INTERACTION OF THERMONUCLEAR ALPHA PARTICLES WITH LOWER HYBRID WAVES IN A TOKAMAK

A.D. Piliya and **A.N. Saveliev**

A.F.Ioffe Physico-Technical Institute, Russian Academy of Sciences
Politekhnicheskaya 26, St.Petersburg 194021, Russia

1. Introduction

The interaction of lower hybrid (LH) waves with energetic ions is of considerable importance for lower hybrid current drive (LHCD) in large tokamaks. Toroidal geometry of the tokamak magnetic field has a significant influence on this interaction due to the low ion collisionality. For example, the fast ion quasilinear (QL) diffusion becomes a multi-dimensional process which involves an additional particle trapping. Numerical models taking account of these effects are too demanding for routine LHCD simulations. An approach, where toroidal features of the plasma are ignored in the Fokker-Planck calculations has been proposed for this purpose [1],[2]. Despite a simplified treatment of the QL diffusion, the "cylindrical" approximation can be expected to give a reasonably accurate estimation for RF power absorption. The wave damping due to the fast ions is, in fact, determined by a moment of the distribution function which, with a considerable degree of probability, is insensitive to its details. The important effect of "fat bananas" is modelled by a proper modification of the energetic ion profile. Fast calculation of the power deposition is important because variations in the wave damping rate is the only way a small minority of energetic ions affect the wave propagation. Unfortunately, the accuracy of the "cylindrical" approximation cannot be assessed remaining within its framework. In this report we use for comparison an "intermediate" model where particle excursions off the flux surfaces are omitted but considerable toroidal dynamics is still included.

2. Thin banana Fokker-Planck model for the fast ions

The starting point for the Fokker-Planck analysis is the equation easily obtained under standard assumptions of the quasilinear theory

$$\frac{\partial f}{\partial t} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp D_{QL}\frac{\partial f}{\partial v_\perp}\right) + \frac{\partial}{\partial \mathbf{v}}\left(\nu\,\mathbf{v}\,f\right) + Q(\mathbf{r},\mathbf{v}) \qquad (1)$$

Here $f(\mathbf{r},\mathbf{v},t)$ is the fast ion distribution function, the first term in the right hand side the local QL diffusion operator with $D_{QL}(v_\perp,\mathbf{r})$ being the LH diffusion coefficient [1] at the point **r**, the second term results from collisions and Q is a source of fast ions. In the second term ν is the collision frequency and we have omitted here, following Ref. [2], the pitch-angle scattering. Equation (1) is written in variables I_j (j=1,2,…) which are integrals of particle motion. The variables **v** and **r** are functions of I_j and time. Now we make the (generally unjustified) assumption that the fast ion motion is strictly along the magnetic field lines. Then a particle orbit is specified by a flux surface label (which we omit) and a set of four constants I_j (j=1,...4) of 2D motion on the surface. We choose these constants as $v_{\|0}, v_{\perp 0}, \theta_i, \varphi_i$ with first two quantities being the particle parallel and perpendicular velocities at the point of the trajectory where the magnetic field B reaches its minimum value B_{min} and two others the initial (at t=0) values of the poloidal and toroidal angles. In an axisymmetric magnetic configuration $B=B_{min}$ on a line $\theta=const$ and all trajectories cross this line. Unlike θ_i and φ_i, the integrals $v_{\|0}$ and $v_{\perp 0}$ considered as functions of the particle co-ordinates and velocities do not depend on time explicitly:

$$v_\perp = \sqrt{h} v_{\perp 0}, \quad v_\| = \sqrt{v_{\|0}^2 + (h-1) v_{\perp 0}^2} \tag{2}$$

where $h(\theta)=B/B_{min}$. For the fast ions the QL diffusion time τ_{QL} is comparable with the slowing down time and much larger than the particle transit period τ_{tr}. Being interested in F behaviour in the time-scale exceeding τ_{tr}, we can apply $1/T\int_t^{T+t} ...dt$ to both sides of Eq. (1). Here integration is along the particle trajectories and $\tau_{tr} \ll T \ll \tau_{QL}$. Under the last condition, the ions make many toroidal rotations during the time interval T (this is also valid for trapped particles whose net toroidal displacement is due to the precession of their "banana" orbits). Then the time averaging along trajectory is equivalent to the flux surface averaging with a weight found with the use of Eq. (2). It can be easily seen that for an arbitrary function $\Phi(v_\perp, v_\|, \theta, \varphi)$ its time and surface averaged values, $\overline{\Phi}$ and $<\Phi>$, respectively, are related by $\overline{\Phi} = <J\Phi>/<J>$ where $J = (v_{\|0}/v_\|)h$ is the Jacobian of the transformation given by Eq. (2). Here < > is defined as the averaging over a small shell volume enclosed between two close flux surfaces. It should be noted that $<\Phi>$ is a function of only two variables, $v_\perp$ and $v_\|$. With regard to Eq. (1) this means, not surprisingly, that $f=f(v_\perp, v_\|, t)$. Then, transforming the derivatives in the **v**-space to the $\mathbf{v}_0$ space we obtain, after some algebra:

