

ELECTROMAGNETIC WAVE CONVERSION IN MAGNETIZED PLASMAS

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1. Introduction

Electromagnetic wave scattering is among the most efficient methods of plasma diagnostics in laboratory fusion research devices as well as in the near and outer space. Electromagnetic wave scattering is caused by fluctuations of charged particle density and other plasma parameters: current density, electric and magnetic fields. A peculiarity of electromagnetic wave scattering in plasmas is coherent scattering by collective plasma excitations – combination scattering – that occurs along with Thomson incoherent scattering by individual plasma particles. The phenomenon of electromagnetic wave combination scattering by collective plasma excitations was considered for the first time in [1]. A detailed theory of electromagnetic wave scattering in plasmas was developed in [2–4]. Increased activities in the controlled fusion research and employment of magnetic plasma confinement systems require improved methods of plasma state analysis. In Refs. [5,6], experimental studies of electromagnetic wave conversion under the scattering by magnetic fluctuations in plasmas were shown to be possible. It is obvious that information on the plasma state can be drawn from experimental data only provided the theoretical spectra of magnetic and electric field fluctuations in the plasma are available.

2. Plasma with Strong Magnetic Field

In order to consider a plasma with strong external magnetic field \mathbf{B}_0 we introduce a dimensionless parameter

$$q^2 = \frac{k^2 s^2}{\omega_{B_i}} \ll 1, \quad (1)$$

where s is the electron thermal velocity and $\omega_{B_i} = eB_0/Mc$ is the ion cyclotron frequency. We restrict the consideration to the spectral range of frequencies ω lower than the electron cyclotron frequency $\omega_{B_e} = eB_0/mc$ ($\omega \ll \omega_{B_e}$) and employ an approximate expressions for the plasma dielectric permittivity components $\varepsilon_{ij}(\omega, \mathbf{k})$, obtained under the assumption that $\frac{m}{M} \ll 1$ and expanded in power series of the small parameter q^2 . We introduce the dispersion tensor

$$\Lambda_{ij}(\omega, \mathbf{k}) = \varepsilon_{ij}(\omega, \mathbf{k}) - \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \eta^2, \quad (2)$$

where $\eta^2 = \frac{k^2 c^2}{\omega^2}$ is the square of the wave refraction index and write its determinant in the form

$$\Lambda(\omega, \mathbf{k}) = \varepsilon_{33} \left\{ \left(\eta^2 \cos^2 \vartheta - \varepsilon_{11} \right) \left(\eta^2 - \varepsilon_{22} - \frac{\varepsilon_{23}^2}{\varepsilon_{33}} \right) + \varepsilon_{12}^2 + \frac{\eta^2}{\varepsilon_{33}} \left[(\eta^2 - \varepsilon_{22}) \varepsilon_{11} \sin^2 \vartheta + 2\varepsilon_{12} \varepsilon_{23} \sin \vartheta \cos \vartheta - \varepsilon_{12}^2 \sin^2 \vartheta \right] \right\}, \quad (3)$$

where ϑ is the angle formed by the vectors \mathbf{k} and \mathbf{B}_0 . The large quantity ε_{33} is put before the curly brackets for convenience. The two first addends between the curly brackets are of the order of one, all the other terms are of the order of q^2 .

The dispersion equation that determines the plasma eigenwaves frequencies and damping rates reduces to the condition

$$\Lambda(\omega, \mathbf{k}) = 0. \quad (4)$$

We neglect the imaginary part of (4) and thus find the squared eigenfrequencies to be given by

$$\omega_{\pm}^2 = \frac{1}{2}k^2v_A^2 \left\{ 1 + (1 + \xi^2) \cos^2 \vartheta \pm \sqrt{[1 + (1 + \xi^2) \cos^2 \vartheta]^2 - 4 \cos^2 \vartheta} \right\}, \quad (5)$$

where $\xi^2 = \frac{k^2v_A^2}{\omega_{B_i}^2}$ ($v_A = \frac{B_0}{\sqrt{4\pi n_0 M}}$). The solution ω_+^2 corresponds to the fast magnetosonic wave (S), whereas the solution ω_-^2 is associated with the Alfvén wave (A). In the long-wave length limit, for small values of the parameter ξ^2 ($\xi^2 \ll 1$), we have

$$\omega_+ \equiv \omega_S = kv_A, \quad \omega_- \equiv \omega_A = \omega v_A \cos \vartheta. \quad (6)$$

