REGION–OF–INTEREST TOMOGRAPHY OF FINE STRUCTURES IN PLASMAS

V. Pickalov[1], A. Likhachov[1], G. Fuchs[2], A. Ohl[3], G. Röttig[3] and I. Bandlow[4]

[1]*Institute of Theoretical and Applied Mechanics, 630090 Novosibirsk, Russia*

[2]*Institut für Plasmaphysik Association–Euratom, Trilateral Euregio Cluster Forschungszentrum Jülich GmbH D–52425 Jülich Germany*

[3]*Institute of Low-Temperature Plasma Physics, 17489 Greifswald, Germany*

[4]*Institut für Mathematik und Informatik, Greifswald, 17487 Germany*

1. Introduction

Tomography is a key tool to diagnose different types of plasmas, e.g. such of low or high temperature. In plasma physics there are well–developed techniques for side–on observation such as emission sinograms [6], interferometry, schlieren, etc. However, many experimental handicaps can make an interpretation difficult, in particular if the information about the object under investigation is limited [2]. In cases when the information is distributed unevenly in space, tomographic methods extract information for particular regions of interest more accurately as compared to others.

There are methods to solve what is called the exterior Radon problem [7], i.e. when data have been measured for all angles, however, for impact parameters which lay outside a circle with radius R. It has been found that the solution of this problem is unique but very unstable; the internal regions do have no influence on the resulting integrals. The opposite case, the internal Radon problem, asks for a reconstruction of an image inside a circle with radius R. In this case the data are by far more uncertain, because external regions do have influence on any line through the interior. Possible approaches to solve internal region–of–interest tomography (ROIT) problems are: – to take up a priory information and to use extrapolation procedures to simulate projections from plasma emission in those regions, where projections could not be measured; – not to restore the exact values of local emission, and instead to try to get what is called a local tomogram exposing the main geometrical features of the exact one. These inversion methods rely on integral measurements for different lines that are common to a given point. As a first step to solve the ROIT problem, we propose to use local tomograms, and in particular the Vainberg– and summation– image. The validity of these approaches is checked using real experimental data for situations where some kind of additional information is available for the plasma region under investigation. It turns out that local tomograms do give us an approximate image of the plasma cross–section and they do show the main geometrical features of the phenomena thereby including information about the location of steep gradients and fine structures.

2. Local tomography approach: Vainberg image

Most tomography problems can be illustrated by the Radon transform. The Radon transform of two-dimensional functions $g(x, y)$ is composed of projection integrals over the domain along straight lines [7, 8]:

$$f(p, \xi) = \int_{-\infty}^{\infty} g(x, y) \, dl_{p,\xi} \tag{1}$$

For simplicity, we will treat $g(x, y)$ as a distribution of the local emissivity, and $f(p, \xi)$ as an integral emission of the two-dimensional plasma cross-section along lines with impact parameter p and angle ξ to the axis X. The Radon transform has an analytic inversion, namely (2D case):

$$g(x, y) = -\frac{1}{2\pi^2} \int_0^\pi d\xi \int_{-\infty}^{\infty} \frac{f(p, \xi) \, dp}{(p - p_0)^2}, \quad p_0 = -x \sin \xi + y \cos \xi. \tag{2}$$

Thus, to reconstruct the plasma image we need experimental data of projections of the emission $f(p, \xi)$ made for angles $\xi \in [0, \pi]$ and impact parameters $p \in [-R_0, R_0]$ inside the region given by a radius R_0. Although Eq. (2) is not usually used for practical tomography, it shows all the problems of our inverse problem. Because of singularity of the internal integral the noise in the data could be significantly enhanced by the reconstruction. Usually tomography is global in the sense that the Radon inversion at the point (x_0, y_0) depends upon all integrals $f(p, \xi)$ of the object $g(x, y)$ (see Eq. (2)). This nonlocality requests the use of all data in the standard algorithm even if we are interested in a small region of the object only. Local methods could reduce the data acquisition time storage. In [1] a Λ-operator was introduced, it gives an edge detection algorithm and a local tomogram. Earlier this operator has been used for practical tomographic devices by E.Vainberg [9], such tomograms are called Vainberg images. Faridani and co-workers have used a combination of the Λ operator and its inverse Λ^{-1} (which is also local and occurs to be a summation image):

$$g^F = \Lambda \, g + \mu \, \Lambda^{-1} \, g. \tag{3}$$

The convolution in Eq. (2) is the source of nonlocality, and Vainberg has changed this filtration with a generalized function $1/p^2$ for the second derivative of f:

$$g^V(x, y) = -\frac{1}{2\pi^2} \int_0^\pi \frac{d^2 \, f_\xi(p)}{d \, p^2}\Big|_{p=p_0(\xi)} \, d\xi, \quad g^V \equiv 2\Lambda \, g. \tag{4}$$

It differs from the real reconstruction first of all by the property that the convolution in the Radon inversion (2) is equivalent to the multiplication of $\tilde{f}(\nu)$ in the Fourier domain by the function $|\nu|$, whereas the second derivative gives us $|\nu|^2$ as multiplier for $\tilde{f}(\nu)$. For a given scalar α, the Riesz potential operator is defined by [7]:

$$\mathcal{F}(I^\alpha g)(\boldsymbol{\nu}) = |\boldsymbol{\nu}|^{-\alpha} \tilde{g}(\boldsymbol{\nu}), \tag{5}$$

where F(g)(ν) is a Fourier transform. For the n-dimensional case: if $\alpha < n$, then the general inverse Radon transform is [7]:

$$g = \frac{1}{2} I^{-\alpha} R^{\#} (I^{\alpha+1-n} R_\xi g), \tag{6}$$

where $R_\xi g$ is the n-dimensional Radon transform, and $R^{\#}$ is the backprojection operator. For the Laplace operator Δ we have $I^{-2} = -\frac{1}{4\pi^2} \Delta$, and

$$\Delta g = -4\pi^2 I^{-2} g = -2\pi^2 I^{-\alpha-2} R^{\#} (I^{\alpha+1-n} R_\xi g). \tag{7}$$

