RAPID EQUILIBRIUM RECONSTRUCTION ON W7-AS USING FUNCTION PARAMETERIZATION

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1. Introduction

Equilibrium reconstruction on W7-AS is normally only possible via a lengthy iterative procedure. Using initial estimates derived from the experiment, the pressure and current density profiles are varied in numerical calculations of the free-boundary equilibrium until consistency with available experimental data is reached. This is extremely time consuming, however, since solving 3-D MHD equilibria is a computationally intensive task requiring CPU times of roughly 1 hour on a Cray J-90 using the NEMEC [1] code for standard W7-AS conditions. Due to these constraints, reconstructions are generally limited to specific time-points of certain discharges.

In order to overcome this, we apply function parameterization [2] (FP) techniques to equilibrium recovery on W7-AS. FP determines simple functional representations of plasma parameters in terms of raw diagnostic information (magnetic signals in our case) by statistical regression on a database of numerically calculated equilibria. This provides a rapid, direct mapping of magnetic data onto physical parameters of interest.

We develop FP models for global plasma parameters, profile quantities and 3-D flux surface geometry in terms of uncorrelated linear combinations of the simulated diagnostic signals, which are constructed using principal component analysis [3]. The impact of varying levels of signal noise on the robustness of the recovery is also examined.

2. Equilibrium database

Our database consists of *circa* 400 equilibria with vanishing net toroidal current calculated with NEMEC. 8 input parameters were randomly varied over ranges appropriate for W7-AS, namely 3 ratios of the 4 external coil currents (since a global scaling does not change the vacuum flux surfaces), a variable limiter position and 4 free parameters in the plasma pressure profile, which was parameterized as follows:

$$p(s) = p_0(1-s)^2 \exp(as + bs^2 + cs^3),$$

where s is the normalized toroidal flux, p_0 is the pressure at the magnetic axis and a, b, c are shape parameters. The database represents configurations bounded by an upper/lower limiter and encompasses plasmas with a wide variety in both physical size and β values.

The DIAGNO code [4] was used to simulate the responses of the available magnetic diagnostics for each equilibrium. These include 6 flux loops (of which only 3 are independent due to the machine symmetry), zeroth and first order poloidal cosine moments from an array of 12 poloidal field coils, a $\cos(2\theta)$ coil and two diamagnetic coils. These diagnostics begin sampling only when the currents in the external coils have reached steady values and thus measure only signals due to the plasma.

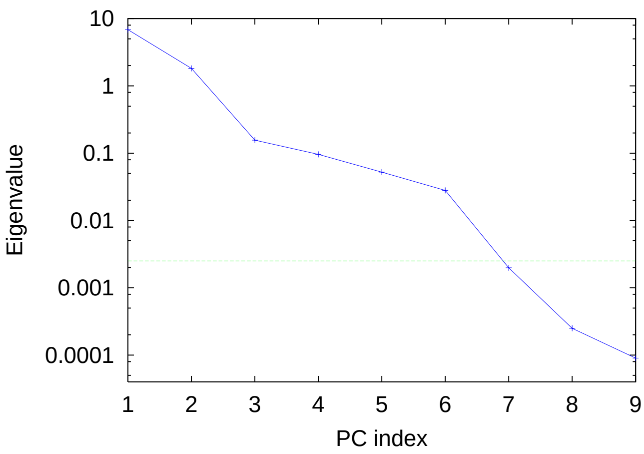

Fig. 1: Log plot of eigenvalue (variance) versus PC index

In order to condition the diagnostic data, in particular to remove any redundancy (collinearity) and thereby reduce the dimensionality involved, a principal component analysis (PCA) was performed on the magnetic signals. The resulting pattern of principal component (PC) eigenvalues (or variances) versus PC index is shown in Fig. 1. Knowledge of expected noise levels in the diagnostic signals enables the selection of a cut-off point, beyond which PC's can be neglected, since any information they hold is too small to be reliably resolved from the noise. The dashed line in Fig. 1 indicates 5% random signal noise, which we take as a realistic estimate of the signal errors. The leading 6 PC's account for 99.97% of the total variance in the magnetic signals, with a signal-to-noise ratio of 11 for the weakest retained PC.

