DIFFRACTION EFFETCS AND THE SPECTRAL GAP FOR LOWER HYBRID CURRENT DRIVE IN LOW-DENSITY, HIGH-SAFETY-FACTOR AND HIGH-ASPECT-RATIO TOKAMAK PLASMAS

P. Rodrigues and J.P.S. Bizarro

Centro de Fusão Nuclear, Associação Euratom-IST,
Instituto Superior Técnico, Lisbon, Portugal

Lower hybrid (LH) waves are widely employed, at present, in experiments on plasma heating and current drive in tokamaks. In spite of the good results achieved with this technique in the field of non-inductive current generation, certain of its features remain to be fully understood, so that the extrapolation to large, reactor-sized tokamaks still involves some risks. The main unsolved issue is the so-called spectral gap problem, which is related with the fact that a LH spectrum launched by a typical antenna array has a very-high parallel phase velocity. Hence, the power it carries can only be absorbed by the plasma after significant slowing down, in order for the phase velocity to become comparable with the electron thermal speed. It is precisely the physical mechanism which provides this slowing down of the phase velocity that has not yet been made clear.

According to the most accepted views, the main cause responsible for LH spectrum distortion and shifting is the poloidal inhomogeneity in the refractive index of LH waves propagating in tokamak plasmas. Likewise, several other mechanisms have been proposed, reflecting the complexity of the problem and the lack of a universal solution. Notwithstanding the importance of such mechanisms in a real experiment, their effects on LH spectra must always be set against the evolution predicted by the LH wave equation itself [1]. In order to avoid the difficulties related with solving the full wave equation in the LH frequency range, multidimensional WKB methods can be used [1,2]. Indeed, they allow a wave description along the direction perpendicular to the group velocity, providing thus an alternative way for wave effects to act on the LH spectrum evolution, in addition to the phenomena characteristic of classical geometrical optics.

However, the introduction of such wave effects does not always lead to sufficient slowing down of LH spectra, as shown here for LH experiments performed in TRIAM-1M.

These plasmas are representative of low-density, high-safety-factor and high-aspect-ratio tokamak plasmas for which no plausible explanation has yet been found for the spectral-gap problem [3,4]. To assert this, we compute the evolution of a given LH wave spectrum for typical TRIAM-1M plasma parameters, firstly by means of a multidimensional WKB method, or beam tracing, and then by using standard ray-tracing techniques. Both computations are done for the LH slow (electrostatic) mode in toroidal geometry, with a simplified magnetic equilibrium in which magnetic surfaces lies on concentric torii, and also without accounting for power absorption.

Beam-tracing calculations are carried out following the general prescription given in [1]. We compute the reference ray parameters a, θ, S_0 and $S_1 = m/n$ as functions of the parametric variable τ, integrating the differential system (29) in [1]. These parameters are, respectively, the radial and poloidal positions, the eikonal function and the poloidal component of its gradient, evaluated at the reference ray. The latter component can be also written as the ratio between the poloidal and toroidal wave numbers. Simultaneously, we integrate the differential system (30) in [1], obtaining the functions $\nu_1(\tau)$ and $S_2(\tau)$, which describe the launched wave-packet width and the wave-front curvature, respectively. In both systems, initial conditions are extracted from the boundary condition for the electrostatic-potential distribution, $\Phi(a = a_0,\theta)=\Phi_0(\theta)$, launched by the LH antenna at the tokamak minor radius a_0. With these six functions, we are able to evaluate the asymptotic potential distribution $\Phi(a,\theta)$, as well as its spectral distribution over radial surfaces, namely $\Phi(a,m) = 1/\sqrt{2\pi}\int_{-\infty}^{+\infty}\Phi(a,\theta)\exp(-im\theta)d\theta$, following expressions (33) and (37) in [1].