The operator $I^{-1} = \frac{1}{2\pi} \sqrt{-\Delta}$ is called Lambda operator (it is also known as Calderón's operator):

$$\Lambda g = I^{-1} g = \frac{1}{2} I^{-\alpha-1} R^{\#} (I^{\alpha+1-n} R_\xi g), \tag{8}$$

and taking $\alpha = -1$ we have

$$\Lambda g = \frac{1}{2} R^{\#} (I^{-n} R_\xi g), \text{ and its inverse } \Lambda^{-1} g = \frac{1}{2} R^{\#} (I^{2-n} R_\xi g). \tag{9}$$

For our case of two-dimensional tomography $n = 2$ Equations (9) yield:

$$\Lambda g = \frac{1}{2} R^{\#} (I^{-2} R_\xi g), \quad \Lambda^{-1} g = \frac{1}{2} R^{\#} R_\xi g. \tag{10}$$

Now we have the relations between Λ-tomography, summation image:

$$g^S(x, y) = \frac{1}{2\pi} \int_0^\pi f_\xi(p_0(\xi)) \, d\xi, \tag{11}$$

and Vainberg image (4): $V g = g^V = 2\Lambda g, \quad g^S = \frac{1}{2\pi} \Lambda^{-1} g. \tag{12}$

Finally, for the Faridani image g^F we have:

$$g^F = \frac{1}{2} g^V + 2\pi\mu g^S. \tag{13}$$

It should be emphasized here that in the general case an exact reconstruction of $g(x, y)$ from only line integrals which pass through the region of interest is impossible [7]. Instead we put the problem of approximate estimation of this function or of some special features, such as locations of discontinuities, general geometrical characteristics, etc. Some results of real experimental data treatment are presented in Fig. 1. Here Fig. 1(a) is a tomogram of plasma emission (measurements are performed in plasma reactor in INP, Greifswald) reconstructed by IPR method for irregular covered sinogram space [3], and Fig. 1(b) shows the local Vainberg
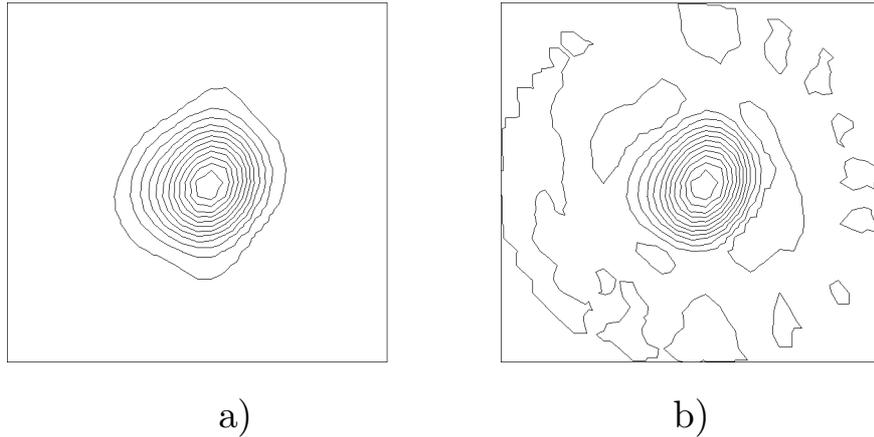
Figure 1. Experimental plasma emission tomogram (a) and its Vainberg image (b)

image for the same plasma cross-section, reconstructed from 11 projections. The similarities between images are clear.

3. Conclusions and outlook

As it has been shown, ROI tomography can give good results for both plasma and plasma flow tomography. Future development of tomographic reconstruction could take more and more advantage of wavelet analysis [5]. Wavelet decomposition can for instance be employed to reduce noise by using the 'shrinkage approach' [4], or to implement contrast filtering on the projections before they are being further processed by the usual tomography filters.

References

[1] Faridani A., Ritman E.L. and Smith K.T.: SIAM J. Appl. Math. **52**, 459 (1992)

[2] Fuchs G. and Pickalov V.: Plasma Phys. Control Fusion **40**(1), 91 (1998)

[3] Ingesson L.C. and Pickalov V.V.: J. Phys. D: Appl. Phys. **29**(12), 3009 (1996)

[4] Kolaczyk E.D.: J. Amer. Stat. Assoc. **91**(9), 1079 (1996)

[5] Louis A.K., Maaß P. and Rieder A.: Wavelets: Theory and Applications. Chichester, Wiley, 1997

[6] Melnikova T.S. and Pickalov V.V.: Beitr. Plasmaphysik **24**(5), 431 (1984)

[7] Natterer F.: The Mathematics of Computerized Tomography. N.Y., Wiley, 1986

[8] Pickalov V.V. and Melnikova T.S.: Plasma Tomography. Nauka, Novosibirsk, 1995 (*in Russian*)

[9] Vainberg E.I., Kazak I.A. and Kurozaev V.P.: Soviet J. Non-dest. Test. **17**, 415 (1981)