Having taken into account the imaginary part of Eq. (4), we obtain the expression for the Cherenkov damping rates of the magnetosonic and Alfvén waves, i.e.,

$$\gamma_S = \frac{1}{2}\omega_S \Psi''(\omega_S), \quad \gamma_A = \frac{1}{2}\omega_A P''(\omega_A), \quad (7)$$

where the notation is introduced: $\Psi'' = \frac{\sqrt{\pi}}{2} \frac{m}{M} \text{tg}^2 \vartheta \frac{e^{-z^2}}{z}$, $z^2 = \frac{3}{2} \frac{\omega^2}{k^2 s^2 \cos^2 \vartheta}$,

$$P'' = \frac{\sqrt{\pi}}{3} \frac{m}{M} q^2 \left\{ \frac{\omega^4}{(k^2 v_A^2 - \omega^2)^2} + \frac{k^2 v_A^2}{\omega^2} \frac{\cos^2 \vartheta}{[1 - \varphi(z)]^2 + \pi z^2 e^{-2z^2}} \right\} z e^{-z^2}.$$

As the eigenwave frequency ω approaches the ion-cyclotron frequency ω_{B_i} , the refraction index of the Alfvén wave tends to infinity, whereas the refraction index of the magnetosonic wave remains finite. For $\omega \rightarrow \omega_{B_i}$, the cyclotron damping of the Alfvén wave becomes so strong that propagation of this wave becomes impossible.

3. Electric and Magnetic Field Fluctuations in Equilibrium Plasmas

According to the fluctuation-dissipation theorem, the spectral distribution of electric field fluctuations is given by

$$\langle E_i E_j \rangle_{\mathbf{k}\omega} = 4\pi i \frac{T}{\omega} \left\{ \Lambda_{ji}^{-1} - (\Lambda_{ij}^{-1})^* \right\}, \quad (8)$$

where $\Lambda_{ij} = \frac{\lambda_{ij}}{\Lambda}$ is the inverse of the dispersion tensor Λ_{ij} . In the limiting case $q^2 \ll 1$, only the components λ_{11} and λ_{22} are of the same order of magnitude (all the other components λ_{ij} are of higher orders of smallness). Therefore, the spectral distribution of electric field fluctuations in the range of frequencies much lower than the ion cyclotron frequency ($\omega \ll \omega_{B_i}$) is determined by the expression [7]:

$$\langle E_i E_j \rangle_{\mathbf{k}\omega} = 8\pi T \omega \frac{v_A^2}{c^2} \left\{ e_i^A e_j^A \frac{\omega^2 P''}{(\omega^2 - k^2 v_A^2 \cos^2 \vartheta)^2 + (\omega^2 P'')^2} + e_i^S e_j^S \frac{\omega^2 \Psi''}{(\omega^2 - k^2 v_A^2)^2 + (\omega^2 \Psi'')^2} \right\}, \quad (9)$$

where \mathbf{e}^A and \mathbf{e}^S are the electric polarization vectors of the Alfvén and magnetosonic waves. The spectral distribution of long-wavelength fluctuations ($kv_A \ll \omega_{B_i}$) of magnetic field in a plasma with strong external magnetic field is given by

$$\langle B_i B_j \rangle_{\mathbf{k}\omega} = 8\pi \frac{T}{\omega} k^2 v_A^2 \left\{ b_i^A b_j^A \frac{\omega^2 P'' \cos^2 \vartheta}{(\omega^2 - k^2 v_A^2 \cos^2 \vartheta)^2 + (\omega^2 P'')^2} + b_i^S b_j^S \frac{\omega^2 \Psi''}{(\omega^2 - k^2 v_A^2)^2 + (\omega^2 \Psi'')^2} \right\}, \quad \omega \ll \omega_{B_i}, \quad (10)$$