$$<J> \frac{\partial f}{\partial t} = \frac{\partial}{\partial \mathbf{v}_0}\left(\overleftrightarrow{\mathrm{D}} \frac{\partial f}{\partial \mathbf{v}_0} + \nu \mathbf{v}_0 <J> f \right) + <JQ> \tag{3}$$

Here $\overleftrightarrow{\mathbf{D}}$ is the QL diffusion tensor with the elements $D_{11} = \langle \tilde{D} \rangle$, $D_{12} = D_{21} = (v_{\perp 0}/v_{\|0})\langle (h-1)\tilde{D} \rangle$ and $D_{22} = \left(v_{\perp 0}/v_{\|0}\right)^2 \langle (h-1)^2 \tilde{D} \rangle$, the subscripts 1 and 2 refer to the perpendicular and parallel directions respectively and $\tilde{D} = (v_{\|0}/v_\|) D_0$. Equation (3) describes the high energy part of the distribution function where adopted collision model is valid. For steady state solutions a sink at lower velocities is assumed. The LH wave damping is usually treated in the approximation of unmagnitised ions. Then the RF power absorbed by the fast ions is proportional to

$$\int_{\omega/k_\perp}^{\infty} dv_\perp v_\perp^2 \left(\frac{\omega}{k_\perp v_\perp} \right)^3 \left(1 - \frac{\omega^2}{k_\perp^2 v_\perp^2} \right)^{-1/2} \frac{dF}{dv_\perp} \tag{4}$$

where ω is the wave frequency, $k_\perp$ the perpendicular wave vector, and $F(v_\perp, \theta)$ the 1D distribution function in the **v** space: $F(v_\perp, \theta) = \int_{-\infty}^{\infty} F\{v_{\perp 0}(v_\perp, v_\|, \theta), v_{\|0}(v_\perp, v_\|, \theta)\} dv_\|$. In the "cylindrical" approximation F is independent of θ and found directly from the equation

$$\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp\left[D\frac{\partial F}{\partial v_\perp}+\nu\, v_\perp F\right]\right)=Q(v_\perp) \qquad (1)$$

which can be obtain from Eq.(3) by putting $h = 1$ and integrating over $v_{\|}$.

3. Numerical results

Comparison of two above models has been performed using a numerical code developed for LHCD simulations in the presence of thermonuclear α-particles. Its ray tracing package is based on the Fast Ray Tracing code [3]. The α-particle package calculates the diffusion tensor elements and solves one of FP equations (3), (4) repeatedly within the iterative cycle. In the 2D case Eq. (3) is solved with the use of the Monte-Carlo method. Numerical simulations of a typical ITER-like tokamak discharge has been performed with thermonuclear α-particles as the energetic ion minority. The central electron density is $n_{e0}=10^{20}m^{-3}$, central electron temperature T_{e0}=8 KeV, central ion temperature T_{i0}=15 KeV; The parabolic radial temperature and density profiles are taken. To model the spread of their spatial distribution an amplification factor 2 is included into the source. Figure 1 shows 2D distribution function. Its characteristic feature is the poloidal angle dependence indicating the α-particle increased concentration in the low field side of the plasma. Figure 2 compares distribution functions in 2D and 1D models. The LH power absorbed by α-particles in 1D and 2D models is 15 and 10 MW, respectively for ITER. Similar results are obtained for other cases. We can conclude that toroidal effects diminish significantly the wave damping due to α-particles and that the 1D model [2] is accurate to a factor of order of unity . Calculation of the 2D distribution function can be speeded up considerably by using 1D model of Ref. [2], in all iterative steps except the last.. Fig. 3 shows illustrates accuracy of this procedure.

