THE COULOMB SCATTERING EFFECT ON THE BOUNCE RESONANCE WAVE DISSIPATION IN TOROIDAL PLASMAS

F.M. Nekrasov, A.G. Elfimov[1], C.A. de Azevedo and A.S. de Assis[2]

Instituto de Física, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil
[1]*Instituto de Física, Universidade de São Paulo, 05315-970, Brazil*
[2]*Instituto de Matemática-GMA, Universidade Federal Fluminense, Niterói, Brazil*

One of the most important characteristic properties of magnetized plasmas in inhomogeneous magnetic fields is a trapped particle effect. In the past, this effect is considered to build the neo-classical transport theory [1] and to analyze boot-strap current in tokamaks. In the presence of the trapped particles the wave dissipation can be changed [2]. To support bounce resonance dissipation, the phase correlation between wave and bouncing particles should be fulfilled at list during some bounce periods of particles. Weak collisions may destroy this phase correlation. So, it is necessary to take into account the weak collision effect. In Ref. [3], weak collision effect is already studied in diffusion approximation of collisional operator which is valid for untrapped particles and where the limits of the electron bounce resonance damping are demonstrated for tokamak plasmas.

Here, we extend our analysis by taking into account the elastic scattering term in the collision operator in the Boltzmann equation, by an approach similar to that of the neo-classical theory [1]. We will analyze the scattering effect on the trapped particle part of the parallel permeability of magnetized toroidal plasmas with circular magnetic surfaces for one travelling wave $\sim \exp \mathrm{i}(n\zeta - \omega t)$, where n is the toroidal mode number. This part of the parallel tensor component may be important for the case of Alfvén waves where the main dissipation is defined by trapped particles [2] if the phase velocity is less than the thermal velocity, $\omega/k_{\|} \ll v_{Te}$.

For trapped electrons with small Larmor radii, the Boltzmann equation for a perturbed distribution function $f(t, r, \theta, \zeta, v, \sigma, \gamma)$ in the drift approach can be reduced to a two-dimensional form with the accuracy of ϵ in standard way [3],

$$\frac{\partial f_s^t}{\partial t} + sk_0 v\sqrt{2\epsilon}\sqrt{\hat{\kappa}^2 - \sin^2\frac{\Theta}{2}}\left(\frac{\partial f_s^t}{\partial \Theta} + \mathrm{i}nq_t f_s^t\right) - \hat{S}t[f_s^t] =$$
$$= -\frac{sev\sqrt{2\epsilon}F_M}{T_e}\sqrt{\hat{\kappa}^2 - \sin^2\frac{\Theta}{2}}\,\bar{E}_3(\Theta) \qquad (1)$$

where $k_0 = B_\theta/rB$, sign $s = \pm 1$ mean the positive or negative directions of the velocity of particles along the magnetic field, and the variable $\hat{\kappa}$ is represented via the magnetic moment λ,

$$\hat{\kappa}^2 = \frac{1+\epsilon-\lambda}{2\epsilon},\;\; 0 \le \hat{\kappa} \le 1,\;\; \lambda = \sin^2\gamma(1+\epsilon\cos\theta),\;\; \cos\gamma = s\sqrt{1-\frac{\lambda}{1+\epsilon\cos\theta}}.$$

The new poloidal variable Θ is $\theta + (\epsilon/2)\sin\theta$, and the distribution function, the parallel oscillating field and the current, $\bar{f}, \bar{E}_3, \bar{j}_3$, are represented via the following transformations:

$$\begin{aligned} &f = \bar{f}\exp\left(\mathrm{i}\,\frac{3\epsilon}{2}nq_t\,\sin\Theta\right),\;\; E_3 = \bar{E}_3(1+\epsilon/2\cos\Theta)\exp\left(\mathrm{i}\,\frac{3\epsilon}{2}nq_t\,\sin\Theta\right), \\ &\bar{E}_3 = \sum_{m=-\infty}^{\infty} E_m\exp(\mathrm{i}m\Theta);\;\; j_3 = \bar{j}_3\exp\left(\mathrm{i}\,\frac{3\epsilon}{2}nq_t\,\sin\Theta\right)/(1+\epsilon\cos\Theta). \end{aligned} \tag{2}$$