where \mathbf{b}^A and \mathbf{b}^S are the magnetic polarization vectors of the Alfvén and magnetosonic waves. Similarly to the spectral distribution of electric field fluctuations, the spectral distribution of magnetic field fluctuations (10) has two maxima given rise to by Alfvén and magnetosonic fluctuation oscillations and a low-frequency maximum associated with incoherent fluctuations. Like in the case of electric field fluctuations, differing polarizations of Alfvén and magnetosonic waves are responsible for the fact that relevant magnetic field fluctuation maxima are manifested in different components of the spectral distribution tensor. The maximum associated with the Alfvén excitations appears in the spectral distribution of the component $\langle B_2^2 \rangle_{\mathbf{k}\omega}$ whereas the maximum corresponding to the magnetosonic fluctuations is manifested in the spectral distributions of $\langle B_1^2 \rangle_{\mathbf{k}\omega}$, $\langle B_3^2 \rangle_{\mathbf{k}\omega}$, and $\langle B_1 B_3 \rangle_{\mathbf{k}\omega}$. As distinct from the case of electric field fluctuations, the maximum of incoherent fluctuations of the magnetic field corresponds to the zero frequency. Bearing in mind that ion motion should be allowed for in the case of very low frequencies ($z^2 \ll \frac{m}{M}$), we obtain the expressions for the spectral distributions of incoherent Alfvén and magnetosonic fluctuations, i.e.,

$$\langle B_2^2 \rangle_{\mathbf{k}\omega}^A = \frac{1}{4} \sqrt{\frac{\pi}{6}} \frac{m}{M} W q^2 \frac{v_A}{s} \sin \vartheta \operatorname{tg} \vartheta e^{-\frac{M}{m} z^2}, \quad \omega \ll k v_A \cos \vartheta, \quad (11)$$

$$\langle B_i B_j \rangle_{\mathbf{k}\omega}^S = b_i^S b_j^S \sqrt{\frac{2\pi}{3}} \frac{m}{M} W \frac{S}{v_A} \sin \vartheta \operatorname{tg} \vartheta e^{-\frac{M}{m} z^2}, \quad \omega \ll k v_A, \quad (12)$$

where $W = 8\pi \frac{T}{k v_A}$. The widths of the distributions (11) and (12) are determined by the ion thermal velocity. The spectral distributions of short-wavelength fluctuations of electric and magnetic fields have maxima for frequencies (5). As distinct from the long-wavelength fluctuations, the short-wave Alfvén and magnetosonic fluctuation excitations are not separated in individual components of the spectral distribution. In the ion cyclotron resonance range, $((\omega_{B_i} - \omega)/\omega_{B_i})^2 \ll \frac{2}{3} \frac{m}{M} q^2 \cos^2 \vartheta$, both electric and magnetic field fluctuation spectra have a maximum associated with incoherent cyclotron fluctuations. It is given by

$$\langle E_1^2 \rangle_{\mathbf{k}\omega} = 2\pi \sqrt{6\pi} \frac{M}{m} \frac{T}{\omega_{p_i}} \frac{\omega_{B_i}^2}{\omega_{p_i}^2} \frac{c}{s} \phi(\xi) e^{-z_1^2}, \quad \langle E_2^2 \rangle_{\mathbf{k}\omega} = \cos^4 \vartheta \langle E_1^2 \rangle_{\mathbf{k}\omega}, \quad (13)$$

where $z_1^2 = \frac{3}{2} \frac{M}{m} \frac{(\omega_{B_i} - \omega)^2}{k^2 s^2 \cos^2 \vartheta}$ and $\phi(\xi) = \frac{1}{\xi^5 \cos^5 \vartheta}$, for $\xi^2 > 1$.

4. Fluctuations in Nonequilibrium Plasmas

To find the spectral distributions of electric and magnetic field fluctuations in a nonequilibrium plasma, we have to know the spectral distribution of the Langevin current along with the dielectric permittivity tensor. The spectral distribution of current density fluctuations in the plasma disregarding the Coulomb interaction of charged particles is usually taken for the Langevin current $\langle j_i j_j \rangle_{\mathbf{k}\omega}^0$. The spectral distributions of electric and magnetic field fluctuations in a nonequilibrium plasma is determined by the expressions:

$$\langle E_i E_j \rangle_{\mathbf{k}\omega} = \frac{8\pi^2}{\omega} T(\omega, \mathbf{k}) e_i^* e_j \operatorname{Sp} \lambda \delta(\Lambda'), \quad (14)$$

$$\langle B_i B_j \rangle_{\mathbf{k}\omega} = \frac{8\pi^2}{\omega} T(\omega, \mathbf{k}) \eta^2 b_i^* b_j \operatorname{Sp} \lambda \delta(\Lambda'), \quad (15)$$

where $T(\omega, \mathbf{k})$ is the effective temperature

$$T(\omega, \mathbf{k}) = \frac{2\pi}{\omega} \frac{\lambda_{kl}}{\Lambda''} \langle j_k j_l \rangle_{\mathbf{k}\omega}^0. \quad (16)$$

We note that the effective temperature depends on the frequency and the wave vector. It is obvious that the value of the effective temperature for the frequency equal to the eigenoscillation frequency should be regarded as the temperature of the relevant eigenoscillations: $T_A = T(\omega_A, \mathbf{k})$, $T_S = T(\omega_S, k)$.

