FAST MAGNETOSONIC AND FAST ALFVÉN WAVES IN MAGNETIZED ELLIPTIC PLASMAS

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Besides computational results, the circular plasma cylinder model was among the most popular for the purpose of theoretical investigation of fast magnetosonic waves (FMSW) and fast Alfvén waves (FAW). As a lot of devices have an elliptic plasma cross-section it is interesting to investigate the influence of ellipticity at RF field distribution in plasma. Earlier this problem was considered in [1], where the small deviation from a circle was supposed.

The paper presented concerns with analytical studies of the excitation and propagation of FMSW and FAW in an elliptic plasma cylinder with axial magnetic field. Also azimuth surface waves (ASW) [2] were considered. As a first step we studied the excitation of eigenmodes of elliptic plasma cylinder with axial magnetic field $B_0 \parallel Oz$. We used the elliptic cylinder coordinate system (ξ, φ, z). It consists of a system of confocal ellipses $\xi = \xi_i$ (ξ - analog of a radial coordinate) and hyperboles $\varphi = \varphi_i$ (φ - analog of an azimuth coordinate). The focal distance of all ellipses ($2h$) is equal $2h = 2a\varepsilon = 2a / ch\xi$, $\varepsilon = \sqrt{1 - b^2 / a^2}$. Here a - large (b - small) axes of an ellipse, ε - eccentricity, $\varepsilon \to 0$ $(\xi \to \infty)$ corresponds to a circle, $\varepsilon \to 1$ $(\xi \to 0)$ corresponds to a narrow slot. Let's notice that even a small deviation from a circle ($a = 100cm,\ b = 99cm$) leads to $\varepsilon = 0{,}14$ and for JET device $\varepsilon = 0{,}7$. We assumed that plasma consists of two species of ions and electron to ion mass ratio equals to zero ($E_z = 0$). The RF field takes the form $\vec{B} \sim \vec{B}(\xi, \varphi) e^{ik_{\parallel} z}$. Then we have for B_z component

$$\frac{\partial}{\partial \xi}\left(\frac{1}{N_\perp^2} \frac{\partial B_z}{\partial \xi} \right) - i \frac{\partial}{\partial \xi}\left[\frac{\varepsilon_2}{N_\perp^2 \left(\varepsilon_1 - N_\parallel^2 \right)} \right] \frac{\partial B_z}{\partial \varphi} + \frac{1}{N_\perp^2} \frac{\partial^2 B_z}{\partial \varphi^2} + \frac{\omega^2 h^2}{c^2} \left(\mathrm{ch}^2 \xi - \cos^2 \varphi \right) B_z = 0 \qquad (1)$$

here $$N_\perp^2 = N_\perp^2(\xi) = \frac{\left(\varepsilon_1 - N_\parallel^2 \right)^2 - \varepsilon_2^2}{\left(\varepsilon_1 - N_\parallel^2 \right)}, \qquad \varepsilon_1 = 1 - \frac{\omega_{p1}^2}{\omega^2 - \omega_{c1}^2} - \frac{\omega_{p2}^2}{\omega^2 - \omega_{c2}^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2},$$

$$\varepsilon_2 = -\frac{\omega}{\omega_{c1}} \frac{\omega_{p1}^2}{\omega^2 - \omega_{c1}^2} - \frac{\omega}{\omega_{c2}} \frac{\omega_{p2}^2}{\omega^2 - \omega_{c2}^2},$$

$\omega_{p1}, \omega_{p2}, \omega_{pe}$ - ions and electron plasma frequencies, $\omega_{c1}, \omega_{c2}, \omega_{ce}$ - ions and electron cyclotron frequencies.