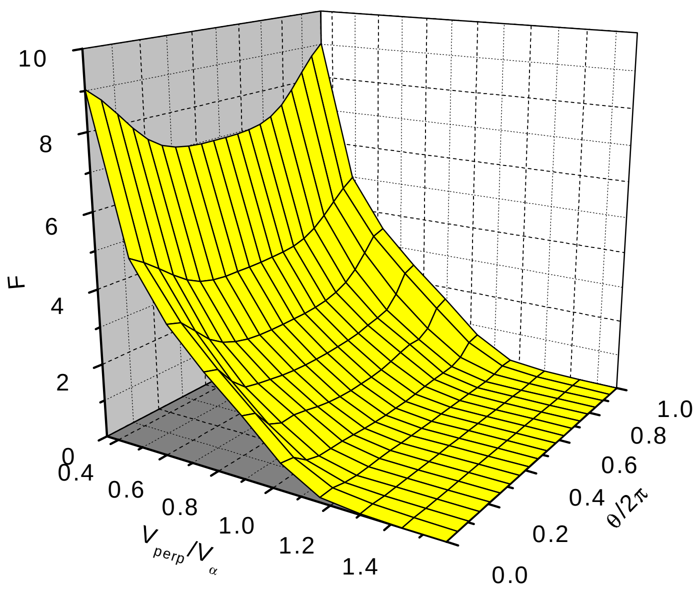

Figure 1. Normalised 2D α-particle distribution function.

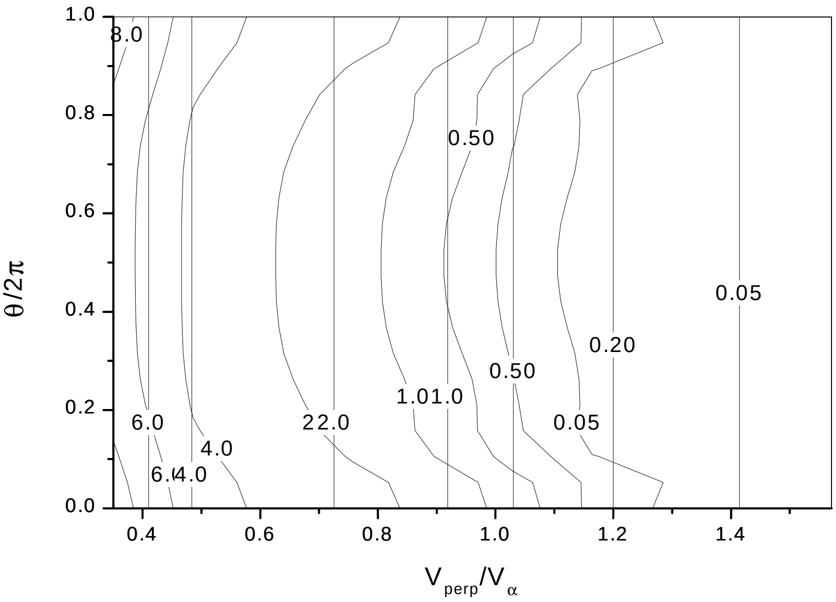

Figure 2. Comparison of 2D and 1D distributions.

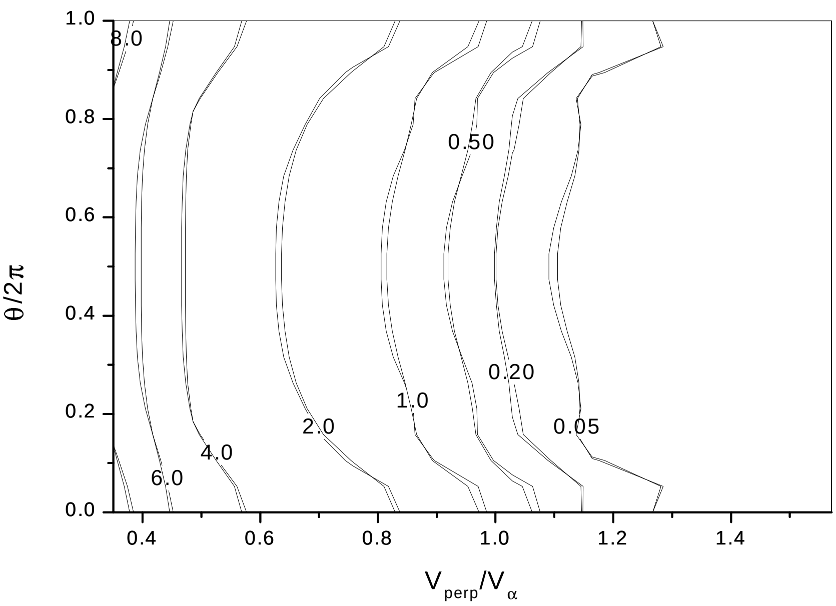

Figure 3. Comparison of full 2D and 1D ray-tracing +2D Fokker-Planck calculations.

References

[1] E. Barbato, F. Santini: Nuclear Fusion **31**, 673 (1991)

[2] N.J. Fisch, J.M. Rax: Nuclear Fusion **32,** 549 (1992)

[3] A. Esterkin, A. Piliya: Nuclear Fusion **36,** 1501 (1996)