Here, we propose to use only the scattering part of the collisional operator because the scattering processes are more important than diffusion (see, for example, [4]). In this case, the scattering coefficient, represented in the simplified form,

$$c_e(v) = \frac{2}{3\sqrt{\pi}(1+d_0v/v_{Te})};\;\; d_0 = 0 \text{ if } v \ll v_{Te} \text{ and } d_0 = \frac{2\sqrt{2}}{3\sqrt{\pi}} \text{ if } v \gg v_{Te}.$$

Finally, the collision operator is

$$\hat{S}t[f] = \nu_e\frac{v_{Te}^2}{v^2(1+d_0v/v_{Te})}\frac{1}{\sin\gamma}\frac{\partial}{\partial\gamma}\left(\sin\gamma\frac{\partial f}{\partial\gamma}\right), \tag{3}$$

where $\nu_e = (4/3)\sqrt{2\pi}e^4\Lambda N_e/\sqrt{m_eT_e^3}$ is the electron collision frequency, N_e is the plasma density, T_e is the electron temperature, and Λ is the Coulomb logarithm.

Now, we change variables $(\hat{\kappa}, \Theta)$ and introduce a new variable w via Jacobi function,

$$\hat{\kappa} = \tilde{\kappa},\;\; \sin\tilde{\Theta} = \frac{1}{\hat{\kappa}}\sin\frac{\Theta}{2},\;\; w(\hat{\theta}) = \int_0^{\tilde{\Theta}}\frac{\mathrm{d}\eta}{\sqrt{1-\tilde{\kappa}^2\,\sin^2\eta}}, \tag{4}$$

where $\tilde{\Theta}$ is in the interval $-\pi/2 \le \tilde{\Theta} \le \pi/2$. After changing the distribution function in Eq. (1), $f_s^t = \tilde{f}_s^t\exp\left[-2\mathrm{i}nq_t\tilde{\kappa}\int_0^w \mathrm{d}\eta\mathrm{cn}\eta\right]$, and on expanding the right hand side of the obtained equation in Fourier series over variable w,

$$\tilde{\kappa}\,\mathrm{cn}w\,\exp\left[2\mathrm{i}(m+nq_t)\tilde{\kappa}\int_0^w \mathrm{d}\eta\mathrm{cn}\eta\right] = \sum_{r=-\infty}^{\infty} C_{m,r}^t\exp\left(\mathrm{i}\,r\frac{\pi w}{2K}\right), \tag{5}$$

we get the equation

$$\begin{aligned} -\frac{\nu_e v_T^2}{2\epsilon\tilde{\kappa}^2(1-\tilde{\kappa}^2)}\frac{1}{v^2(1+d_0v/v_{Te})}\frac{\partial^2\tilde{f}_s^t}{\partial w^2} + sk_0v\sqrt{\frac{\epsilon}{2}}\frac{\partial\tilde{f}_s^t}{\partial w} + \frac{\nu_e v_T^2 n^2 q_t^2\,\tilde{f}_s^t}{2\epsilon(1-\tilde{\kappa}^2)v^2(1+d_0v/v_{Te})} - \\ -\mathrm{i}\omega\tilde{f}_s^t = -s\sqrt{2\,\epsilon}\frac{e\,vF_M}{T_e}\sum_{m=-\infty}^{\infty}E_m\sum_{r=-\infty}^{\infty}C_{m,r}^t\exp\left(\mathrm{i}\,r\frac{\pi w}{2K}\right). \end{aligned} \tag{6}$$