In the case of homogeneous plasma the equation (1) allows a separation of variables $B_z(\xi,\varphi)=u(\xi)\cdot v(\varphi)$ and we have two Mathieu's equations for field in plasma

$$\frac{\partial^2 u}{\partial \xi^2}-(C-2q\cdot \mathrm{ch}\,2\xi)\cdot u=0\,, \qquad \frac{\partial^2 v}{\partial \xi^2}-(C-2q\cdot \mathrm{ch}\,2\xi)\cdot v=0 \tag{2}$$

where $q=(N_\perp \omega h/2C)^2=(h/2\lambda_\perp)^2$ is main parameter of the problem. Let's assume for a simplicity that the plasma is surrounded by metal wall with $\xi=\xi_0$. (Generalization to the plasma - vacuum layer - metal wall is straightforward.). So the boundary condition is

$$-i\varepsilon_2\left.\frac{\partial B_z}{\partial \varphi}\right|_{\xi=\xi_0}+\left(\varepsilon_1-N_\parallel^2\right)\left.\frac{\partial B_z}{\partial \xi}\right|_{\xi=\xi_0}=0. \tag{3}$$

The solution of Eq.(2) can be written as

$$B_z(\xi,\varphi)=\sum_{n=0}^{\infty}C_m\,\mathrm{Ce_m}(\xi,q)\mathrm{ce_m}(\varphi,q)+\sum_{n=0}^{\infty}S_m\,\mathrm{Se_m}(\xi,q)\mathrm{se_m}(\varphi,q). \tag{4}$$

Here $\mathrm{ce_m},\mathrm{se_m}$ - ordinary and $\mathrm{Ce_m},\mathrm{Se_m}$ - modified Mathieu's functions [3]. At small values of $q/4$ it is possible to separate one azimuthal harmonic in (4) (neighbouring harmonics are the order of $q/4$). Then using (3) we get the dispersion equation of an elliptical cylinder eigenmodes

$$\frac{\varepsilon_2^2\cdot m^2}{\left(\varepsilon_1-N_\parallel^2\right)^2}\frac{\mathrm{Ce_m}(\xi_0,q_i)}{\mathrm{Ce'_m}(\xi_0,q_i)}=\frac{\mathrm{Se'_m}(\xi_0,q_i)}{\mathrm{Se_m}(\xi_0,q_i)}\ . \tag{5}$$

which defines the eigenvalues q_i. Taking three principal terms in expansion of $\mathrm{Ce_m},\mathrm{Se_m}$ through Bessel functions we obtain from (5) (see Fig.1, curve "qtheo")

$$\sqrt{q_l}=\frac{(k_{\perp l}r)_{cir}}{2}\varepsilon\left(1+\varepsilon^2/4\right), \quad \text{or} \quad (k_{\perp l}a)_{el}=(k_{\perp l}r)_{cir}\left(1+\varepsilon^2/4\right), \tag{6}$$

where $(k_{\perp l}r)_{cir}$ - solutions of Eq.(5) for circular cylinder. The relation (6) implies the completely remarkable fact: modification of plasma cylinder form at preservation of cross-sectional area does not change the eigenvalues $k_{\perp l}$ (or eigenfrequencies $\omega_l^{(0)}$)! Beyond the limits of expansion Eq.(5) was solved numerically (Fig.1, curve "q1mode'). Analyzing (4) together with (3) we can conclude that at $q\geq 1$ it is a gang of elliptic harmonics. Thus the solution of dispersion equation we get from equality to zero of determinant which originate from (3) (Fig. 1, curve "qdet"). Comparing curves "q1mode' and "qdet", we see that Eq.(5) gives a right solution even outside the limits of applicability.

Now we study azimuthal dependence of obtained solutions. Formally this dependence differs completely from running on an azimuth waves in circular cylinder. (We have for (4) $S_m=i\dfrac{\varepsilon_2}{\varepsilon_1-N_\parallel^2}\cdot m\cdot C_m\dfrac{A_m^m}{B_m^m}\dfrac{\mathrm{Ce_m}(\xi,q)}{\mathrm{Se'_m}(\xi,q)}$ instead of $S_m=\pm i\cdot C_m\dfrac{A_m^m}{B_m^m}\dfrac{\mathrm{Ce_m}(\xi,q)}{\mathrm{Se_m}(\xi,q)}$ for running waves, where A_m^m, B_m^m - coefficients of Fourier expansion ce_m ,se_m on $\cos l\varphi$ and $\sin l\varphi$ accordingly.) The dependencies on ε of normalized to basic harmonic satellite azimuthal harmonics amplitudes ($m = +1$ is basic harmonic) are shown in Fig.2a, 2b for the first and seventh radial modes. The azimuthal behaviour of wave field is shown in Fig.3 (a-1st mode, b,c-7th mode). For the first radial mode there is no an essential modification of an azimuthal structure of oscillations as compared to circular cylinder. But for the seventh radial mode the azimuthal structure cardinally changed - there was a transition of field maxims field to small axes of ellipse.

