Theoretical analysis of two-point collective scattering correlation functions using a drift wave model

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Drift waves are believed to be responsible for a great part of the loss of particles and energy in fusion experiments. Because of this, the measurement of these waves is important. Collective light scattering is one of the methods used[1], and a two-point collective light scattering configuration is currently being used at the W7-AS experiment for this purpose[2].

The basic idea in the two-point scheme is this[3]: A small-scale structure is registered at one point using a collective light scattering system. The structure then moves with the flow to the measurement point of another similar system. Using the crosscorrelation of the photocurrents from the two systems to recognise the structure, the time-of-flight can be found. This is then used to estimate the group velocity of the large-scale structures in the flow.

We have calculated the expected crosscorrelation function, using a drift wave dispersion model. This must be regarded as only a crude model, since the existence of a well-defined dispersion relation for drift waves in a fusion plasma can be discussed.

In the experiment at W7-AS, the plasma is confined to an approximate torus-volume, as shown in Fig. 1. There is a large magnetic field in the toroidal direction, and the density is largest at the centerline of the torus. The drift waves are transverse waves, propagating in the poloidal plane perpendicular to the magnetic field, and the beams are therefore placed in this plane.

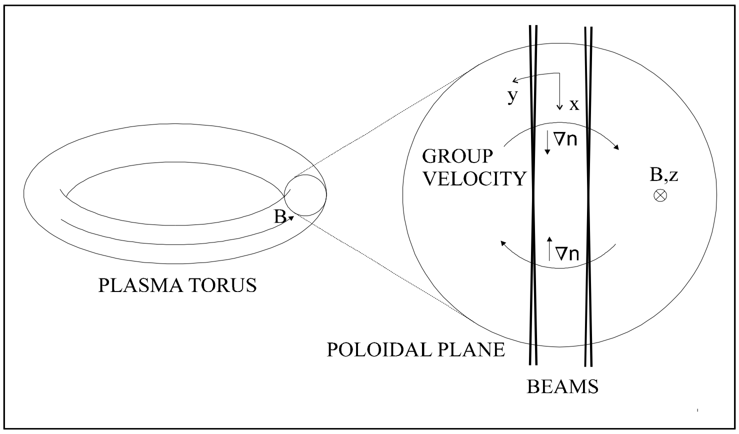

Figure 1: *The geometry of drift wave measurements in the W7-AS stellarator. The beams all lie in the poloidal plane, and are aligned along the x-direction. The main magnetic field is along z. This setup is sensitive to drift waves propagating in the y-direction. The measurement volume is so elongated in the beam direction that group velocities in both the positive and negative y-direction contribute to the signal.*

Drift waves cause plasma density fluctuations, which in turn give rise to variations in the refractive index of the plasma, and a beam of light propagating through the medium will therefore experience position-dependent phase changes ϕ.

The experimental setup is sensitive to spatial variations in the y and z directions. We assume that the variations in the toroidal z direction are much slower than variations along y. Hence, the depence on the wavenumber k_y is of interest.

Assuming the existence of a well-defined dispersion relation, the spatial Fourier transform S_ϕ of the space-time autocorrelation of the phase changes can be written as

$$S_\phi(k_y, \tau) \quad \propto \quad \left\langle |\hat{n}(k_y)|^2 \right\rangle \langle \exp\left[-i\omega(k_y)\tau\right] \rangle \,, \tag{1}$$

where τ is the correlation time, $\left\langle |\hat{n}(k_y)|^2 \right\rangle$ is the spectrum of the density variations, and the frequency $\omega(k_y)$ is given by the dispersion relation.

The function S_ϕ is used to find the crosscorrelation of the photocurrents from the two systems. The normalized crosscorrelation coefficient can be found from the expression

$$\rho_{i,c}(\tau) \quad \propto \quad \int \mathrm{d}k_y \, S_\phi(k_y, \tau) \exp\left[-\frac{w^2(k_y - k_0)^2}{4} + ik_y d_0\right]. \tag{2}$$

Here, k_0 is the mean wavenumber of the measured fluctuations, w is the waist radius of the laser beams, and d_0 is the distance between the two measurement points. This expression shows how the experimental setup allows a certain spectral range of fluctuations to contribute to the signal. The greater the beam width, the narrower the spectral bandwidth around the measurement wavenumber k_0.