The vacuum magnetic field information is specified by the experimentally known external coil current ratios and limiter position. These, together with the retained PC's of the magnetic signals, constitute the set of predictor variables used in FP models, which we label $X_1, \ldots, X_N$ for convenience.

3. FP models

We wish to recover scalar plasma parameters such as volume and energy content, profile parameters such as flux surface effective radius and 3-D flux surface geometry. Examining the RMS recovery error of these parameters in the database for various models, we find that their behaviour is inadequately described by linear models in the FP predictors. Also, the small improvement in fit of a full cubic model does not justify the much larger model size. A quadratic dependence appears to be the optimum trade-off between model size and recovery accuracy.

Defining X_0 to be unity, the dependence of an arbitrary scalar parameter y on the fitted parameters X_i takes the form:

$$y = \sum_{\substack{i=0 \\ j=i}}^{N} a_{ij}\, X_i\, X_j$$

and there are $(N+1)(N+2)/2$ parameters in the model.

We can easily extend the above model to recover profile quantities by exploiting their generally smooth radial behaviour, which can be characterized by a fourth order polynomial in

$\rho = \sqrt{s}$, which varies like the normalized flux surface effective radius. We thus obtain a global model which is fitted for all configurations and all radii:

$$y = \sum_{k=0}^{4} \sum_{\substack{i=0 \\ j=i}}^{N} a_{ijk}\, X_i\, X_j\, \rho^k,$$

with $5(N+1)(N+2)/2$ fitted parameters. 3-D flux surface geometry can similarly be modelled since each coefficient in its Fourier decomposition can be treated as an independent profile quantity.

4. Recovery results

Below we plot the recovery error expressed as a percentage of the parameter spread for models with a baseline of the vacuum information and varying numbers of PC's of the magnetic signals (up to the maximum of 9), and also for simulated random Gaussian noise from 0–20%.

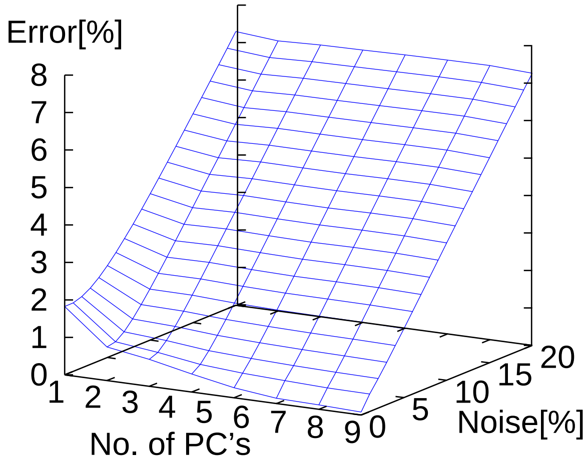

Fig. 2: Recovery RMSE for $\mathrm{W_{kin}}$

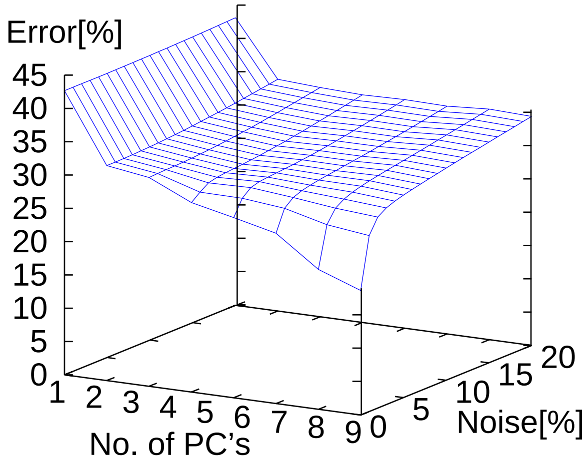

Fig. 3: Recovery RMSE for β on-axis

Fig. 2 shows the error for $\mathrm{W_{kin}}$, the energy content of the plasma. Global quantities such as this, which depend on the volume-integrated magnetic field, are generally well recovered in the database. Indeed, the error is relatively low even for few added PC's, which is consistent with the fact that $\mathrm{W_{kin}}$ has an almost linear dependence on the plasma diamagnetic signal.

