INFLUENCE OF EXTERNAL FIELDS ON ROTATING FIELD CURRENT DRIVE IN SPHERICAL PLASMAS

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Abstract. Non-inductive current drive in a spherical or compact plasma can be achieved by using a rotating magnetic field to drive a steady Hall current. Recent results in rotamak experiments indicate that the addition of a toroidal field to a combination of the rotating field (for current drive) and the steady vertical field (to hold the MHD equilibrium) can lead to increased toroidal current. This paper will consider the results of a model, based on the MHD equations with neglible ion flow, to calculate the driven current. The problem is reduced by expansion in vector spherical harmonics, resulting in a coupled series of non-linear differential equations which are solved using iterative techniques.

Experiments on the Rotamak have been conducted for a number of years [1]. The early experiments on a spherical device showed that a rotating magnetic field could be used to drive substantial currents and create a compact torus magnetic field configuration. The theoretical analysis of the spherical rotamak [2,3; 4] has been essentially confined to this class. Recent experiments on the Flinders Rotamak-ST [5] have included a toroidal field, produced by a current-carrying central rod, with encouraging results; for it has been shown that an enhanced current can be driven with this configuration. This paper will be devoted to a theoretical and computational analysis of this situation in spherical geometry.

We use a model where the rotating magnetic field is applied to a spherical plasma, with the rotating field oriented parallel to the equatorial plane, taken to be the $x-y$ plane. In our model the ions form a uniform background and the frequency of the rotating field is very much less than the electron cyclotron frequency (with respect to the rotating field strength) and very much greater than the ion cyclotron frequency. This condition is satisfied by the rotamak experiments.

The basic model equations are Maxwell's equations (without the displacement term) and Ohm's law

$$\mathbf{E} \;=\; \eta \mathbf{J} + \frac{1}{ne}\mathbf{J}\times\mathbf{B} \tag{1}$$

where η is the resistivity and n the electron number density. The current drive comes from the introduction of the non-linear Hall term : $\mathbf{J}\times\mathbf{B}$. The externally applied field is

$$\mathbf{B}_{app} \;=\; B_\omega[\sin\theta\cos(\phi-\omega t)\hat{\mathbf{r}} + \cos\theta\cos(\phi-\omega t)\hat{\boldsymbol{\theta}} - \sin(\phi-\omega t)\hat{\boldsymbol{\phi}}]$$

$$+ \quad B_z(\cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}) + B_t\frac{a}{r\sin\theta}\hat{\boldsymbol{\phi}}. \tag{2}$$

where ω is the angular frequency of the applied field and B_t is the toroidal field at a radius a from the central rod. We express the equations in non-dimensional form by introducing the dimensionless parameters

$$\gamma = \frac{B_\omega}{ne\eta} = \frac{\omega_{ce}}{\nu_{ei}}, \tag{3}$$

$$\lambda = \sqrt{\frac{\omega\mu_0 a^2}{2\eta}} = \frac{a}{\delta}; \tag{4}$$

here λ is the ratio of the plasma radius a and the classical skin depth δ and γ is the ratio between the electron cyclotron frequency, ω_{ce}, and the electron-ion collision frequency, ν_{ei}. The magnetic field parameters, b_z and b_t, are scaled with respect to the amplitude of the rotating field B_ω.

We consider here only the steady state and express the equations in dimensionless form as a set for the reduced magnetic field $\mathbf{B}$, obtained by subtracting the singular terms originating explicitly from the central rod current and the (dimensionless) current $\mathbf{J} = \nabla\times\mathbf{B}$. Thus:-

$$-2\lambda^2\frac{\partial\mathbf{B}}{\partial\phi} = \nabla\times\nabla\times\mathbf{B} + \gamma\nabla\times\Big((\nabla\times\mathbf{B})\times\mathbf{B}\Big) + \frac{\gamma b_t}{r^2\sin^2\theta}(\frac{\partial\mathbf{J}}{\partial\phi} + 2J_z\hat{\boldsymbol{\phi}}). \tag{5}$$

