{pe}^2 k_\perp^2}{\omega_{ce}^2 - \omega^2} + \frac{2\omega_{pi}^2}{\omega_{ci}^2 - \omega^2}\left[\frac{h(\lambda_i)\omega_{ci}^2)}{v_{\theta i}^2}\right] = 0 \tag{6}$$

where $\eta_{0e} = (\omega/k_\|)/\sqrt{2}v_{\theta\|e}$, $h(\lambda_i) = e^{-\lambda_i}I_1(\lambda_i)$ and $\lambda_i = k_\perp^2 v_{\theta i}^2/\omega_{ci}^2$, e referring to electrons and i to ions, v_θ is the thermal velocity, and $I_1(x)$ is the Modified Bessel's function of order 1.

From this equation we can get the Landau damping for the case of $\omega/k_\| > 2v_{\theta e}$:

$$\gamma_k(k) = \frac{\sqrt{\pi}\omega^4}{\sqrt{8}\,|\,k_\|\,|\,v_{\theta e}^2 A}e^{-\frac{\omega^2}{2(k_\| v_{\theta e})^2}} \tag{7}$$

where $A = k_\|^2 v_{\theta e}^2 + 2h(\lambda_i)\omega_{ci}^2$.

For the case of our lower hybrid waves, the middle term of equation (5), K_{13}, is negligible, so the relation written in equation (3) for the cold plasma case is still valid. This means that in the $\vec{k}$-space the resonant wave is bound to the neighborhood of the plane, call it ζ-plane, given by equation (3), that is to say that in the direction perpendicular to this plane the wave spectrum is a δ-function. With the assumption that the wave spectrum lies on the x-z-plane, the problem calculating the quasilinear diffusion coefficient (2), becomes an integration on one dimension only, namely on the intersection of ζ-plane and $y = 0$-plane, called σ-axis. This can be performed rather easily.

Rotating the reference system around y-axis by an angle θ_0 given by the relation $\tan\theta = (-K_3/K_1)^{1/2}$ and assuming that in the direction perpendicular to this angle the spectrum is a δ-function and fitting $\gamma_k(k)$ by a polynominal of order 6, we can perform the analytical integration of equation (2) using the software $Mathematica$, version 3.0. The result is shown in Figure 1.

3. Numerical Technique

We have developed a FORTRAN code to solve the 2-dimensional Fokker-Planck equation, equation (1), in $(v, cos\theta)$-space, non-relativistic, with the collisional term as given by Landau. We have followed the technique used by Schoucri and Shkarofsky (1993,1993) adding the quasilinear term as mentioned in the previous section. The details of the code will be written in another context.

4. Results and Conclusions

In this study, we have adopted the JT-60 parameters, see Ide (1992,1994): density $(0.317\ 10^{14} cm^{-3})$, electron temperature $(T_e = 5KeV)$, magnetic field strength B_0 $(80KGauss)$, $\omega_{LH}(2\pi 3.7GHz)$, range for the resonant velocity $(3.5 \leq v/v_{\theta e} \leq 5.5)$, and $\gamma_k(k)$ as indicated in equation (7). Figure 1 shows the "sharp" and "wing" profiles for the quasilinear coefficient $\mathbf{D}_{LH}$.

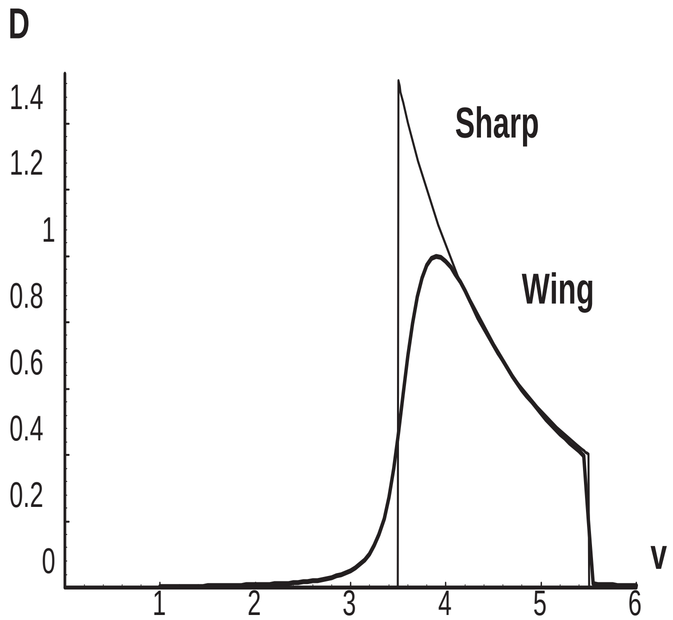

Figure 1. Sharp and winged profiles for the quasilinear coefficient.

Using the 2-dimensional code developed by us, calculated the change in the driven current and the power density deposited into plasma effected by the introduction of the "wing" in the quasilinear coefficient as compared to the case without the "wing", that is, a "sharp" profile obtained taking the δ-function approximation for the resonant function. The effect is very large as seen in the following table, Table 1:

Model	Density	Current	Power Deposition	Efficiency
Sharp	1.00745	0.038439	0.02049	1.8761
Wing	1.02570	0.184246	0.10779	1.7093

Table 1. Change observed in the driven current when the wing is introduced

It is important to note that the difference between the sharp and the winged diffusion coefficient profile is small. However this small "wing" is enough to bring a lot of electrons into the resonant region because it is located in the region of much larger particle density - a Gaussian distribution at about $v/v_{\theta e} = 3.5$.

If we take the plasma cross section of JT-60 and assume that only 5% of its area is used for the generation of the driven current, we still have about 2.5 MA of driven current. This value approaches that of the experimentally observed values for the case of $T_e = 2$ KeV.

We conclude that the δ-function approximation for the resonant function in the lower hybrid current drive parameters is not a proper approximation, and that the correct use of the resonant function results in a large enhancement of the driven current, approaching the experimentally observed values.

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