Fig. 3 shows the error for a parameter local to the plasma core, the β value at the magnetic axis. In contrast with $\mathrm{W_{kin}}$, the error saturates at around 30% for models with 2 or more PC's. This is due to the fact that the local β value is highly dependent on the pressure profile shape, which is difficult to infer from remote magnetic measurements. This behaviour is also true of other parameters linked to the pressure profile shape, a particular example being the rotational transform ι, which depends on the local pressure gradient. The sharp rise in error for 0–2% noise that is evident for models with 5 or more PC's is due to the rapid swamping of the information contained in the weaker PC's.

In general, both scalar parameters and also coefficients in the Fourier decomposition of the flux surface geometry continue to show improved recovery errors up to 6 retained PC's. Based on this model, we show a reconstruction of an equilibrium outside the database using

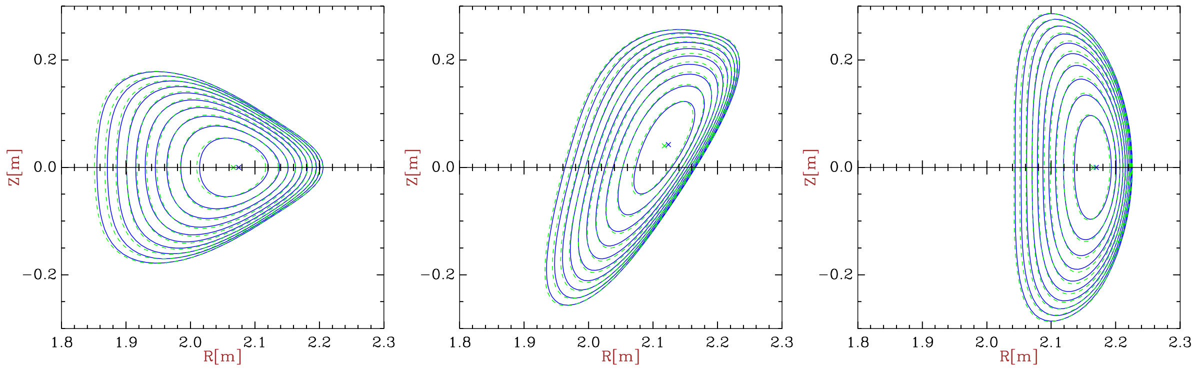

Fig. 4: Left to right: flux surface reconstruction in $\phi = 0°, 18°, 36°$ planes
(FP surfaces solid, NEMEC surfaces dashed)

its simulated magnetic data. Fig. 4 compares the FP flux surface geometry to that of NEMEC for 3 different values of the toroidal angle ϕ. The RMS deviation of the flux surfaces are representative of the average in the database, which decreases from 7mm at the centre to 3mm at the edge. The time taken for reconstruction of 11 flux surfaces (for s = 0.0, 0.1, . . ., 1.0) is less than 20ms on a 143MHz UltraSPARC workstation.

5. Discussion

Even with the relatively limited number of magnetic signals at our disposal, equilibrium recovery on W7-AS using FP is feasible and gives flux surface reconstructions accurate to roughly 5mm (average RMS error). Further improvement in accuracy could be obtained for flux surface geometry with additional magnetic diagnostics, the optimal locations of which are currently under investigation. Direct recovery of the pressure profile and related parameters such as ι is not suited to this method, but can be performed with an interpretive method based on FP for the timepoint at which the Thomson diagnostic operates [5].

The method provides a speed advantage of some 5–6 orders of magnitude over conventional methods and is thus an ideal candidate for routine post-shot equilibrium reconstructions accompanying the experiment. Since the magnetic diagnostics operate continuously, the time evolution of discharges from birth to extinction can be followed. Certain plasma parameters can also be calculated in real-time, since their functional representations are simple and thus trivial to evaluate.

References

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