The solutions of this equation in the plasma region have to be matched through the vacuum region outside $r = a$ to the externally applied fields at infinity. The final term includes the interaction of the toroidal field with the plasma currents. Following the previous analysis [2] we express the magnetic field in terms of two sets of scalar functions of r , $a_{\ell m}(r)$ and $b_{\ell m}(r)$, which are the coefficients of the expansion of $\mathbf{B}$ in vector spherical harmonics:

$$\mathbf{B}(r,\theta,\phi,t) = \sum_{\ell=0}^{L}\sum_{m=-\ell}^{\ell}\Bigg[\frac{a_{\ell m}(r,t)}{r}\mathcal{Y}_{1\ell m} + \frac{1}{\sqrt{\ell(\ell+1)}}\frac{1}{r}\frac{\partial}{\partial r}\Big(r a_{\ell m}(r,t)\Big)\mathcal{Y}_{2\ell m} + \frac{1}{\sqrt{\ell(\ell+1)}}b_{\ell m}(r,t)\mathcal{Y}_{3\ell m}\Bigg]. \tag{6}$$

The vector spherical harmonics $\mathcal{Y}_{1\ell m}, \mathcal{Y}_{2\ell m}$ and $\mathcal{Y}_{3\ell m}$ form a othonormal set of vector functions of the spherical polar angles θ and ϕ.

The vector equations can be expressed as a set of scalar equations by taking the scalar product of the Eq. (5) with $\mathcal{Y}^*_{\alpha\ell m}$ and integrating over the surface of the unit sphere. We find the final linear term needs to be expessed in terms of the coefficients

$$G(\alpha,\beta,\ell,\ell',m) = 2\pi\int_0^\pi\Big(im\mathcal{Y}^*_{\alpha\ell m}\cdot\mathcal{Y}_{\beta\ell m} + 2(\mathcal{Y}^*_{\alpha\ell m}\cdot\hat{\boldsymbol{\phi}})(\mathcal{Y}_{\beta\ell m}\cdot\hat{\mathbf{z}})\Big)\frac{d\theta}{\sin\theta} \tag{7}$$

which have been calculated using the expressions for the vector spherical harmonics listed in [2]. The non-linear term needs to be expressed in terms of a triple vector product integral (a generalised Clebsch-Gordan coefficient) :-

$$D(\alpha_1 l_1 m_1; \alpha_2 l_2 m_2; \alpha_3 l_3 m_3) = \int_0^{2\pi} \int_0^{\pi} \mathcal{Y}_{\alpha_1 l_1 m_1} \cdot (\mathcal{Y}_{\alpha_2 l_2 m_2} \times \mathcal{Y}_{\alpha_3 l_3 m_3}) \sin\theta d\theta d\phi. \tag{8}$$

Thus, using only the non-zero G-coefficients,

$$2im\lambda^2 a_{\ell m} + \nabla^2 a_{\ell m} + \frac{\gamma b_t}{r^2} G(1,1,\ell,\ell,m) b_{\ell m} = \gamma \sqrt{\ell(\ell+1)}\, c_{3\ell m}, \tag{9}$$

and

$$\begin{aligned} & 2im\lambda^2 b_{\ell m} + \nabla^2 b_{\ell m} - \frac{\gamma b_t}{r^3} \Bigg(\sqrt{\ell(\ell+1)} G(3,1,\ell,\ell',m) b_{\ell' m} \\ + & \sqrt{\frac{\ell(\ell+1)}{\ell'(\ell'+1)}} G(3,2,\ell,\ell',m) \frac{\partial}{\partial r}(r b_{\ell' m}) - \sqrt{\frac{\ell(\ell+1)}{\ell'(\ell'+1)}} G(3,3,\ell,\ell',m) r \nabla^2 a_{\ell' m} \Bigg) \\ = & \gamma \left(\ell(\ell+1) \frac{c_{1\ell m}}{r} - \frac{\sqrt{\ell(\ell+1)}}{r} \frac{\partial}{\partial r}(r c_{2\ell m}) \right), \end{aligned} \tag{10}$$

where $\ell = 1 or 2$; and if $\ell = 1$ then $\ell' = 2$ and visa-versa. The terms $c_{\alpha\ell m}$ are quadratic functions of $a_{\ell m}$ and $b_{\ell m}$ as detailed in Ref [4].

