

1D MODELLING OF PULSE REFLECTOMETRY: DENSITY PROFILE RECONSTRUCTION BASED ON THE OPTIMAL PULSE LENGTH

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1. Introduction

The reflectometry is a widely used tool to reconstruct the density profile in Tokamak plasmas. For fusion plasmas, it is well known that pulse reflectometry gives a bad density profile reconstruction at the edge [1]. We propose here a new technique which improves the profile reconstruction. This technique is based on the optimal pulse length ℓ_{opt} which minimises the reflected pulse length for a given incident wavelength λ_0 . This notion of the optimal pulse length arises from the plasma dispersion effects. That is to say, if the pulse length is much larger than the wavelength its spectrum is very narrow, in this case the dispersion effects are very small and the reflected pulse has the same shape. In contrast a very small pulse length corresponds to a very broad spectrum, the dispersion effects are important and they give a larger width than the incident pulse width. The evolution of the reflected pulse width is shown in Fig. 1. The minimal value of the pulse length is obtained when the broadening of the reflected pulse length due to the dispersion effects becomes of the order of the incident pulse width. Due to the coherence of the pulse, this minimal value is smaller than the sum of the incident pulse width and the broadening due to the dispersive effects. However, at this point, it is possible to obtain a relation between this particular value of the pulse length and the averaged slope of the density profile around the corresponding cut-off. From this dependency, we have set up a new method to reconstruct the density profile in the case of the O-mode. To test this new method, we have implemented a code, which solves the wave propagation equation in one space dimension for the O-mode (although particular cases with 2 space dimensions can be treated)

$$\partial_t^2 E_Z - c^2 \partial_x^2 E_Z + \omega_{\text{pe}}^2 E_Z = 0 \quad (1)$$

where the electric field E_Z is a function of x and t and the plasma pulsation ω_{pe} depends only on x . The assumption of a steady-state density plasma is justified by looking at the time scales, the time of flight is much smaller (some ns) than the time scale of the density perturbations (\approx ms). This code describes the electromagnetic wave in an isotropic plasma as well. The numerical scheme and some details are given in [3]. We have also written a code for the X mode, which exhibits also the existence of an optimal length.

Usually, in plasma physics experiments, the pulse width is greater than the optimal length, the reflected pulse looks like the incident pulse. Then the only information given by the reflected pulse is the time of flight for a finite number of frequencies; using Abel inversion or linear piecewise reconstruction [2] the density profile can be reconstructed. However it is well-known that these methods give a bad profile at the plasma edge. We propose a new method which gives a better profile at the edge of the usual tokamak plasma.

For a linear profile $n_e(x) = \alpha x$, the optimal length ℓ_{opt} is given by

$$\ell_{\text{opt}} = 8 \sqrt{\frac{2\pi m_e \epsilon_0 f}{e^2 \alpha}} = \frac{4(n_c^+ - n_c^-)}{\alpha}$$

f is the frequency and n_c^+ and n_c^- are respectively the cut-off densities of the upper and lower frequency bounds associated to the pulse with the optimal length. For an arbitrary profile, the rightmost expression can be seen as relating ℓ_{opt} to the density gradient α^{-1} averaged over the pulse bandwidth.

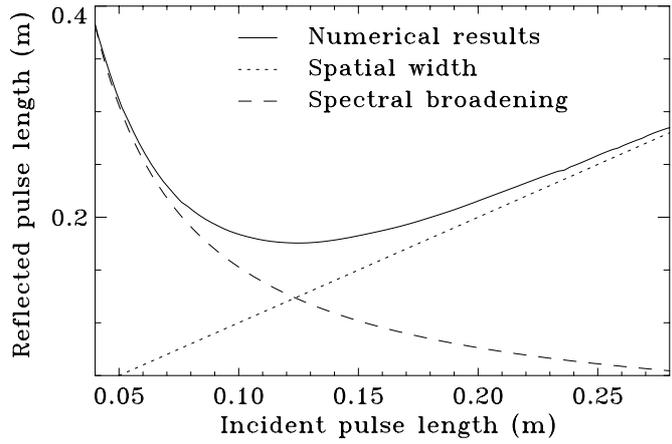


Fig. 1. Reflected pulse length versus incident pulse length

2. Profile reconstruction

To compare the different profile reconstruction methods, we bear in mind the common methods. In the case of the time of flight measurement, the profile is reconstructed using the following algorithm (equivalent to Shevchenko's scheme [2] with $k > 0$).