We use the dispersion relation obtained from the linearized Hasegawa-Wakatani equations[4]. Assuming infinite plasma conductivity, the dispersion relation for drift waves is given by

$$\omega(k_x, k_y) \quad = \quad \frac{V_{De} k_y}{1 + (\rho_s k_\perp)^2} - i\frac{\mu_\perp}{\rho_s^2} \frac{(\rho_s k_\perp)^4}{1 + (\rho_s k_\perp)^2}, \tag{3}$$

where $k_\perp$ is the magnitude of the wavevector perpendicular to the magnetic field, given by $k_\perp^2 = k_x^2 + k_y^2$, and $\mu_\perp$ is the transverse ion viscosity. The effective ion gyroradius ρ_s is the characteristic spatial scale of the problem and is given by

$$\rho_s \quad = \quad \sqrt{\frac{k_B T_e}{m_i}} \frac{1}{\omega_{ic}} \quad = \quad \frac{\sqrt{k_B T_e m_i}}{e B_0}, \tag{4}$$

where k_B is Boltzmann's constant, T_e is the electron temperature, B_0 is the magnetic field, m_i is the ion mass, and $\omega_{ic} = eB_0/m_i$ is the ion cyclotron frequency. The electron diamagnetic drift velocity V_{De}, given by

$$V_{De} \quad = \quad \frac{k_B T_e}{e B_0} \frac{\partial \ln n_0}{\partial x}, \tag{5}$$

is the wave propagation speed at small perpendicular wavenumbers, $\rho_s k_\perp \ll 1$. The quantity n_0 in this expression is the equilibrium plasma density.

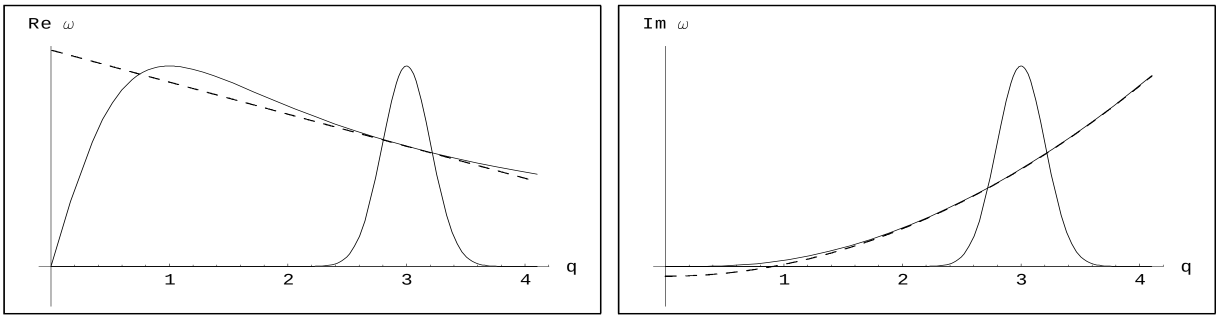

Figure 2: *The real and imaginary parts of the dispersion relation for drift waves, as a function of $q = \rho_s k_y$. The frequency given by Eq. (6) is drawn with a solid line, and the approximations are drawn with a dashed line. The Gaussian curve shows the weighting function for the contribution of fluctuations to the signal, for the case of a measurement wavenumber $q_0 = 3$ and a spectral width given by $\sigma_q = 0.2$, which are typical values for the measurements at W7-AS. The approximations are seen to be good in the spectral region which contributes to the signal.*

Since we measure waves with wavenumber $k_x = 0$, we can let $k_\perp = k_y$. If we define the dimensionless parameter $q = \rho_s k_y$, we can then write

$$\omega(k_y) \quad = \quad \frac{V_{De}}{\rho_s}\frac{q}{1+q^2} - i\frac{\mu_\perp}{\rho_s^2}\frac{q^4}{1+q^2}. \tag{6}$$

To obtain an analytic expression for the crosscorrelation, we approximate this relation by a new function $\tilde{\omega}(k_y)$ in the neighbourhood of the measurement wavenumber k_0, as shown in Fig. 2. This function is linear in the real part and quadratic in the imaginary part. The result is

$$\tilde{\omega}(k_y) \quad = \quad c_{\rm ph}k_0 + c_{\rm gr}(k_y - k_0) - i\gamma_1 - i\gamma_2 k_y^2, \tag{7}$$

where

$$c_{\rm ph} = \frac{\mathrm{Re}\{\omega(k_0)\}}{k_0} = \frac{V_{De}}{1+q_0^2}, \qquad c_{\rm gr} = \left.\frac{\partial \mathrm{Re}\{\omega\}}{\partial k_y}\right|_{k_0} = V_{De}\frac{1-q_0^2}{(1+q_0^2)^2}, \tag{8}$$