The values of T_A and T_S can differ considerably from the equilibrium temperature T . The condition of plasma stability with respect to collective excitations reduces to the requirement that the damping rate of the relevant collective excitation must be greater than zero. The effective temperature of relevant excitations grows infinite at the stability boundary.

The spectral distribution of incoherent nonequilibrium fluctuations of the magnetic field is maximum for zero frequency. Separating the incoherent fluctuations of Alfvén and magnetosonic types and comparing them to the relevant equilibrium fluctuations, we introduce the effective temperatures of the incoherent fluctuations, i.e.

$$T_A^N = 8 \sqrt{6\pi \frac{M}{m} \frac{\omega_{B_i}^2}{\omega_{p_i}^2} \frac{\omega_{B_i}^2}{k^2 v_A^2} \frac{\langle j_1^2 \rangle_{\mathbf{k}_0}^0}{ks} \frac{1}{\sin^2 \vartheta \cos \vartheta}}, \quad T_S^N = \sqrt{6\pi \frac{M}{m} \frac{\omega_{B_i}^2}{\omega_{p_i}^2} \frac{\langle j_2^2 \rangle_{\mathbf{k}_0}^0}{ks} \frac{\cos \vartheta}{\sin^2 \vartheta}}.$$

We note that effective temperatures of incoherent fluctuations are always finite, as distinct from the temperatures of collective Alfvén and magnetosonic fluctuations. Thus, it is convenient to describe the state of a nonequilibrium plasma in terms of the sets of temperatures. Thus, the diagnostics of states of a nonequilibrium plasma with strong external magnetic field may be reduced to the calculation of these temperatures.

5. The Conversion – Wave – Inducing Current

The main problem in the calculation of scattering cross sections is to find the current resulting from the nonlinear interaction of the incident wave with longitudinal field fluctuations (charge density fluctuations) and with transverse magnetic fluctuations. This current determines the scattered wave field. For cold plasmas, for which thermal corrections to the dispersion of both incident and scattered electromagnetic waves may be ignored, the inducing current is given by

$$J_{i\mathbf{k}\omega} = -i\omega\kappa_{ij}(\omega) \left\{ \alpha_{jk} \frac{\delta n_{\mathbf{q}\Delta\omega}}{n_0} + \gamma_{jkl} \frac{\delta B_{l\mathbf{q}\Delta\omega}}{B_0} \right\} E_k^0, \quad \mathbf{q} = \mathbf{k} - \mathbf{k}_0, \quad (17)$$

where the quantities α_{jk} and γ_{jkl} , for the scattering with small change in frequency, are defined by

$$\alpha_{jk} = \delta_{jk} - \frac{4\pi}{q^2} q_j k_{0l} \kappa_{lk}(\omega_0), \quad \gamma_{jkl} = -4\pi \frac{\omega_0 \omega_B}{\omega_p^2} \epsilon_{jml} \kappa_{mk}(\omega_0), \quad \Delta\omega \ll \omega_0. \quad (18)$$

Electromagnetic wave scattering by magnetic fluctuations is accompanied by wave conversion, i.e., transformation of an ordinary electromagnetic wave into an extraordinary one or vice versa. If the incident wave propagates perpendicularly to the direction of the magnetic field, $\mathbf{k}_0 \perp \mathbf{B}_0$, then $\alpha_{jk} = \delta_{jk}$. Therefore, the transformation of the ordinary wave into the extraordinary one occurs due to the interaction with the transverse magnetic fluctuations only. Though the scattering cross-section is much greater than the conversion cross-section, under certain conditions one can distinguish conversion from scattering and find the spectral distribution of converted waves. Studies of converted wave spectral distributions provide information on plasma parameters other than those associated with wave scattering, in particular, on the magnetic field fluctuation intensity distribution etc.

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