The amplitude of the oscillating current, which is induced by trapped electrons, is

$$j^t_{3,p} = -4e\,\epsilon \sum_s s \int_0^\infty v^3 \mathrm{d}v \int_0^1 \mathrm{d}\tilde{\kappa}\tilde{\kappa}^2 \int_{-K}^{K} \mathrm{cn}w\, f^t_s \exp\left[-2\mathrm{i}(p+nq_t)\tilde{\kappa}\int_0^w \mathrm{d}\eta \mathrm{cn}\eta\right] \mathrm{d}w. \quad (7)$$

Using expansions of Jacobi function in q-series, we can present the solution of Eq. (6) as

$$\tilde{f}^t_{\pm 1} = -s\sqrt{2\epsilon}\frac{e\,vF_M}{T_e}\sum_{r,m}\frac{E_m C^t_{m,\pm r}(\tilde{\kappa})}{\Delta^s_r}\exp\left(\mathrm{i}\,r\frac{\pi w}{2K}\right), \quad C^t_{m,r} \approx \frac{r\pi\, J_r[8\sqrt{q}(m+nq_t)]}{4K(m+nq_t)} \quad (8)$$

where F_M is the Maxwell's distribution function and the resonance denominator is

$$\Delta^s_r = -\mathrm{i}\left(\omega - s\,r\bar{\omega}_b\frac{v}{v_T}\right) + \frac{\nu_e v_T^2}{2\epsilon v^2(1+d_0 v/v_T)}\frac{1}{(1-\tilde{\kappa}^2)}\left[\frac{\pi^2 r^2}{4\tilde{\kappa}^2K^2} + n^2q_t^2\right] \quad (9)$$

and $\bar{\omega}_b = k_0 v_T\sqrt{\epsilon/2}(\pi/2K)$. Then, on substituting Eq. (8) in Eq. (7), we obtain

$$\varepsilon^{p,m}_{33(t)} = \mathrm{i}\frac{\epsilon\sqrt{\pi\epsilon}\omega_{pe}^2}{\omega}\sum_{r=1}^{\infty} r^2 \int_0^1 \frac{\tilde{\kappa}\mathrm{d}\tilde{\kappa}}{2K}\int_{-\infty}^{\infty} e^{(u^2/2)}u^4\,\mathrm{d}u \frac{J_r[8\sqrt{q}(p+nq_t)]J_r[8\sqrt{q}(m+nq_t)]}{(p+nq_t)(m+nq_t)(1+d_0|u|)\,\Delta^s_r} \quad (10)$$

For $|m+nq_t| < 1$, we have $J_1 \approx 4\sqrt{q}(m+nq_t)$; the other Bessel functions are small. Further, for $n^2q_t^2 \gg 1$, using $q \approx \tilde{\kappa}^2/16$, $K \approx \pi/2$, we can easily integrate Eq. (10) and obtain imaginary part of this dielectric tensor component in the form

$$\mathrm{Im}\,\varepsilon^{p,m}_{33(t)} = \frac{\epsilon\sqrt{\epsilon}\,\omega_{pe}^2}{2\,\omega\bar{\omega}_b}\,I_m(\nu_0); \quad I_m(\nu_0) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} u^6 \mathrm{d}u\,(1+d_0|u|)\,\frac{\Phi(\tau)}{\nu_0\tau^2}\exp\left(-\frac{u^2}{2}\right) \quad (11)$$

where ν_0 is the normalized collision frequency, $(\nu_e/2\omega_b)\,n^2q_t^2/\epsilon$, and

$$\Phi(\tau) = \frac{1}{2}\ln(1+\tau^2) + \frac{\arctan|\tau|}{|\tau|} - 1, \quad \tau = \frac{u^2}{\nu_0}\left(u - \frac{\omega}{\omega_b}\right)(1+d_0|u|). \quad (12)$$