$$x_{ck} = \frac{n_k - n_{k-1}}{\alpha_k} + x_{ck-1} \text{ where } \frac{1}{\alpha_k} = \left(\frac{c\tau(f_k)}{4n_{ck}} - \sum_{i=0}^{k-2} \frac{1}{\alpha_{i+1}} \left(\sqrt{1 - \frac{n_{ci}}{n_{ck}}} - \sqrt{1 - \frac{n_{ci+1}}{n_{ck}}} \right) \right) / \sqrt{1 - \frac{n_{ck-1}}{n_{ck}}}$$

where $\tau(f_j)$, x_{cj} , n_{cj} and α_{cj} are respectively the time of flight, the cut-off position, the density at the cut-off and the local slope of the linear profile associated to the frequency f_j . In the case of $k=1$ the sum is neglected to compute the first value of α . Here we assume that the plasma edge position is known but the problem is that this position is generally unknown. To improve the profile reconstruction it is possible to use some fits or interpolation to add more points to compute the profile. But one has to extrapolate the plasma edge position. It is well-known that the time of flight series are generally noisy and it is not easy to evaluate correctly this position. We have also tested the Abel inversion using an analytic evaluation of the integral. To test these methods, 8 or 12 different frequencies are used with and without density perturbations in the plasma. Some results on the effects of coherent density perturbations using these methods can be found in ICPIG proceedings [3].

3. Optimal pulse length profile reconstruction

If the incident pulse length can be varied, the minimum value of the width of the reflected pulse ℓ_{opt} can be used to measure the slope of the density profile averaged between two consecutive cut-off layers. To find the optimal length, we have tested different methods, which can be used in an experiment. It is possible to determine the optimal length with 3 different pulse lengths by using interpolation but in practice one needs at least 4 different widths per frequency. The amplitude of the reflected pulse changes with the pulse width. For the optimal length, in the case of a linear density profile, we find that the amplitude is approximately $A_0/\sqrt{2}$. For wider pulses the amplitude of the reflected pulse becomes quickly of the order of the amplitude of the incident pulse. This fact could be used to evaluate the

optimal length, although the accuracy would be very poor in a real experiment. It is better to deduce it from the direct measurement of the reflected pulse widths.

From the set of measurements obtained for a set of 8 or 12 different frequencies, an algorithm can be used to determine the averaged gradient length at each position. As in the other methods, we assume that the plasma is in a steady state configuration during the measurement. The first step is to measure the slope of the density profile at the edge of the plasma. We assume at this stage that the density profile begins at a position x_0 (arbitrarily chosen). From the expression of the optimal length we can deduce the α value that is to say the first part of the density profile reconstruction. Then a connection point is chosen to build the next step; there is some arbitrariness in the choice of the connection point, usually one takes the point x_{c+} corresponding to the upper frequency bound. From the optimal length at the next frequency we evaluate the slope of the next linear part of the reconstructed profile. The process is iterated over all frequencies and at the end the density profile is reconstructed. This method can be described by the following algorithm (for $k>0$)

$$x_k = x_{k-1} + \frac{n_k - n_{k-1}}{\alpha_k} \quad \text{where } C_k = n_c^+ \sqrt{1 - \frac{n_{k-1}}{n_c^+}} - n_c^- \sqrt{1 - \frac{n_{k-1}}{n_c^-}}$$

$$\frac{C_k}{\alpha_k} = \frac{\ell_{\text{opt}}}{4} - \sum_{i=0}^{k-2} \left(\frac{n_c^+}{\alpha_{i+1}} \left(\sqrt{1 - \frac{n_i}{n_c^+}} - \sqrt{1 - \frac{n_{i+1}}{n_c^+}} \right) - \frac{n_c^-}{\alpha_{i+1}} \left(\sqrt{1 - \frac{n_i}{n_c^-}} - \sqrt{1 - \frac{n_{i+1}}{n_c^-}} \right) \right)$$