At the centre of the plasma, $r \to 0$, the regularity of the field quantities dictates that both a_{lm} and b_{lm} must go as r^l. Both a_{lm} and $\dfrac{\partial a_{lm}}{\partial r}$ must be continuous across the plasma-vacuum boundary. This leads to the following boundary conditions at $r = 1$:

$$\begin{aligned} \frac{\partial a_{10}}{\partial r} + \frac{(l+1)}{r} a_{10} &= 3\sqrt{\frac{4\pi}{3}} b_z \\ \frac{\partial a_{11}}{\partial r} + \frac{(l+1)}{r} a_{11} &= -3\sqrt{\frac{2\pi}{3}} \end{aligned} \tag{11}$$

otherwise $a_{\ell m} = 0$ and also $b_{\ell m} = 0$ for all ℓ and m.

The Eqs. (9) and (10) can be solved by using an iterative process coupled with a finite difference analysis. They can then be written in the form

$$\mathcal{L}u = \mathcal{R}(u) \tag{12}$$

where the operator $\mathcal{L}$ represents a linear block tri-diagonal matrix operator.The iterative process is:

$$\begin{aligned} \mathcal{L}u^* &= \mathcal{R}(u^{k-1}) \\ u^k &= \sigma u^* + (1-\sigma) u^{k-1} \quad k = 1,2,3... \end{aligned} \tag{13}$$

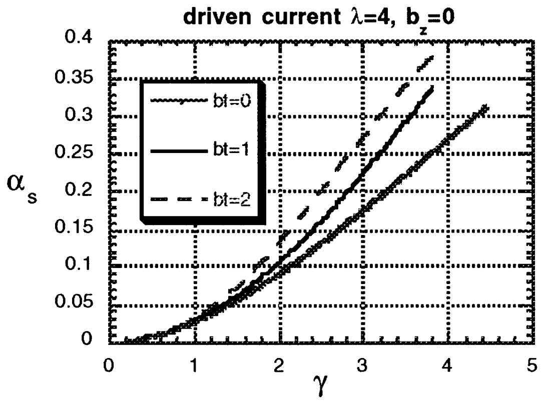

Figure 1. Driven current as a function of strength of rotating field

The value of σ can be adjusted so that the process converges. The solution of the block tri-diagonal linear equations is accomplished by using the method outlined in Ref [6].

A significant parameter is α_s, which describes the ratio of *actual toroidal current driven* to the *maximum possible toroidal current*. This parameter measures the basic efficiency of the current drive. We can express α_s in terms of the time independent coefficients $a_{\ell 0}$:

$$\begin{aligned} \alpha_s &= \frac{6\gamma}{4\lambda^2\sqrt{4\pi}} \sum_{l=odd} \frac{\sqrt{2l+1}}{l(l+1)} \Big[(2l+1)\sqrt{\frac{4\pi}{3}} b_z \delta_{l1} \\ &- \Big(l a_{l0}(r=1) + (l+1) a_{l0}(r=0) + l(l+1) \int_0^1 \frac{a_{l0}}{r} dr \Big) \ Big]. \end{aligned} \tag{14}$$

Figure 1 shows the driven current in terms of the parameter α_s for $b_z = 0.0$, $\lambda = 4$ and $b_t = 0.0, 1.0$ and 2.0. The current increases significantly as the rotating field is increased and for this case ($b_z = 0$) increasing b_t leads to increasing driven current.

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