For $\tau_m > 25$ and $d_0 = 0$, where $\tau_m = 4\omega^3/27\nu_0\bar{\omega}_b^3$ is the minimum value of τ for the cubic parabola in Eq. (12), we have

$$\mathrm{Im}\,\varepsilon^{p,m}_{33(t)} = \frac{\sqrt{\pi}}{4}\frac{\epsilon\sqrt{\epsilon}\,\omega_{pe}^2}{\omega\bar{\omega}_b}\left[\left(\frac{\omega}{\bar{\omega}_b}\right)^4\exp\left(-(\omega/\bar{\omega}_b)^2\right) + A_1\nu_0\ln\nu_0\right] \quad (13)$$

where A_1 has the order of unity. For $\nu_0 \to 0$, we recover the result of Ref. [2]. In the collisional case $(\omega = 0)$, we can find an asymptotic of the integral $I_m(\nu_0)$ in Eq. (11): $5/\sqrt{2}\,\nu_0$ for strong $(\nu \gg 1)$, and $\approx \sqrt{2}\,\nu_0\,\ln(1/\nu_0)$, for weak collisions $(\omega/\bar{\omega}_b \ll \nu_0 \ll 1)$. An intermediate variation of the integral over parameter ν_0 is shown in **Fig. 1**.

In Fig. 1, we show the dependence of $I_m(\nu_0)$, which is proportional to the wave dissipation, on the parameter ν_0, for various ω.

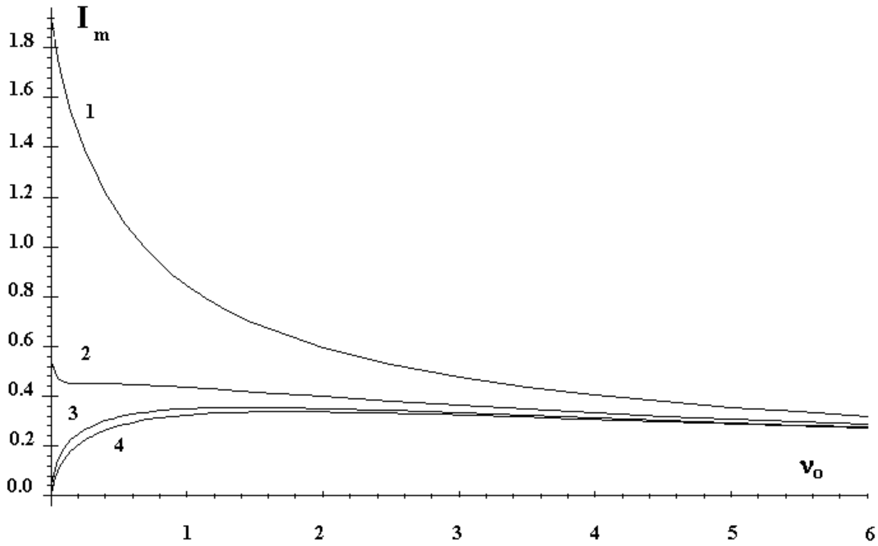

Figure 1. $I_m(\nu_0)$ for various ω. The curves, numbered from 1 to 4, are calculated for $\omega/\bar{\omega}_b = 2,\ 1,\ 0.5$, and 0, respectively.

In conclusion, on the basis of the study of the imaginary part of parallel plasma permittivity in tokamaks with a circular cross section of magnetic surfaces, we demonstrate that the weak collisions can strongly affect the collisionless wave dissipation. We find three different characteristic kinds of dissipation: collisionless dissipation ($\nu_0 = 0$), weak collision dissipation ($\nu_0 \ll (\omega/\bar{\omega}_b)^3$) and dissipation with strong collisions $(\nu_0 \gg 1)$.

For low frequency Alfvén waves with ω of the order of ω_b, the sufficient condition for the collisionless wave dissipation is defined by the inequality $\nu_e < \epsilon(2\omega/9)^3/(nq_t\omega_b)^2$.

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