where ℓ_{opt} , x_j and n_j are respectively the optimal length for the k^{th} frequency, the j^{th} connection position, the density at the j^{th} connection point. We find the definition of the optimal length in the case of $k=1$ if the sum is neglected. One of the advantages of this new method is that the density profile reconstruction does not depend on the edge position of the plasma. As no Abel inversion is needed, thus avoiding problems of initialisation; the reconstruction error is minimal near the edge and increases toward the center. Hence, the profile reconstruction is made by a succession of linear segments.

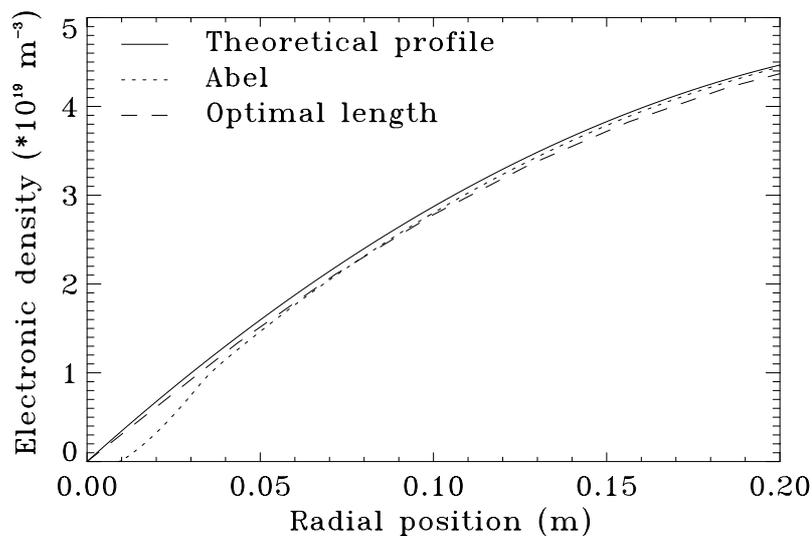


Fig. 2. Comparison between reconstruction by Abel-inversion and Optimal length method

Obviously the plasma position stays unknown if we use only the optimal lengths to reconstruct the density profile. In fact it is possible to combine the usual time of flight measurement method with the optimal pulse length technique to improve the density profile reconstruction. This technique permits to determine the plasma edge position without any further assumption. The principle is to use the time delay to determine the value of the connection point in the range (x_c^+, x_c^-) . However, different algorithms are needed to reconstruct any type of density profile and for simplicity we restrict here to the optimal reconstruction method.

The technical requirements for fusion devices are the following. In the case presented here the optimal length ranges from 0.15 ns at 18 GHz to 0.75 ns at 60 GHz. For a tokamak plasma with $n_e=6 \cdot 10^{19} \text{ m}^{-3}$ and a minor radius = 0.8 m (Tore Supra) the range is 0.23 ns at 18 GHz to 0.95 ns at 65 GHz. For bigger tokamaks the optimal length increases with the minor radius of the plasma (that is to say α decreases in the optimal length expression) and it is easier to apply this method.

4. Effects of density perturbations on the profile reconstruction

The effect of the density perturbations is different according to whether they have a large wavenumber (resonant Bragg backscattering) [4] or whether they are near the cut-off with a small wavenumber. For the Bragg resonant perturbation, it is easy to discriminate the backscattered pulse from the reflected one because the backscattered pulse width has no minimum whatever the incident pulse length. However, if the perturbation amplitude is of the order of 10% the reflected pulse length can be modified.

For density perturbations of small wavenumber, $2\pi/x_c < k_f < k_A$ (Airy wavenumber), near the cut-off, the variation of the optimal length becomes important when density perturbation wavelength is of the order of $(x_c^+ - x_c^-)$ (cut-off positions of the upper and lower bounds of the k-spectrum of the incident pulse). As in the usual methods, significant errors then may occur.

References

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