Now, we must concern ourselves with the problem of establishing how the wave-packet power density is distributed over the parallel index domain $N_{\|} = ck_{\|}/\omega$. To do this, we chart the quantity

$$P(a,N_{\|}) = \vec{E}(a,m)\cdot\vec{E}^*(a,m)\Big/\int\vec{E}(a,m)\cdot\vec{E}^*(a,m)dN_{\|} \qquad (1)$$

which is close to the normalised spectral wave energy density. Here, $\vec{E}(a,m) = \{-\partial\Phi(a,m)/\partial a; -im\Phi(a,m)/a\}$ stands for the electric field vector and $\vec{E}^*$ for its complex conjugate, with $N_{\|}$ being given by

$$N_{\|} = \frac{cB_\theta(a,\theta)}{\omega a B(a,\theta)}n\left[S_1 + \frac{aB_\varphi(a,\theta)}{rB_\theta(a,\theta)}\right] \qquad (2)$$

with $r = R_0 + a\cos\theta$, where R_0 is the tokamak major radius. We should notice that, although expression (2) has an explicit dependence on the poloidal coordinate θ, this could be ignored due to the high-aspect-ratio limit, $R_0/a_0 \rightarrow \infty$. Therefore, $N_{\|}$ can be considered roughly constant over each radial surface, depending solely on a and S_1.

In order to make a valid comparison between beam-tracing and ray-tracing results, we must chart a quantity similar to the one defined in (1). As the electric-field computation along the whole ray-tracing integration can turn itself into a cumbersome task, we separate the launched potential distribution, $\Phi(a = a_0,\theta) = \Phi_0(\theta)$, into a discrete series of rays initialised at ($a = a_0$, $\theta = \theta_j$) with $j = 1,2,\ldots,N_j$. Each one of these rays carries a certain amount of power density, given by

$$\tilde{P}_j(a_0,\theta_j) = \left. \frac{\vec{E}(a_0,\theta) \otimes \vec{E}^*(a_0,\theta)}{\int \vec{E}(a_0,\theta) \otimes \vec{E}^*(a_0,\theta) d\theta} \right|_{\theta = \theta_j} \qquad (3)$$

where $\otimes$ denotes an internal product followed by a convolution in the variable θ. The electric field $\vec{E}(a_0,\theta)$ is calculated as the negative gradient of the standard trial function for the electrostatic potential, $\Phi(a,\theta) = A(a,\theta)\exp[inS(a,\theta) + in\varphi]$ with $\left| n\vec{\nabla}[S(a,\theta)+\varphi] \right| >> \left| \vec{\nabla} \ln A \right|$, evaluated at $a = a_0$. Together with the fact that $n\vec{\nabla}S(a_0,\theta) = k_{a_0}(\theta)\vec{u}_a$, since $k_{\theta_0}(\theta) = 0$, with $k_{a_0}(\theta)$ established by the dispersion relation, and further choosing $S(a_0,\theta) + \varphi = 0$, then $\vec{E}(a_0,\theta) \cong -i\Phi_0(\theta)\left[k_{a_0}(\theta)\vec{u}_a + n/r\,\vec{u}_\varphi\right]$.

Remembering that, in the absence of absorption, the power density carried by each ray is kept constant, we apply the standard ray-tracing equations, to propagate the LH wave power density, taking a as the parametric variable. Hence, we are able to define for every a a new power density distribution

$$\tilde{P}_j[a,\theta_j(a)] \overset{def}{=} \tilde{P}_j(a_0,\theta_j) \qquad (4)$$

After performing a discrete Fourier analysis over the discrete variable θ_j, we are left with a quantity

$$\tilde{P}(a,N_{\|}) = Z^{-1} 1/\sqrt{2\pi} \sum_j \tilde{P}_j[a,\theta_j(a)] \exp[-im\theta_j(a)] \Delta\theta_j \qquad (5)$$

Z being a normalising factor, which can be matched directly against $P(a,N_{\|})$ in (1), and $N_{\|}$ as in (2).