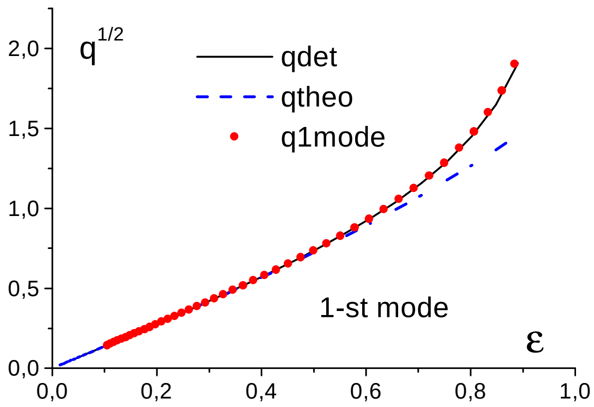

Fig. 1.

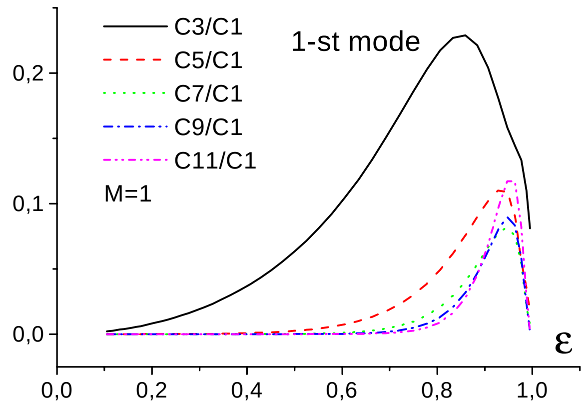

Fig. 2a.

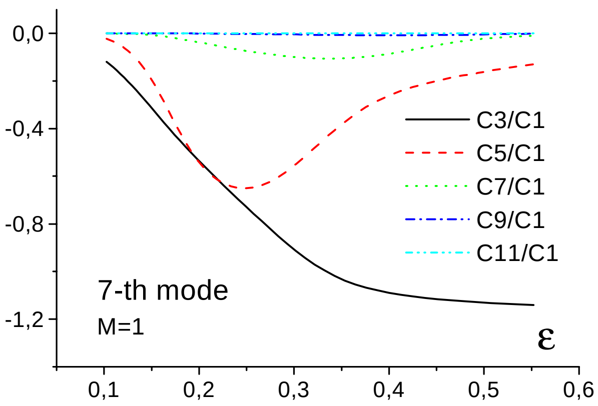

Fig. 2b.

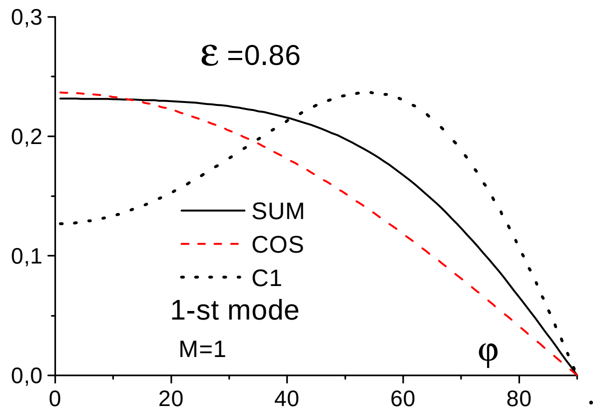

Fig. 3a.