where $q_0 = \rho_s k_0$, and $c_{\rm ph}$, $c_{\rm gr}$ are the phase and group velocities evaluated at k_0, respectively. The other constants are given by

$$\gamma_2 \quad = \quad -\frac{1}{2k_0}\left.\frac{\partial \mathrm{Im}\{\omega\}}{\partial k_y}\right|_{k_0} \quad = \quad \mu_\perp\frac{q_0^2(2+q_0^2)}{(1+q_0^2)^2}, \tag{9}$$

and

$$\gamma_1 \quad = \quad -\mathrm{Im}\{\omega(k_0)\} - \gamma_2 k_0^2 \quad = \quad -\frac{\mu_\perp}{\rho_s^2}\frac{q_0^4}{(1+q_0^2)^2}. \tag{10}$$

Inserting Eq. (7) in Eq. (2), and assuming a flat density power spectrum and no spread in drift wave velocity, we obtain the crosscorrelation

$$\rho_{i,c}(\tau) \quad = \quad \frac{1}{2} + \frac{1}{2}\alpha_\pm^2(1+\gamma_2|\tau|/a)^{-1}\exp\left[-\frac{(c_{\rm gr}\tau \mp d_0)^2/2a + 2\gamma_2 k_0^2|\tau|}{1+\gamma_2|\tau|/a} - 2\gamma_1|\tau|\right], \tag{11}$$

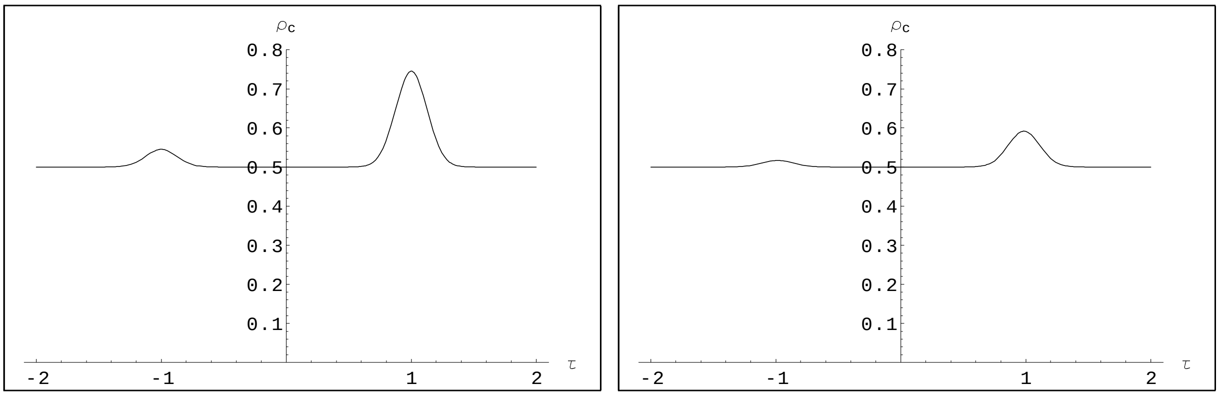

Figure 3: *The crosscorrelation coefficient for typical plasma values, but without turbulence. The time is normalized to the time-of-flight* $d_0/c_{\rm gr}$*. The weight factors are* $\alpha_+ = 0.7$ *and* $\alpha_- = 0.3$*. The plasma parameters are* $\rho_s = 1.6$ mm *and* $V_{De} = 7$ km/s*. The optical parameters are* $w = 5$ mm*,* $d_0 = 2$ cm*, and* $k_0 = 20$ cm^{-1}*, which gives* $q_0 = 3.2$*. The left picture shows the result for the case* $\mu_\perp = 0$ *(no viscous dissipation,) and the right picture shows the result for* $\mu_\perp = 0.0032$ m^2/s.

where α_+ and α_- are the weight factors for waves travelling in the positive and negative x-direction, respectively. This function is shown in Fig. 3.

Work is currently in progress at W7-AS to measure these correlations. It should be realized that the results given in Fig. 3. are idealized. A spread in the drift wave group velocity at different positions along the x coordinate, as well as plasma shear, will contribute to a reduction in the possible observed correlation. Estimates indicate that a spread in group velocity of only several percent may hinder observation of correlations. On the other hand the presence of localized coherent structures in the plasma turbulence[5] may lead to significant, and observable two-point correlations.

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