The two quantities (1) and (5) are plotted in Fig. 1, where it is easily seen the two approaches produce rather similar results, except for the central region of the plasma. Indeed, ray tracing predicts a focusing of the wave packet near the plasma centre, whereas beam tracing effectively handles the diffraction broadening that occurs when the width of the wave packet becomes comparable with the wavelength.

However, as far the spectral-gap problem is concerned, neither approach is able to account for sufficient changes in the LH spectrum. According to estimates established for minimal Landau damping in plasmas typical of TRIAM-1M [3,4], it is necessary that significant amounts of power can be transferred to $N_{\|} \gtrsim 4.9$, which clearly is not the case. In fact, even for beam tracing, which is the most favourable approach, one hardly gets meaningful power density above $N_{\|} \gtrsim 2.5$, the latter value being approximately half the minimum required for the spectral gap to be bridged.

In conclusion, we may say that, very much like geometrical optics, diffraction effects cannot be held responsible for sufficient modification of LH wave spectra in order to solve the spectral-gap problem in low-density, high-safety-factor and high-aspect-ratio tokamak plasmas. The basic reason for this is that the poloidal inhomogeneity in this tokamaks is very weak [3,4], so a LH spectrum launched with $m \approx 0$ remains essentially unchanged [2].

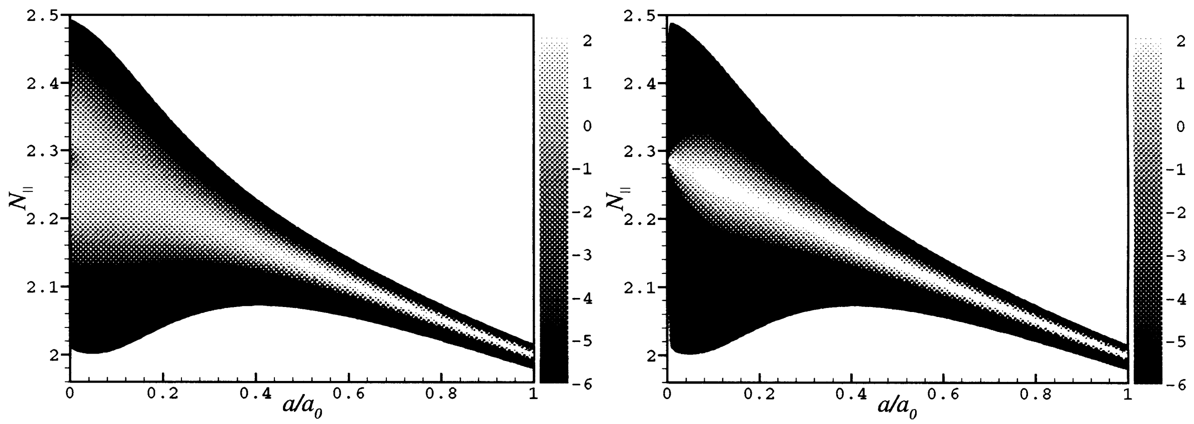

Fig. 1. *Plots of* $\log_{10}[\, P\,(a/a_0,N_{\|})\,]$ *(left) and* $\log_{10}[\, \tilde{P}\,(a/a_0,N_{\|})\,]$ *(right) for the launched electrostatic potential* $\Phi_0(\theta) = \exp[\,-1/2\,(\theta/\sigma_\theta)^2\,]$, *with* $\sigma_\theta = 16.48^{\mathrm{o}}$ *the antenna half-width. Other parameters are* $f = 8.2\mathrm{GHz}$, $B_0 = 7\mathrm{T}$, $a_0 = 0.12\mathrm{m}$, $R_0 = 0.84\mathrm{m}$, $n_e = 2{\times}10^{19} - 1.25{\times}10^{21}\, a^2\ (\mathrm{m}^3)$ *and* $q = 1 + 9.72{\times}10^2\, a^2$. *At launching, the spectrum has* $N_{\|} = 2.0$ *for* $\theta = 0$.

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