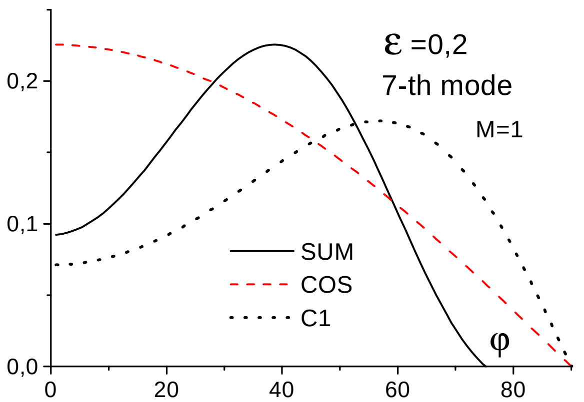

Fig. 3b.

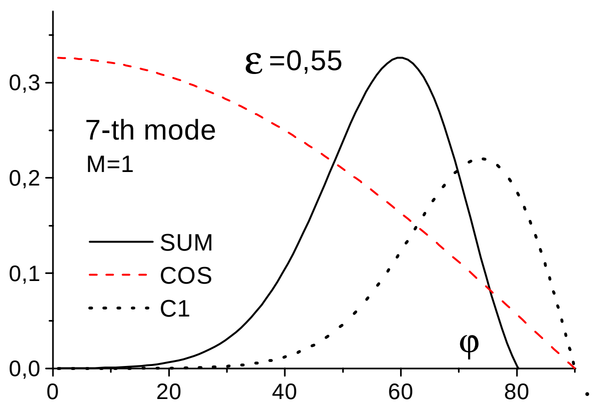

Fig. 3c.

Let's write out corrections to frequencies of eigenwaves caused by ellipticity. For $N_{\|}^2 >> N_A^2$ FAW there excites at $\omega \approx \omega_{ci}$, $\omega < \omega_{ci}$ with

$$\omega^{(0)} = \omega_{ci}\left(1 - \frac{\omega_{ci}^2}{k_{\|}^2 v_A^2}\frac{k_{\|}^2 + k_{\perp}^2/2}{k_{\|}^2 + k_{\perp}^2}\right), \quad \text{and} \quad \omega_{el} = \omega^{(0)}\left(1 + \frac{k_{\perp}^2/k_{\|}^2}{1 + k_{\perp}^2/k_{\|}^2}\frac{\varepsilon^2}{4}\right) \tag{7}$$

For FMSW ($N_A^2 = c^2/v_A^2 >> N_{\|}^2$, v_A - Alfvén velocity, $\omega^{(0)} = kv_A$) we have

$$\omega = \omega^{(0)}\left(1 + \varepsilon^2/4\right) \tag{8}$$

As shown in[2] at $N_{\|} = 0$ ASW with $m > 0$ in $\omega_{LH} < \omega < |\omega_{-e}|$, frequency band there exist (ω_{LH} - low hybrid frequency). It's worth to notice that the similar waves can be in plasma with two species of ions at $\omega_{ii} < \omega < \omega_{12}$, where ω_{ii} - ion-ion hybrid frequency, at ω_{12} - $\varepsilon_1 = -\varepsilon_2$). Their dispersion equation in elliptic coordinates is similarly to (5) but changing q to $-q$ and $N_{\perp}^2 = (\varepsilon_2^2 - \varepsilon_1^2)/\varepsilon_1$. At $q \le 1$ we have

$$\frac{\Delta\omega}{\omega_{ASW}^{(0)}} = -\frac{\left[k_{\perp}^2 a^2 - m^2(\varepsilon_2^2/\varepsilon_1^2 - 1)\right]}{m\omega\partial(\varepsilon_2/\varepsilon_1)/\partial\omega}\frac{\varepsilon^2}{4} .$$

References

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[3] Handbook of Mathematical Functions, Ed. By Abramowitz M. & Stegun I.A. (1964).